Using Math in Calculations

Unit: Math & Measurement

MA Curriculum Frameworks (2016): SP5

Mastery Objective(s): (Students will be able to…)

- Substitute values for variables in equations and solve them.

Success Criteria:

- Values are substituted for the correct variables.
- Equations are correctly solved for the missing variable using basic algebra.
- Answers have the correct units and are rounded to the appropriate number of significant figures.

Language Objectives:

- Set up and solve word problems relating to the concepts described in this section.

Notes:

Unlike biology, chemistry is a physical science. Among other things, this means chemistry involves calculations, which means you need to be comfortable with algebraic expressions.

Variables and Units

Unlike expressions in math class, which make a clear distinction between constants (the numbers you know the value of) and the variables,

- Equations in chemistry are written as all variables, because each equation works the same way no matter which quantity (or quantities) you are looking for.
- Each of the variables is a letter that relates to the quantity that it represents. For example, volume is V, mass is m, temperature is T, and the number of moles of substance is n. In chemistry, the same quantity *always* uses the same variable.

Use this space for summary and/or additional notes:

- Almost all quantities are measured and have units. . These units are your key to what kind of quantity the numbers describe. For example, in the quantity 12.5 mL, the mL means "milliliters." In the quantity 37.21 g, the g means "grams." In the quantity 21.5 °C, the °C means "degrees Celsius."
  - The unit is part of the quantity. For example, you can't say your height is 1.62. (1.62 what?) You would need to say that your height is 1.62 m (1.62 meters).
  - The unit tells you which type of quantity, and the type of quantity tells you the variable. For example, in the quantity 12.5 mL, the mL (milliliters) is a volume, which uses the variable "*V*." This means that if you have an equation in which the letter *V* represents volume, you would *replace* the letter *V* with the quantity 12.5 mL.
  - Be careful! In many cases, the same letter can be a unit or a number. For example, the letter "m" next to a number means the unit "meters" (a distance), but the variable "*m*" in an equation means "mass".

Some Quantities Used in Chemistry

Quantity	**Unit**	**Variable**	**Quantity**	**Unit**	**Variable**
mass	g	m	temperature	°C, K	T
length	m, cm	ℓ	velocity	$\frac{\text{m}}{\text{s}}$	v
area	m^2	A	heat	J	q*
volume	mL	V	energy	J	E
number of moles	mol	n	pressure	bar, atm, kPa	P
density	$\frac{\text{g}}{\text{mL}}$	ρ†	time	s	t
concentration	$\frac{\text{mol}}{\text{L}}$	c	equilibrium constant	—	K
distance	m	d, ℓ	charge	C	q*

* Notice that *q* is used for both heat and electrical charge. You need to figure out which quantity is meant from context.

† Some chemistry books use the Roman letter "D" for density, but the Greek letter "ρ" ("rho") is preferred. Be careful not to confuse it with the letter "P" (pressure).

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Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$\rho = \dfrac{m}{V}$ and you're given: m = 12.3 g and V = 2.8 cm^3

simply substitute 12.3 g for m, and 2.8 cm^3 for V, giving:

$$\rho = \frac{12.3\ \text{g}}{2.8\ \text{cm}^3} = 4.4\ \tfrac{\text{g}}{\text{cm}^3}$$

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Equations

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word	**meaning**	**word**	**meaning**
and, more than (but not "is more than")	+	is	=
less than (but not "is less than")	−	is at least	≥
of	×	is more than	>
per	÷	is at most	≤
percent	÷ 100	is less than	<
change in *x*, difference in *x*	Δx*		

Suppose you were given the equation:

$$\rho = \frac{m}{V}$$

Using the table on page 82, we can see that *m* is mass and *V* is volume, which means the equation says "density is mass divided by volume". This means that if we knew that the mass of an object was 10.5 g and its volume was 23.7 mL, we could substitute those numbers into the equation to find the density:

$$\rho = \frac{m}{V} = \frac{10.5\ \text{g}}{23.7\ \text{mL}} = \frac{10.5}{23.7}\frac{\text{g}}{\text{mL}} = 0.443\,\tfrac{\text{g}}{\text{mL}}$$

* Note: The Greek letter Δ (delta) is attached to a variable to indicate the change in that variable. For example, ΔT represents a change in temperature. ΔT is one variable in the equation, even though it uses two symbols.

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We can use the same approach no matter which variable we are looking for.

Sample Problems:

Q: An object has a volume of 17.7 mL and a density of $2.35\,\frac{\text{g}}{\text{mL}}$. What is its mass?

