Dimensional Analysis

Unit: Math & Measurement

MA Curriculum Frameworks (2016): SP5

Mastery Objective(s): (Students will be able to…)

- Use compound units to infer conversion factors.
- Infer equations from compound units.

Success Criteria:

- Equations relate quantities in the same way that units do.
- Word problems involving conversions that use inferred quantities are correct, including the correct units, and are rounded to the appropriate number of significant figures.

Tier 2 Vocabulary: dimension, unit, conversion

Language Objectives:

- Explain the process of using a compound unit as a conversion factor using appropriate vocabulary.

Notes:

dimensions: the units that something is measured in.

dimensional analysis: using the relationship between units (dimensions) and equations in order to analyze (and solve) problems. This can involve either:

- Using units to predict the relationships between quantities (and sometimes the equations that relate them)
- Using an equation to determine what the dimensions (units) should be.

Remember that units are like variables. They can be multiplied and divided by their coëfficients, and by each other. If you divide meters (a unit of distance) by seconds (a unit of time) you end up with the units $\frac{m}{s}$ (a unit of velocity*). Because this works for one set of units, it must also work for other units that measure the same dimensions. Because we divided the units of distance by the units of time and got units of velocity, this means we can divide *any* distance (regardless of the units) by *any* time (regardless of the units) to get velocity.

* There is a subtle difference between velocity and speed, which you will study if you take physics.

Use this space for summary and/or additional notes:

For example, density measures how heavy something is for its size. The density of iron is $7.87\,\frac{\text{g}}{\text{cm}^3}$. Because grams measure mass and cm^3 measure volume, it must be true that:

$$\text{density units} = \frac{\text{mass units}}{\text{volume units}} \quad \text{which means} \quad \text{density} = \frac{\text{mass}}{\text{volume}}$$

This must mean that *any* mass unit divided by *any* volume unit gives a density unit (which is, in fact, true).

Some of the units that density can be measured in (mass units divided by volume units) include:

$$\frac{\text{g}}{\text{mL}} \quad \frac{\text{g}}{\text{L}} \quad \frac{\text{g}}{\text{cm}^3} \quad \frac{\text{kg}}{\text{L}} \quad \frac{\text{kg}}{\text{m}^3} \quad \frac{\text{tonne}}{\text{m}^3} \quad \frac{\text{lb.}}{\text{ft.}^3}$$

You can use dimensional analysis in the same manner to help you figure out how to solve many problems that include units of measure. In fact, dimensional analysis has led to new scientific discoveries. Scientists can sometimes determine a relationship between two quantities based on the units, and are then able to prove that relationship in the laboratory!

However, there are pitfalls. For example, mechanical work and torque have the same units $(\text{N}\cdot\text{m})$, but they describe different kinds of quantities and cannot be used interchangeably. (In fact, in a physics problem involving tightening a bolt with a wrench, $36\ \text{N}\cdot\text{m}$ of torque ended up doing $19\ \text{N}\cdot\text{m}$ of work on the bolt!)

Also, dimensional analysis cannot predict constants that might appear in an equation. For example, the unit for energy is a joule, which equals one $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$.

Because mass is measured in kg and velocity in $\frac{\text{m}}{\text{s}}$, this suggests that the equation for kinetic energy should be mv^2. However, the equation is actually $\frac{1}{2}mv^2$. The factor of $\frac{1}{2}$ is a necessary part of the equation, but cannot be discovered by dimensional analysis.

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Making Compound Units Into Conversion Factors

Any time you have a number with a compound unit, you can make a "temporary" conversion factor out of it. For example, if a car is driving $60\,\frac{\text{mi.}}{\text{hr.}}$, then:

$$60\,\tfrac{\text{mi.}}{\text{hr.}} = 60 \times \frac{\text{mi.}}{\text{hr.}} = \frac{7.87}{1} \times \frac{\text{mi.}}{\text{hr.}} = \frac{60\,\text{mi.}}{1\,\text{hr.}}$$

In other words, the quantity $60\,\frac{\text{mi.}}{\text{hr.}}$ gave us the conversion factor 60 mi. = 1 hr. (Notice that the coëfficient of 60 ended up with miles, which is the unit that was on top.) This means that for this problem—as long as the car keeps going the same speed—60 miles takes 1 hour. That means you can use this conversion factor *in this problem* to convert miles to hours, or hours to miles.

