Name: ______________________

Block: ______

Mean Free Path

Introduction

The kinetic theory of gases, developed by James C. Maxwell and Ludwig Boltzmann, attempts to predict how the molecules in a gas behave. The theory assumes that:

- Gases are made of molecules.
- Gas molecules move randomly according to Newton's Laws of motion, and the distribution (range) of their speeds is not changing.
- There are a large number of molecules.
- The average distance between the molecules is much larger than the size of the molecules.
- The molecules undergo *elastic* collisions with each other and the walls of the container, but do not react or exert any other attractive or repulsive forces on each other.

In kinetic theory, the *mean free path* of a molecule is the average distance that the molecule travels between collisions. The mean free path, λ, of the molecules is given by the formula:

$$\lambda = \frac{V}{\pi\sqrt{2}d^2 n} \tag{1}$$

where n is the number of molecules, V is the volume of the container, and d is the diameter of each of the molecules.

According to this equation, if the volume of the container (V) increases, then the average distance that the molecules travel (λ) also increases. If the number of molecules (n) or their size (d) increases (or both), then the average distance that they travel (λ) decreases. There is a good discussion of the derivation of this formula on the internet at:

`http://hyperphysics.phy-astr.gsu.edu/hbase/kinetic/menfre.html`

In this activity, you and your classmates will imitate gas molecules. You will be taken to different sized "containers" (areas) within the school, where you will be allowed to move randomly and "collide" with each other. The number of "molecules" in the container (students participating) will remain the same, but the volume* of the "container" will change.

You will estimate the mean free path in the different containers by walking in straight lines and counting the number of steps between collisions with other molecules or the walls of the container.

*Because our human "molecules" are moving in only two dimensions, (No jumping on each other's heads!) the height of the container is constant, so the change in volume equals the change in area times a constant height.

Procedure

1. When the teacher says "go," start walking in a straight line and count the number of steps you take.
2. When you bump into another "molecule" or a wall (or reach an invisible boundary), stop and write down the number of steps you took.
3. Close your eyes, and spin around so you are facing in a new, random direction.
4. Open your eyes and start walking/counting again.
5. If somebody collides with you while you are writing, it doesn't count for you (but it does count for the person who was moving).
6. When you have recorded 20 collisions, keep walking and colliding (but not writing) so that the other students can finish the activity with the same number of "molecules" in the "container."

Rules

1. Human "molecules" collide *gently*.
2. Molecules aren't attracted to each other. Molecules move in *straight* lines at constant speeds. Molecules don't change their direction or speed so they can collide with (or avoid) that "special someone."
3. Molecules stay inside the boundaries of the container.

Calculations

We will make the following assumptions:

- The standard unit of measure is one "ceiling tile," which measures $2' \times 2'$.
- The average diameter of a person is about $\frac{3}{4}$ "ceiling tile". *I.e.,* $d = 0.75$, so $d^2 = 0.56$.
- Each person's stride is about one ceiling tile. (The units of λ will be "ceiling tiles.")
- Molecules are moving in two dimensions only, so we will arbitrarily declare that the height of the room is 1 ceiling tile. This means we can calculate the volume V of the "container" by counting ceiling tiles and multiplying length $\times$ width.

Substituting 0.75 for d, $\pi\sqrt{2}d^2 = (\pi\sqrt{2})(0.75)^2 = (\pi\sqrt{2})(0.56) = 2.50$. Thus, equation (1) becomes:

$$\lambda = \frac{V}{2.50\,n} \tag{2}$$

Data

Area of Container #1 in square ceiling tiles (V_1): ____________

Area of Container #2 in square ceiling tiles (V_2): ____________

Number of students (n_1): ____________

Number of students (n_2): ____________

Data for Container #1				Data for Container #2			
Collision	# steps	Collision	# steps	Collision	# steps	Collision	# steps
1		11		1		11	
2		12		2		12	
3		13		3		13	
4		14		4		14	
5		15		5		15	
6		16		6		16	
7		17		7		17	
8		18		8		18	
9		19		9		19	
10		20		10		20	

Average number of steps for Container #1 (λ_1): ____________

Average number of steps for Container #2 (λ_2): ____________

Analysis

Predicted mean free path ($\lambda_1 = \frac{V_1}{2.50\, n_1}$) for Container #1: ____________

Measured mean free path (average # of steps) for Container #1: ____________

Predicted mean free path ($\lambda_2 = \frac{V_2}{2.50\, n_2}$) for Container #2: ____________

Measured mean free path (average # of steps) for Container #2: ____________

Questions

How well do your data agree with the prediction from the formula for Container #1?

How well do your data agree with the prediction from the formula for Container #2?

Which aspects of kinetic-molecular theory (KMT) may not have been good assumptions in either or both containers? Explain.