A: Start with the equation and substitute:

$$\rho = \frac{m}{V}$$
$$2.35\,\tfrac{\text{g}}{\text{mL}} = \frac{m}{17.7\text{ mL}}$$

Now we have to do algebra. We want to get m by itself, which means we need to move 17.7 mL to the other side. Because it's in the denominator (on the bottom), we have to multiply both sides by it.

$$(17.7\ \cancel{\text{mL}})(2.35\,\tfrac{\text{g}}{\cancel{\text{mL}}}) = \frac{m}{\cancel{17.7\text{ mL}}}(\cancel{17.7\text{ mL}})$$
$$41.6\text{ g} = m$$

(Notice that the 17.7 mL cancels on the right because it's in both the numerator and the denominator. Notice also that the mL cancels on the left for the same reason, leaving g, which happens to be the correct unit. This is called "dimensional analysis," and we will study it in more depth in a future section.)

Q: An object has a mass of 44.7 g and a density of $1.68\,\frac{\text{g}}{\text{mL}}$. What is its volume?

A: Again, start with the equation and substitute:

$$\rho = \frac{m}{V}$$
$$1.68\,\tfrac{\text{g}}{\text{mL}} = \frac{44.7\text{ g}}{V}$$

The variable we want is on the bottom. Again, following the rules of algebra, in order to get it off the bottom, we first have to multiply both sides by it to clear the fraction:

$$V \cdot 1.68\,\tfrac{\text{g}}{\text{mL}} = \frac{44.7\text{ g}}{\cancel{V}} \cdot \cancel{V}$$

Then, we can solve for V in a subsequent step:

$$\frac{V \cdot \cancel{1.68\,\tfrac{\text{g}}{\text{mL}}}}{\cancel{1.68\,\tfrac{\text{g}}{\text{mL}}}} = \frac{44.7\ \cancel{\text{g}}}{1.68\,\tfrac{\cancel{\text{g}}}{\text{mL}}}$$
$$V = 26.6\text{ mL}$$

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Note: Whenever you have to solve an equation for a quantity in the denominator, always do it in two steps: clear the fraction first, then divide. If you try to cleverly rearrange the quantities without doing this, you are almost certain to get the wrong answer!

Q: Find the volume taken up by 3.10 mol of a gas at 298 K and 1.25 atm.

A: When you see a problem like this, the first thing you should do is use the units to figure out what quantities you have in the problem, and label them with their variables:

Find the $\overset{V}{\text{volume}}$ taken up by $\overset{n}{3.10}$ mol of a gas at $\overset{T}{298}$ K and $\overset{P}{1.25}$ atm. (For this problem, use $\underset{R}{0.0821}\,\frac{\ell\cdot\text{atm}}{\text{mol}\cdot\text{K}}$ for the gas constant.)

To solve this problem, we need an equation that relates *V*, *n*, *T*, and *P*. This turns out to be the ideal gas law:

$$P\,V = n\,RT$$

Now substitute the numbers in place of the variables in the equation:

$$\underset{}{\overset{P}{(1.25\text{ atm})}}\ \overset{V}{V}\ \overset{=}{=}\ \overset{n}{(3.10\text{ mol})}\ \overset{R}{\left(0.0821\,\tfrac{\ell\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)}\ \overset{T}{(298\text{ K})}$$

Then solve, using algebra. This means we need to divide both sides by 1.25 atm to get the answer.

$$V = \frac{(3.10\,\text{mol})(0.0821\,\frac{\ell\cdot\text{atm}}{\text{mol}\cdot\text{K}})(298\,\text{K})}{1.25\,\text{atm}} = 60.7\,\ell$$

The Problem-Solving Process

1. Identify the quantities in the problem, using the units.
2. Assign variables to those quantities.
3. Make a list of all of your variables (including the one you're looking for), and what they're equal to.
4. Write down an equation that relates all of those variables.
5. Substitute the values of the variables into the equation. You should have only one variable left, which is the one you're looking for.
6. Solve the equation, using algebra.
7. Don't forget to round your answer correctly and include the units!

Use this space for summary and/or additional notes:

Homework Problems

For these problems, use the table of units and variables on page 82 to determine which quantities represent which variables. Then substitute the variables into the equation given. *You do not have to solve the equations.*

1. 375 J of heat is added to a 75 g block of metal that has a specific heat capacity of $C = 0.450 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$. What is the temperature change of the metal?

   $q = mC\Delta T$

   Answer: $(375\text{ J}) = (75\text{ g})(0.450 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}})\Delta T$

2. A rock has a density of $6.4 \frac{\text{g}}{\text{cm}^3}$ and a mass of 1 500 g. What is its volume.

   $\rho = \frac{m}{V}$

3. 2.5 mol of an ideal gas has a pressure of 1.5 bar and a temperature of 325 K. The gas constant is $0.081 \frac{\text{L}\cdot\text{bar}}{\text{mol}\cdot\text{K}}$. What is its volume?

   $PV = nRT$

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