Examples:

- The density of iron (Fe) is $7.87\,\frac{\text{g}}{\text{cm}^3}$. What is the volume of 15.0 g of iron?

  To solve this problem, we recognize that the compound unit $7.87\,\frac{\text{g}}{\text{cm}^3}$ can be written as follows:

$$7.87\,\tfrac{\text{g}}{\text{cm}^3} = 7.87 \times \frac{\text{g}}{\text{cm}^3} = \frac{7.87}{1} \times \frac{\text{g}}{\text{cm}^3} = \frac{7.87\,\text{g}}{1\,\text{cm}^3}$$

  This means 7.87 g = 1 cm^3. We can use this as the conversion factor between grams of iron and cubic centimeters of iron. We will use in this problem to convert grams of iron into cm^3 of iron:

$$\frac{15.0\,\text{g}}{1} \times \frac{1\,\text{cm}^3}{7.87\,\text{g}} = \frac{15.0\,\text{cm}^3}{7.87} = 1.91\,\text{cm}^3$$

Use this space for summary and/or additional notes:

- The concentration of sodium chloride (NaCl) in sea water is about $0.48\,\frac{\text{mol}}{\text{L}}$. How many moles of NaCl are in 55,000 L of sea water?

  To solve this problem, we recognize that the compound unit $0.48\,\frac{\text{mol}}{\text{L}}$ is going to be the conversion factor 0.48 mol = 1 L. This is the temporary conversion factor that we will use in this problem to convert liters of sea water into moles of NaCl:

$$\frac{55\,000\,\cancel{\text{L}}}{1}\times\frac{0.48\,\text{mol}}{1\,\cancel{\text{L}}}=55\,000\times0.48\,\text{mol}=26\,400\,\text{mol}=26\,000\,\text{mol}$$

  (Note that we had to round off the final answer to 2 significant figures.)

- The molar mass of NaOH is $40.00\,\frac{\text{g}}{\text{mol}}$. What is the mass of 2.85 mol of NaOH?

  To solve this problem, we recognize that the compound unit $40.00\,\frac{\text{g}}{\text{mol}}$ is going to be the conversion factor 40.00 g = 1 mol. This is the temporary conversion factor that we will use in this problem to convert moles of NaOH into grams:

$$\frac{2.85\,\cancel{\text{mol}}}{1}\times\frac{40.00\,\text{g}}{1\,\cancel{\text{mol}}}=2.85\times40.00\,\text{g}=114\,\text{g}$$

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Homework Problems

1. An object has a density of $3.65\frac{g}{cm^3}$. If the volume of the object is $12.5\ cm^3$, what is its mass?

   Answer: 45.6 g

2. A liquid solution has a salt concentration of 2.5 mol/l . How many moles of salt are in 0.50 ℓ of the solution?

   Answer: 1.3 mol

3. A car is travelling at a speed of 65 mi./hr. How many hours would it take for this car to travel 250 mi.?

   Answer: 3.8 hr

4. Suppose the average temperature of the Earth is rising at a rate of 2.0◦C/100 years . When Mr. Bigler gives this same homework assignment to one of your children 25 years from now, how much warmer will the average temperature of the Earth be?

   Answer: 0.5 °C

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5. If a gas at "standard temperature and pressure" has a molar volume of $22.7\frac{\text{L}}{\text{mol}}$, how many moles of this gas would there be in a 5.5 L balloon?

   Answer: 0.24 mol

6. Suppose you have a job at which you earn $8.00 per hour (which you can write as 8.00 $/hr). How many hours would you have to work to have enough money for a $1200 car?

   Answer: 150 h

7. The continent of South America is drifting away from Africa at a rate of about $2.5\frac{\text{cm}}{\text{year}}$. If South America was once touching Africa, and the speed of the plates was constant, how many years did it take for South America to get to its present location, which is about 5000 km away from Africa?

   (*Hint: don't forget that you will need to convert* km *to* mm.)

   Answer: ?

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