Uncertainty & Error Analysis

Unit: Laboratory

MA Curriculum Frameworks (2016): SP4

Mastery Objective(s): (Students will be able to…)

- Determine the uncertainty of a measured or calculated value.

Success Criteria:

- Take analog measurements to one extra digit of precision.
- Correctly estimate measurement uncertainty.
- Correctly read and interpret stated uncertainty values.
- Correctly propagate uncertainty through calculations involving addition/subtraction and multiplication/division.

Tier 2 Vocabulary: uncertainty, error

Language Objectives:

- Understand and correctly use the terms "uncertainty" and "relative error."
- Correctly explain the process of estimating and propagating uncertainty.

Notes:

In science, unlike mathematics, there is no such thing as an exact answer. Ultimately, every quantity is limited by the precision and accuracy of the measurements that it came from. If you can only measure a quantity to within 10 %, that means any calculation that is derived from that measurement can't be any better than ±10 %.

Error analysis is the practice of determining and communicating the causes and extents of uncertainty in your results. Error analysis involves understanding and following the uncertainty in your data, from the initial measurements to the final calculated and reported results.

Note that the word "error" in science has a different meaning from the word "error" in everyday language. In science, **"error" means "uncertainty."** If you report that you drive (2.4 ± 0.1) miles to school every day, you would say that this distance has an error of ±0.1 mile. This does not mean your car's odometer is wrong; it means that the actual distance *could be* 0.1 mile more or 0.1 mile less—*i.e.,* somewhere between 2.3 and 2.5 miles. ***When you are analyzing your results, <u>never</u> use the word "error" to mean mistakes that you might have made!***

Use this space for summary and/or additional notes:

Uncertainty

The uncertainty or error of a measurement describes how close the actual value is likely to be to the measured value. For example, if a length was measured to be 22.34 cm, and the uncertainty was 0.31 cm (meaning that the measurement is only known to within ±0.31 cm), we could represent this measurement in either of two ways:

(22.34 ± 0.31) cm 22.34(31) cm

The first of these states the variation (±) explicitly in cm (the actual unit). The second is shows the variation in the last digits shown.

What it means is that the true length is approximately 22.34 cm, and is statistically likely* to be somewhere between 22.03 cm and 22.65 cm.

Absolute Error

Absolute error (or absolute uncertainty) refers to the uncertainty in the actual measurement. For the measurement (22.34 ± 0.31) cm, the absolute error is ± 0.31 cm.

Relative Error

Relative error shows the error or uncertainty as a fraction of the total.

The formula for relative error is $\text{R.E.} = \dfrac{\text{uncertainty}}{\text{measured value}}$

For the measurement (22.34 ± 0.31) cm, the relative error would be 0.31 out of 22.34. Mathematically, we express this as:

$$\text{R.E.} = \frac{0.31}{22.34} = 0.0139$$

Note that relative error is dimensionless (does not have any units). This is because the numerator and denominator have the same units, so the units cancel.

Percent Error

Percent error is relative error expressed as a percentage. You can turn relative error into percent error by multiplying by 100.

In the example above, the relative error of 0.0139 would be 1.39 % error.

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* Statistically, the uncertainty is one standard deviation. *I.e.,* if multiple measurements are taken, approximately two-thirds of those measurements will lie within the uncertainty (plus or minus) of the stated value.

Use this space for summary and/or additional notes:

Uncertainty of Measurements

If you have the ability to measure a quantity that is not changing (such as the mass or length of an object), you will get the same value every time you measure it. This means you have only one data point.

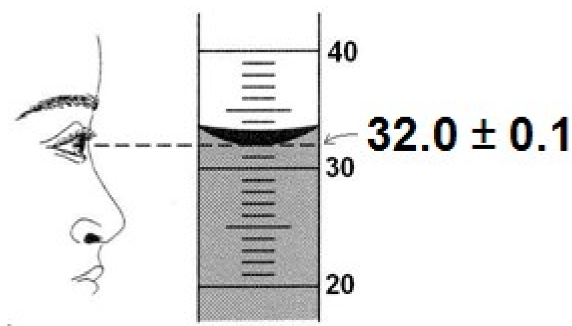
When you have only one data point, the uncertainty is the limit of how well you can measure it. This will be your best educated guess, based on how closely you think you actually measured the quantity. This means you need to take measurements as carefully and precisely as possible, because *every careless measurement needlessly increases the uncertainty of the result*.

Digital Measurements

For digital equipment, if the reading is *stable* (not changing), look up the published precision of the instrument in its user's manual. (For example, many balances used in high schools have a readability of 0.01 g but are only precise to within ± 0.02 g.) If there is no published value (or the manual is not available), assume the uncertainty is± 1 in the last digit.

Analog Measurements

When making analog measurements, always estimate one extra digit beyond the finest markings on the equipment. For example, in the diagram below, the graduated cylinder is marked in 1 mL increments. When measuring volume in this graduated cylinder, you would estimate and write down the volume to the nearest 0.1 mL, as shown:



In the above experiment, you should record the volume as 32.0 ± 0.1 mL. It would be inadequate to write the volume as 32 mL; you *must* write 32.0 mL, or better yet, (32.0 ± 0.1) mL

The zero at the end of 32.0 mL is not extra. It is necessary to show that *you measured the volume to the nearest tenth, not to the nearest one.*

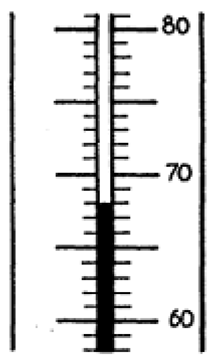
Use this space for summary and/or additional notes:

When estimating, the uncertainty depends on how well you can see the markings, but you can usually assume that the estimated digit has an uncertainty of $\pm \frac{1}{10}$ of the finest markings on the equipment. Here are some examples:

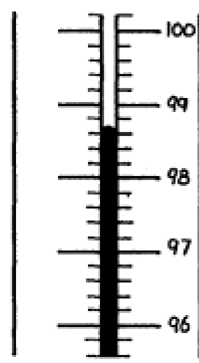
Equipment	Markings	Estimate To	Assumed Uncertainty
ruler	1 mm	0.1 mm	± 0.1 mm
25 mL graduated cylinder	0.2 mL	0.02 mL	± 0.02 mL
thermometer	1 °C	0.1 °C	± 0.1 °C

Homework Problems

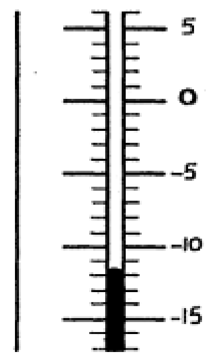
Write the readings that you would record (estimated to one extra decimal place) and the assumed uncertainty for each of the following thermometers and graduated cylinders.



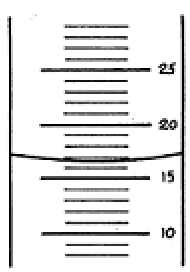
_____ ± ___ °C



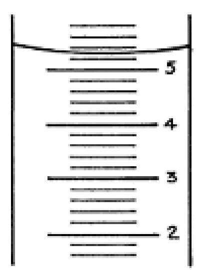
_____ ± ___ °C



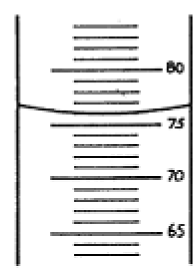
_____ ± ___ °C



_____ ± ___ mL



_____ ± ___ mL



_____ ± ___ mL

Use this space for summary and/or additional notes:

Propagating Uncertainty in Calculations

When you perform calculations using numbers that have uncertainty, you need to propagate the uncertainty through the calculation.

Addition & Subtraction

When quantities with uncertainties are added or subtracted, add the quantities to get the answer, then add the uncertainties to get the total uncertainty.

Sample Problem:

Q: A substance is being heated. You record the initial temperature as (23 ± 0.2) °C, and the final temperature as (84± 0.2) °C. You need to calculate the temperature change (ΔT) with its uncertainty to use in a later calculation. What is the temperature change?

A: To calculate ΔT, simply subtract:

$$\Delta T = T_{final} - T_{initial} = 84 - 23 = 61°\text{C}$$

To calculate the uncertainty, add the individual uncertainties (even though the quantities were subtracted):

$$u = 0.2 + 0.2 = 0.4\ °\text{C}$$

Report the value as: $\Delta T = (61 \pm 0.4)°\text{C}$

Multiplication & Division

Because most calculations that we will perform in chemistry involve multiplication and/or division, you can

For calculations involving multiplication and division, estimate the uncertainty of your calculated answer by adding the relative errors and applying the total relative error to your result.

1. Perform the calculation for the desired quantity.
2. Divide the uncertainty (the ±) for each quantity by its measured value to determine its relative error.

$$\text{R.E.} = \frac{\text{uncertainty}}{\text{measured value}}$$

3. Add up all of the relative errors to get the total relative error.
4. Multiply your calculated result by the total relative error to get its uncertainty (the ± amount).

Use this space for summary and/or additional notes:

Note: ***Most of the calculations that you will perform in chemistry involve multiplication and/or division, so almost all of your uncertainty calculations throughout the course will use relative error.***

Sample Problem #1:

Q: You want to determine the amount of heat released by a chemical reaction. You use the heat from the reaction to heat up some water in an insulated container called a "bomb calorimeter". You will calculate the heat using the equation: $Q = mC\Delta T$.

Suppose you recorded the following data (including uncertainties):

- The mass of the water in the calorimeter is (24.8 ± 0.1) g.
- The temperature change of the water was (12.4± 0.2) °C.
- The specific heat capacity of water is $4.181\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$. (This is a published value. The uncertainty of this value is so small that we can leave it out of our calculations.)

A: The heat released by the reaction is given by the equation:

$$Q = mC\Delta T$$
$$Q = (24.8)(4.181)(12.4)$$
$$Q = 1\,285.74\text{ J}$$

The relative errors for the two quantities that we measured are:

- mass: $\frac{0.1}{24.8} = 0.004\,03$
- temperature change: $\frac{0.2}{12.4} = 0.016\,13$

The total relative error is 0.004 03 + 0.016 13 = 0.020 16

The uncertainty is therefore (0.020 16)(1 285.74) = ±25.92 J

(Note that the absolute uncertainty has the same units as the measurement.)

We would report the measurement as (1 285.74 ± 25.92) J.

Use this space for summary and/or additional notes:

Rounding

In the example above, the uncertainty tells us that our actual result could be different from our calculated value by as much as 25.92 J.

However, we only estimated one digit (which happened to be the tenths place) when we took our measurements. This means we have only one digit of uncertainty. Because we can't report more precision than we actually have, we need to round the calculated uncertainty off, so that we have only one unrounded digit. This means we should report our uncertainty as ± 30 J.

It wouldn't make sense to report our answer as (1 285.74 ± 30) J. Think about that—if the *tens* digit could be different from our calculated value, there is no point in reporting the ones or tenths digits. So we need to round our calculated answer to the same place value as the uncertainty—the tens place.

This means our final, rounded answer should be (1 290 ± 30) J.

Use this space for summary and/or additional notes:

Sample Problem #2:

Q: You need to find the density of a piece of metal. We measure its mass on a balance to be (24.75 ± 0.02) g. You measure its volume in a graduated cylinder using water displacement, and you find the volume to be (7.2 ± 0.1) mL. Calculate the density, including its uncertainty.

A: 1. Calculate the density.

$$\rho = \frac{m}{V} = \frac{24.75\text{ g}}{7.2\text{ mL}} = 3.4375\,\tfrac{\text{g}}{\text{mL}}$$

2. Calculate the relative errors of your two measurements:

$$R.E._{mass} = \frac{\text{uncertainty}}{\text{measured value}} = \frac{0.02}{24.75} = 0.000808$$

$$R.E._{volume} = \frac{0.1}{7.2} = 0.013889$$

3. Add the individual relative errors together to get the total R.E.:

$$0.000808 + 0.013889 = 0.014697$$

4. Multiply the total R.E. by the density to get the uncertainty:

$$3.4375 \times 0.014697 = 0.050521$$

Because you only estimated one decimal place of uncertainty, you need to round the uncertainty off to ± 0.05.

Because uncertainty is rounded to the hundredths place, you need to also round your answer to the hundredths place:

$$\rho = (3.44 \pm 0.05)\,\tfrac{\text{g}}{\text{mL}}$$

Use this space for summary and/or additional notes:

Homework Problems

Because the answers are provided, you must show sufficient work in order to receive credit.

1. You need to combine three liquids and calculate the total volume. You measured the individual volumes as (12.36 ± 0.02) mL, (37.4 ± 0.2) mL, and (61.0 ± 0.1) mL. What is the total volume, including the uncertainty?

   Answer: (110.8 ± 0.3) mL

2. A sample of (0.517 ± 0.008) moles of a chemical is dissolved to make (1.362 ± 0.005) liters of solution.

   a. What is the concentration in moles per liter? (Divide the amount in moles by the volume in liters.)

   Answer: 0.3796 $\frac{mol}{L}$ (Don't worry about rounding yet.)

   b. What are the relative errors of the number of moles and the number of liters? What is the total relative error?

   Answers: moles: R.E. = 0.015
   volume: R.E. = 0.0037
   total R.E. = 0.019

   c. Calculate the uncertainty of the concentration of the solution and express your answer as the concentration (from part a above) plus or minus the uncertainty that you just calculated, with correct rounding.

   Answer: (0.380 ± 0.007) $\frac{mol}{L}$

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