Gas Laws

Unit: Gases

MA Curriculum Frameworks (2016): HS-PS2-8(MA)

Mastery Objective(s): (Students will be able to…)

- Qualitatively describe the relationship between any two of the quantities: *number of particles*, *temperature*, *pressure*, and *volume* in terms of Kinetic Molecular Theory (KMT).
- Quantitatively determine the *number of particles*, *temperature*, *pressure*, or *volume* in a before & after problem in which one or more of these quantities is changing.

Success Criteria:

- Descriptions relate behavior at the molecular level to behavior at the macroscopic level.
- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Tier 2 Vocabulary: ideal, law

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Understand and correctly use the terms "pressure," "volume," and "temperature," and "ideal gas."
- Explain the placement of each quantity in the ideal gas law.

Labs, Activities & Demonstrations:

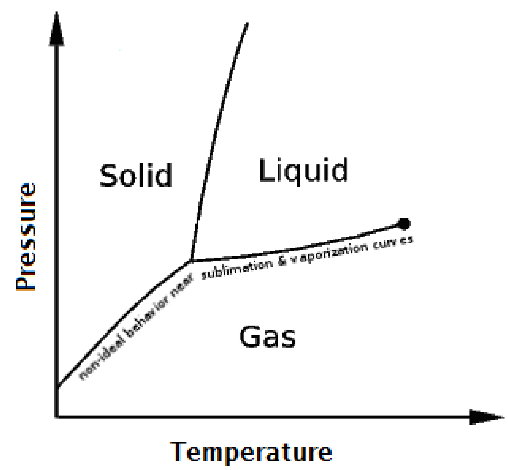
- Vacuum pump (pressure & volume) with:
  - balloon (air vs. water)
  - shaving cream
- Absolute zero apparatus (pressure & temperature)
- Can crush (pressure, volume & temperature)

Use this space for summary and/or additional notes:

Notes:

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost.

Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.,* gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)



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Note about Subscripts and Variables

When variables appear more than once in an equation with different values each time, we use subscripts to group them. You have already seen this a few times in math, *e.g.,* in the formula for the slope of a line and the distance formula:

$$\text{slope: } m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{distance: } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In the above examples, the subscripts "1" and "2" are used to group the *x* and *y* values based on whether they refer to the first point (x_1, y_1), or the second one (x_2, y_2).

In chemistry, we use subscripts the same way. For example, if a gas is heated, that means the temperature is changing. We refer to the starting temperature (temperature #1) as T_1, and the ending temperature (temperature #2) as T_2. The same concept applies to other variables as well, such as moles (*n*), volume (*V*), and pressure (*P*).

Proportionalities

directly proportional: if two quantities are ***directly*** proportional, as one increases, the other increases proportionately.

If x and y are directly proportional, then $x \propto y$ which means $x = ky$ and $\frac{x}{y} = k$ where *k* is a constant. You should notice that *x* and *y* are either ***numerator and denominator*** in a fraction, or are on ***opposite sides*** of the equals sign.

inversely proportional: if two quantities are ***inversely*** proportional, as one increases, the other decreases proportionately.

If x and y are inversely proportional, then $x \propto \frac{1}{y}$ which means $xy = k$ where *k* is a constant. You should notice that *x* and *y* are on the ***same side*** of the equals sign.

Use this space for summary and/or additional notes:

Avogadro’s Principle

In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of particles.

What did we do?	**What happened?**	**What are the molecules doing?**	**Conclusion**
put more (moles of) air into a balloon $n\uparrow$	the volume of the balloon got larger $V\uparrow$	crowding each other → pushing each other farther away	n and V are directly proportional. $\frac{V}{n} = \text{constant}$

If the pressure and temperature are constant, then for an ideal gas:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}\ ^{*}$$

Because it is almost always more convenient to work with moles of gas (n) rather than particles (N), we can rewrite Avogadro’s principle as:

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

* Avogadro’s principle is usually stated $\frac{n_1}{V_1} = \frac{n_2}{V_2}$. I have inverted it in these notes so that the quantities in the numerator and denominator are the same as the quantities in the numerator and denominator of the combined gas law.

Use this space for summary and/or additional notes:

Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional.

What did we do?	**What happened?**	**What are the molecules doing?**	**Conclusion**
decrease pressure by putting a balloon in a vacuum chamber $P\downarrow$	the volume of the air inside the balloon increased $V\uparrow$	colliding with less force → pushing each other less far away	*P* and *V* are inversely proportional. PV = constant

Therefore, if the temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1V_1 = P_2V_2$$

(Note that by convention, gas laws use subscripts "1" and "2" instead of "i" and "f".)

Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional.

What did we do?	**What happened?**	**What are the molecules doing?**	**Conclusion**
cool gas by putting a soda can full of very hot air into cool water $T\downarrow$	the volume of the air got smaller and crushed the can $V\downarrow$	moving more slowly → pushing each other less far away	*V* and *T* are directly proportional. $\frac{V}{T}=$constant

If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Use this space for summary and/or additional notes:

Gay-Lussac's Law

In 1702, French physicist Guillaume Amontons discovered that there is a relationship between the pressure and temperature of a gas. However, precise thermometers were not invented until after Amontons' discovery, so it wasn't until 1808, over a century later, that French chemist Joseph Louis Gay-Lussac confirmed this law mathematically. The pressure law is most often attributed to Gay-Lussac, though some texts refer to it as Amontons' Law.

What did we do?	What happened?	What are the molecules doing?	Conclusion
increase temperature by heating a metal sphere full of air $T\uparrow$	the pressure of the air increased $P\uparrow$	moving faster → colliding with more force	P and T are directly proportional. $\frac{P}{T} = \text{constant}$

If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

The Combined Gas Law

We can combine each of the above principles. When we do this (keeping P and V in the numerator and n and T in the denominator for consistency), we get following relationship for an ideal gas:

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} = \text{constant}$$

Note, however, that in most problems, the number of moles of gas remains constant. This means $n_1 = n_2$ and we can cancel it from the equation, which gives:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

This equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

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When using the combined gas law, any quantity that is not changing may be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.)

This brings us to an important point about science problems: *If something is not mentioned in a problem, **always** assume that it doesn't affect the problem.* On a standardized test like MCAS or an AP® test, it's usually best to state those assumptions explicitly, because if your assumption is valid and you do the rest of the problem correctly, you will almost always receive some credit, *even if your assumption was different from what the person who wrote the problem intended*.

For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel *T* from both sides of the combined gas law, you get:

$$\frac{P_1V_1}{\cancel{T_1}} = \frac{P_2V_2}{\cancel{T_2}} \text{ which simplifies to } P_1V_1 = P_2V_2 \text{ (Boyle's Law)}$$

Solving Problems Using the Combined Gas Law

You can use this method to solve any "before/after" gas law problem:

1. Determine which variables you have
2. Determine which values are *initial* (#1) *vs.* *final* (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures *must* be in Kelvin!)

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Sample problem:

Q: A gas has a temperature of 25 °C and a pressure of 1.5 bar. If the gas is heated to 35 °C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25 °C and 35 °C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):	**Time 2 ("after")**:
P_1 = 1.5 bar	$P_2 = P_2$
T_1 = 25 °C + 273 = 298 K	T_2 = 35 °C + 273 = 308 K

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Gay-Lussac's Law)}$$

4. Plug in our values and solve:

$$\frac{1.5\,\text{bar}}{298\,\text{K}} = \frac{P_2}{308\,\text{K}} \quad \rightarrow \quad \boxed{P_2 = 1.55\text{ bar}}$$

Use this space for summary and/or additional notes:

Homework Problems

Solve these problems using one of the gas laws in this section. Remember to convert temperatures to Kelvin!

1. A sample of oxygen gas occupies a volume of 250. mL at a pressure of 740. torr. What volume will it occupy at 800. torr?

   Answer: 231.25 mL

2. A sample of O_2 is at a temperature of 40.0 °C and occupies a volume of 2.30 L. To what temperature should it be raised to occupy a volume of 6.50 L?

   Answer: 612 °C

3. H_2 gas was cooled from 150. °C to 50. °C. Its new pressure is 750 torr. What was its original pressure?

   Answer: 980 torr

4. A 2.00 L container of N_2 had a pressure of 3.20 atm. What volume would be necessary to decrease the pressure to 98.0 kPa?

(*Hint: notice that the pressures are in different units. You will need to convert one of them so that both pressures are in either* atm *or* kPa.)

   Answer: 6.62 L

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5. A sample of air has a volume of 60.0 mL at S.T.P. What volume will the sample have at 55.0 °C and 745 torr?

   Answer: 73.5 mL

6. N_2 gas is enclosed in a tightly stoppered 500. mL flask at 20.0 °C and 1 atm. The flask, which is rated for a maximum pressure of 3.00 atm, is heated to 680. °C. Will the flask explode?

   Answer: P_2 = 3.25 atm. Yes, the flask will explode.

7. A scuba diver's 10. L air tank is filled to a pressure of 210 bar at a dockside temperature of 32.0 °C. When the diver is breathing the air underwater, the water temperature is 8.0 °C, and the pressure is 2.1 bar.

   a. What volume of air does the diver use?

      Answer: 921 L

   b. If the diver uses air at the rate of 8.0 L/min, how long will the diver's air last?

      Answer: 115 min

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