Molecular Speed & Effusion

Unit: Gases

MA Curriculum Frameworks (2016): N/A (optional topic)

Mastery Objective(s): (Students will be able to…)

- Determine the velocities of gas molecules from their molar masses.

Success Criteria:

- Solutions use the equation appropriate for the information given.
- Solutions have the correct quantities substituted for the correct variables.
- Algebra and rounding to appropriate number of significant figures is correct.

Tier 2 Vocabulary: root, mean, square

Language Objectives:

- Describe the pairing of each gas with its mole fraction and pressure.

Notes:

effusion: the escape of a gas through a small opening.

Graham's Law of Effusion: lighter molecules move faster and heavier molecules move more slowly. If you have a small opening, such as a leak in a tank, the lighter molecules will escape faster.

temperature: a measure of the average kinetic energy of the molecules in a substance

root mean square speed: the average speed of the gas molecules. "Root mean square" means this average is calculated by determining the average of the squares of the speeds, then taking the square root of the result.

Use this space for summary and/or additional notes:

Root Mean Square Speed of Gas Molecules

The root mean square velocity (v_{rms}) of gas molecules is given by the formula*:

$$v_{rms} = \sqrt{\frac{3000\,RT}{M}}$$

where M is the molar mass of the gas in $\frac{g}{mol}$, R is the gas constant $(8.31\,\frac{J}{mol\cdot K})$, and T is the temperature in Kelvin. The quantity v_{rms} comes out in units of $\frac{m}{s}$.

Sample Problem:

A tank is filled with a mixture of helium and oxygen at a temperature of 25 °C. Calculate the root mean square speed (v_{rms}) of the oxygen molecules. (Note: oxygen has a molar mass of $32\,\frac{g}{mol}$.)

$$T = 25^\circ\mathrm{C} + 273 = 298\,\mathrm{K} \quad \text{and} \quad M_{O_2} = 32\,\tfrac{g}{mol}$$

$$v_{rms} = \sqrt{\frac{3000\,RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3000\,(8.31\,\frac{J}{mol\cdot K})(298\,\mathrm{K})}{32\,\frac{g}{mol}}}$$

$$v_{rms} = \sqrt{232{,}161} = 482\,\tfrac{m}{s}$$

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* This formula is usually given as $v_{rms} = \sqrt{\frac{3\,RT}{M}}$, but this requires that M be expressed in $\frac{kg}{mol}$. Because we are not deriving the formula in this class, it makes sense to change the factor from 3 to 3000 in order to keep molar mass expressed in the more familiar units of $\frac{g}{mol}$.

Use this space for summary and/or additional notes:

Graham's Law

From physics, the formula for kinetic energy is:

$$E_{kinetic} = \tfrac{1}{2}mv^2$$

where m is the mass and v is the velocity (speed).

If two molecules have the same temperature, then they have the same kinetic energy. If molecule #1 has mass m_1 and velocity v_1 and molecule #2 has mass m_2 and velocity v_2, then the kinetic energies are:

$$E_{kinetic} = \tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}m_2v_2^2$$

If we cancel the ½ from both sides, and rearrange so the masses are on one side and the speeds are on the other, we get:

$$\frac{v_2^2}{v_1^2} = \frac{m_1}{m_2}$$

Taking the square root of both sides:

$$\frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}}$$

This formula is called Graham's Law, named after Thomas Graham who first proposed it.

For the purpose of these calculations, the mass of a molecule is its molar mass, which is simply the same number of grams as the molecule's atomic mass.

Because the rate of effusion (r) is the velocity divided by time, the units of time cancel, giving $\frac{r_2}{r_1} = \frac{v_2}{v_1}$

Use this space for summary and/or additional notes:

Sample Problem:

If the speed of oxygen molecules in a tank is $482\,\frac{\text{m}}{\text{s}}$, calculate the speed of helium molecules in the same tank. (O_2 has a molar mass of $32.0\,\frac{\text{g}}{\text{mol}}$ and helium has a molar mass of $4.01\,\frac{\text{g}}{\text{mol}}$.)

$$M_{O_2} = 32.0\,\tfrac{\text{g}}{\text{mol}} \text{ and } M_{He} = 4.01\,\tfrac{\text{g}}{\text{mol}}$$

$$\frac{v_{\text{He}}}{v_{\text{O}_2}} = \sqrt{\frac{m_{\text{O}_2}}{m_{\text{He}}}}$$

$$\frac{v_{\text{He}}}{482} = \sqrt{\frac{32.0}{4.01}}$$

$$\frac{v_{\text{He}}}{482} = \sqrt{8.00} = 2.82$$

$$v_{\text{He}} = (2.82)(482) = 1360\,\tfrac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes:

Homework Problems

1. The temperature of the air in a room is 25.0 °C. Determine the root mean square speed of the nitrogen molecules (molar mass $28.0\,\frac{\text{g}}{\text{mol}}$) in the air in that room.

   Answer: $515\,\frac{\text{m}}{\text{s}}$

2. If the N_2 molecules (which have a molar mass of $28.0\,\frac{\text{g}}{\text{mol}}$) in a sample of air have a root mean square speed of $450\,\frac{\text{m}}{\text{s}}$, what is the speed of the O_2 molecules (which have a molar mass of $32.0\,\frac{\text{g}}{\text{mol}}$)?

   Answer: $421\,\frac{\text{m}}{\text{s}}$

3. A balloon is filled with 1.0 mole each of helium (molar mass $4.0\,\frac{\text{g}}{\text{mol}}$) and oxygen (molar mass $32\,\frac{\text{g}}{\text{mol}}$). If half of the gas in the balloon is allowed to escape, how many moles each of helium and oxygen are left in the balloon?

   (*Hint: Find the ratio of the speeds of the two molecules. This will be the same as the ratio of the number of moles of each gas that escapes. The reciprocal will be the ratio of the number of moles of each gas left in the balloon.*)

   Answer: 0.26 mol He, 0.74 mol O_2

Use this space for summary and/or additional notes: