Conversions (Factor-Label Method)

Unit: Math & Measurement

MA Curriculum Frameworks (2016): N/A

Mastery Objective(s): (Students will be able to…)

- Use algebra and units to create a strategy for problem-solving.

Success Criteria:

- Conversion factors are arranged so that the numerator and denominator are equal.
- Units in conversion factors are arranged so they cancel unwanted units and provide desired units.
- Answers are correct with the correct units.

Tier 2 Vocabulary: unit, convert, conversion

Language Objectives:

- Understand and explain that a conversion factor is two quantities (including their units) that are equal.

Notes:

A conversion is based on the idea that you can express the same quantity using different numbers and units.

For example, Mr. Bigler is 5 feet 4 inches tall. We could express this as 64 inches, $5\frac{1}{3}$ feet, $1.\overline{7}$ yards, 0.001 mile, 163 cm, 1.63 m, or 5.3×10^{-13} parsecs.*

The process of getting from one of these numbers to another is called a unit conversion.

Conversions are based on two strategies:

1. Canceling units you don't want and replacing them with units you do want.
2. Repeatedly multiplying by fractions that equal 1 (*i.e.,* the numerator equals the denominator), so the actual quantity doesn't change.

* A parsec is a distance of about 3.26 light years, or about 3×10^{13} km.

Use this space for summary and/or additional notes:

To show how this works, consider the following math problem:

$$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\cdots\times\frac{99}{100}=?$$

The answer is $\frac{1}{100}$, because everything else cancels:

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\cdots\times\frac{\cancel{99}}{100}=\frac{1}{100}$$

As you may know from algebra, this also works with numbers and variables:

$$\left(\frac{4\cancel{w}}{1}\right)\left(\frac{3\cancel{x}}{\cancel{w}}\right)\left(\frac{\cancel{y}}{2\cancel{x}}\right)\left(\frac{5z}{2\cancel{y}}\right)=\frac{\cancel{4}\cdot 3\cdot 5\cdot z}{\cancel{2}\cdot\cancel{2}}=15z$$

Units work just like variables, so the algebra that you can do with a variable also works with a unit:

$$\frac{2\,\cancel{\text{yd.}}}{1}\times\frac{3\,\cancel{\text{ft.}}}{1\,\cancel{\text{yd.}}}\times\frac{12\,\text{in.}}{1\,\cancel{\text{ft.}}}=\frac{2\times 3\times 12\,\text{in.}}{1}=72\,\text{in.}$$

Notice also that each time we multiplied by a fraction, the numerator was equal to the denominator. (3 ft. = 1 yd. and 12 in. = 1 ft.) This means we were multiplying by 1 each time. That's why 72 in. *must be* the <u>*same*</u> distance as 2 yd. We converted by multiplying:

2 yd. × "1" × "1" = 72 in.

(The "1"s are in quotes because the fractions derived from the conversion factors are all equal to one, even if they don't look like it.)

Some chemistry teachers prefer to use a table with lines to keep the conversion factors neat. The following are two equivalent ways to represent the same calculation. Note that conversion factors (fractions that equal 1) are in vertical columns:

$$\frac{\begin{array}{c|c|c}2\,\cancel{\text{yd.}} & 3\,\cancel{\text{ft.}} & 12\text{ in.}\\ \hline 1 & 1\,\cancel{\text{yd.}} & 1\,\cancel{\text{ft.}}\end{array}}{} = \frac{2\times 3\times 12\,\text{in.}}{1\times 1}=72\,\text{in.}$$

$$\frac{2\,\cancel{\text{yd.}}}{1}\times\frac{3\,\cancel{\text{ft.}}}{1\,\cancel{\text{yd.}}}\times\frac{12\,\text{in.}}{1\,\cancel{\text{ft.}}}=\frac{2\times 3\times 12\,\text{in.}}{1}=72\,\text{in.}$$

Use this space for summary and/or additional notes:

To convert a quantity from one unit to another:

1. Write down the number *and* *units* that you're starting with
2. Find a conversion factor that contains the unit you want to get rid of.
3. Turn the conversion into a fraction and arrange it to cancel the unit you want to get rid of. (If the unit you want to cancel is in the numerator, the same unit needs to be in the denominator in the fraction.)
4. Repeat steps 2 & 3 until you end up with the unit you want.
5. After canceling units, multiply and divide the numbers in the numerator & denominator and simplify the expression.

Working Example:

What is the mass (in grams) of 2.75 moles of sodium chloride (NaCl)?

Conversion factor for NaCl:

$$1\ \text{mol} = 58.44\ \text{g}$$

Solution:

1. We are starting with 2.75 moles (2.75 mol) of chlorine. This means we need to write 2.75 mol in fraction form, as $\frac{2.75\,\text{mol}}{1}$.

2. We want to multiply $\frac{2.75\,\text{mol}}{1} \times 1$ (so we don't change the actual amount).

   This will become $\frac{2.75\,\text{mol}}{1} \times \frac{\quad}{\quad}$ and the unknown fraction needs to equal 1.

3. We know we need to cancel moles, so moles will end up on the bottom of the next fraction. This gives us:

$$\frac{2.75\,\text{mol}}{1} \times \frac{\quad}{\text{mol}}$$

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4. Our conversion factor was: 1 mole NaCl = 58.44 g NaCl which means our conversion factor is:

$$\frac{1\,\text{mol}}{58.44\,\text{g}} = \frac{58.44\,\text{g}}{1\,\text{mol}} = 1$$

We need moles on the bottom, which means we need to use the second fraction. Multiplying our original 2.75 mol by this fraction gives us:

$$\frac{2.75\,\cancel{\text{mol}}}{1} \times \frac{58.44\,\text{g}}{1\,\cancel{\text{mol}}}$$

5. Now, the only unit left is the one we want (grams), so we're ready to solve the problem:

$$\frac{2.75\,\cancel{\text{mol}}}{1} \times \frac{58.44\,\text{g}}{1\,\cancel{\text{mol}}} = \frac{2.75 \times 58.44\,\text{g}}{1 \times 1} = \frac{160.71\,\text{g}}{1} = 160.71\,\text{g}$$

6. Because we had only 3 significant figures in the original number, we need to round our answer to 3 "sig figs". This gives us our final, rounded-off answer of $\boxed{161\,\text{g}}$.

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Homework Problems

Perform each of the following conversions.

1. 23.6 cm = _____ m
2. 15.9 L = _____ mL
3. 0.89 km = _____ mm
4. 7.31×10^{24} mmol = _____ mol
5. 15.0 gallons = _____ L
6. 3.65 miles = _____ km
7. 64 inches = _____ cm
8. 183 pounds = _____ kg
9. $65 \frac{\text{miles}}{\text{hour}}$ = _____ $\frac{\text{m}}{\text{s}}$
10. $13.2 \frac{\text{g}}{\text{cm}^3}$ = _____ $\frac{\text{pounds}}{\text{foot}^3}$
11. $3.65 \frac{\text{dollars}}{\text{gallon}}$ = _____ $\frac{\text{cents}}{\text{L}}$
12. $32 \frac{\text{miles}}{\text{gallon}}$ = _____ $\frac{\text{km}}{\text{L}}$

Conversion Factors

1 gallon = 3.785 L	1 mile = 1.61 km	$(1 \text{ inch})^3 \equiv (2.54 \text{ cm})^3$	I hour ≡ 60 min.
1 pound = 454 g	1 inch ≡ 2.54 cm	$(1 \text{ foot})^3 \equiv (12 \text{ inch})^3$	1 min. ≡ 60 s
	1 foot ≡ 12 inches		

Use this space for summary and/or additional notes: