Colligative Properties

Unit: Solutions

MA Curriculum Frameworks (2016): HS-PS2-7(MA)

Mastery Objective(s): (Students will be able to…)

- Calculate boiling point elevation, freezing point depression, vapor pressure lowering and osmotic pressure.
- Calculate the molar mass of a solute, based on the grams of a solute added and its effect on the freezing or boiling point of water.

Success Criteria:

- Solutions use the equation appropriate for the information given.
- Solutions have the correct quantities substituted for the correct variables.
- Algebra and rounding to appropriate number of significant figures is correct.

Tier 2 Vocabulary: depression, elevation

Language Objectives:

- Explain why solutes cause changes in freezing and boiling point.

Notes:

colligative properties: properties of a solution that depend on the physical number of particles dissolved, but not on the chemical properties of those particles.

Solutes can affect the physical properties of a solution by "getting in the way" of the solvent molecules.

molality (m): the concentration of a solution measured in grams of solute per 1 000 g of solvent.

Notice that the molality depends only on the masses of the solute and solvent, not on the volume.

van't Hoff factor (*i*): the number of particles of solute that you get when the solute dissolves. For example, when you dissolve sodium phosphate (Na_3PO_4) in water, it breaks up into three Na^+ ions and one PO_4^{3-} ion, for a total of four particles. This means the van't Hoff factor for Na_3PO_4 is 4.

Note that the van't Hoff factor (*i*) is a measured quantity. If an ionic compound dissociates completely, the value of *i* can be approximated from the chemical formula. However, if a compound is a weak electrolyte (dissolves only partially), the value of *i* must be measured empirically.

Use this space for summary and/or additional notes:

Freezing Point Depression

When a solute is added to a solvent, the solvent particles must "push" the solute particles out of the way in order to form a solid, which requires energy. This means that in order to make the solution freeze, the temperature must be *lower*, in order to increase the amount of energy *given off* when the solution forms a solid.

This is why we put salt on ice in winter—the salt particles get in the way of the water freezing, which means the temperature has to be lower in order for the salt water to freeze. As long as the temperature is above this new freezing point, the solution stays liquid (*i.e.,* the ice melts).

$$\Delta T_f = i\,m\,K_f$$

where:

i = van't Hoff factor

m = molality of the solute $\left(\frac{\text{mol solute}}{\text{kg solvent}}\right)$

K_f = freezing point depression constant. For H_2O, $K_f = 1.86\,\frac{^\circ\text{C}}{\text{m}}$ (degrees Celsius per molal)

Sample problem:

What is the freezing point of 25 g Na_2SO_4 dissolved in 500 g of H_2O?

$$\Delta T_f = i\,m\,K_f$$

i = 3 (because $Na_2SO_4 \rightarrow 2\,Na^+ + SO_4^{2-}$, which is a total of 3 ions)

$$m = \frac{25\text{ g Na}_2\text{SO}_4}{500\text{ g H}_2\text{O}} \times \frac{1\text{ mol Na}_2\text{SO}_4}{142.05\text{ g Na}_2\text{SO}_4} \times \frac{1000\text{ g H}_2\text{O}}{1\text{ kg H}_2\text{O}} = 0.352\text{ m}$$

$K_f = 1.86\,\frac{^\circ C}{m}$

$$\Delta T_f = i\,m\,K_f$$

$$\Delta T_f = (3)\,(0.352\text{ m})\,(1.86\,\tfrac{^\circ\text{C}}{\text{m}})$$

$$\Delta T_f = 1.96\ ^\circ\text{C}$$

The normal boiling point of H_2O is 0 °C, and we just calculated that the freezing point is *lowered* by 1.96 °C. Therefore, the freezing point of the solution is:

$$T_f = -1.96\ ^\circ\text{C}$$

Use this space for summary and/or additional notes:

Boiling Point Elevation

Solute particles attract solvent molecules as they boil and attempt to escape as a gas. The solution needs extra energy (higher temperature) in order to overcome this extra attraction. This is why solutions—liquids with solutes dissolved in them—boil at higher temperatures.

$$\Delta T_b = imK_b$$

where:

i = van't Hoff factor (# solute particles from each molecule, sometimes called the "dissociation factor")

m = molality of the solute $\left(\frac{\text{mol solute}}{\text{kg solvent}}\right)$

K_b = boiling point elevation constant. For H_2O, $K_b = 0.52\,\frac{^\circ\text{C}}{\text{m}}$ (degrees Celsius per molal)

Calculations involving boiling point elevation are done exactly the same way as calculations involving freezing point depression.

Sample problem:

Q: It is often said that salt should be added to boiling water when cooking pasta because the salt will elevate the boiling point of the water, causing the pasta to cook faster. How much would one teaspoon (4 g) of salt raise the boiling point of 4 quarts (about 4 kg) of water?

A: 4 g of NaCl is approximately 0.068 mol.

The molal concentration of salt in the water is therefore $\frac{0.068\text{ mol NaCl}}{4.0\text{ kg }H_2O} = 0.017\text{ m}$.

NaCl dissociates into to ions, so i = 2. $K_b = 0.52\,\frac{^\circ\text{C}}{\text{m}}$. Therefore:

$$\Delta T_b = imK_b$$
$$\Delta T_b = (2)(0.017)(0.52) = 0.018^\circ\text{C}$$

The salt in the water would increase the boiling point from 100 °C to 100.018 °C. We can therefore discount the possibility that boiling point elevation makes any significant contribution to how quickly the pasta cooks.

Use this space for summary and/or additional notes:

Raoult's Law (Vapor Pressure Lowering)

Solute particles attract solvent molecules. This attraction is strong enough to prevent some of those solvent molecules from escaping into the vapor phase.

Vapor pressure is the number of molecules of liquid that can escape into the gas phase at a given temperature, expressed as a pressure. Therefore, the presence of solute particles lowers the vapor pressure of the solvent.

Specifically, Raoult's Law states that the partial pressure of *vapor* "*i*" (P_i) equals the vapor pressure of (pure) "*i*" ($P^{\circ}_{v,i}$) times the mole fraction of *liquid* "*i*" (χ_i) in the mixture:

$$P_i = P^{\circ}_{v,i}\chi_i$$

Sample problem:

A sealed chamber contains a solution of glucose dissolved in water at 22 °C. The vapor pressure of pure water at 22 °C is 2.6 kPa. If the mole fraction of glucose is 0.10, what is the partial pressure of water in the air space above the solution?

Answer:

If the mole fraction of glucose in the solution is 0.10, the mole fraction of water in the solution must be 1 − 0.10 = 0.90. Therefore, the partial pressure of water is:

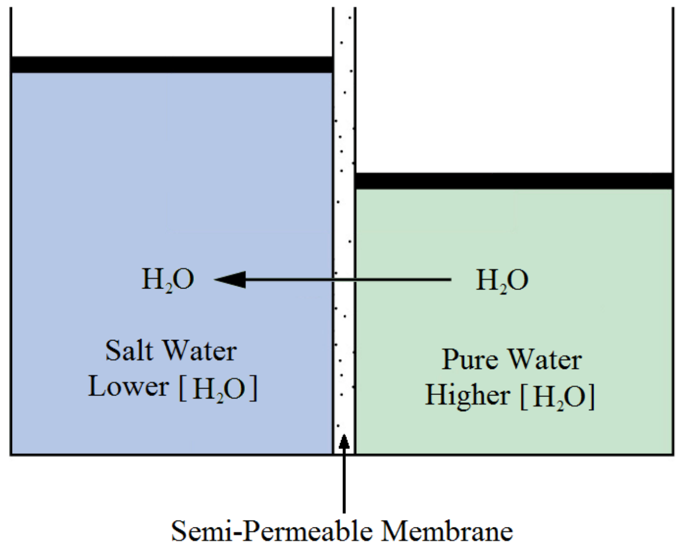
$$P_{\mathrm{H_2O}} = P^{\circ}_{v,\mathrm{H_2O}}\chi_{\mathrm{H_2O}}$$
$$P_{\mathrm{H_2O}} = (2.6\,\mathrm{kPa})(0.90) = 2.3\,\mathrm{kPa}$$

Use this space for summary and/or additional notes:

Osmotic Pressure

Diffusion is the natural flow of molecules from a region of higher concentration to a region of lower concentration.

Recall from biology that osmosis is a form of diffusion in which solvent molecules are able to travel across a semi-permeable membrane (such as a cell membrane), but solute molecules cannot pass through. Therefore, the higher the concentration of solute molecules on one side of the membrane, the more strongly those solute molecules attract solvent molecules from the other side. The force of this attraction can be measured as a pressure.



This is why your skin wrinkles when it gets wet—the solutes inside your skin cells attract the pure water from outside your skin. As the water flows in through the cell membrane, it enlarges your skin cells. As your skin gets larger, the surface area gets larger, which we see as wrinkles. As your skin dries, the water escapes, the cells shrink, and the wrinkles disappear.

Use this space for summary and/or additional notes:

<u>osmotic pressure</u> (π): the observed pressure difference across a semi-permeable membrane because of differences in solute concentration. (Yes, it is awkward to think of the Greek letter π as a variable. Chemists are weird.)

Because osmotic pressure is a pressure, and because we are assuming the solute molecules otherwise obey kinetic-molecular theory (*i.e.,* they move freely, more or less like gas molecules), we can apply the ideal gas law. Note that we need to include the van't Hoff factor because *each* of the ions in solution created by dissolving a compound contributes separately to the osmotic pressure. This gives us the following formula:

$$\pi V = inRT$$

where:

π = osmotic pressure (additional pressure due to osmosis)
V = volume of solution
i = van't Hoff factor
n = moles of solute
R = gas constant
T = temperature (Kelvin)

Because molarity equals the moles of solute (n) divided by the volume of solution (V), the above equation can be simplified to:

$$\pi = iM\mathrm{R}T$$

where M = molarity of the solute, and everything else is as above.

Use this space for summary and/or additional notes:

Homework Problems

1. If 45 grams of sodium chloride were added to 500. grams of water, what would the melting and boiling points be of the resulting solution? $K_b(H_2O) = 0.52\,\frac{^\circ C}{m}$ and $K_f(H_2O) = 1.86\,\frac{^\circ C}{m}$.

   Answer: M.P. = −5.73 °C   B.P. = 101.6 °C

2. What is the vapor pressure of the solution in problem #1 at 250 °C? The vapor pressure of pure water at 250 °C is 3.17 kPa.

   Answer: 3.08 kPa

3. If the solution in problem #1 (which has a density of $1.056\,\frac{g}{mL}$) were placed on one side of a semipermeable membrane, and a 1.00 M solution of NaCl were placed on the other side of the membrane, what would the osmotic pressure be at 27 °C?

   Answer: 12.1 atm

Use this space for summary and/or additional notes:

4. Which solution will have a higher boiling point: a solution containing 105 g of sucrose ($C_{12}H_{22}O_{11}$) in 500. g of water, or a solution containing 35 g of NaCl in 500. g of water?

   Answer: for the sucrose, T_b = 100.32 °C for the NaCl, T_b = 101.40 °C

5. 0.546 g of a compound with a van't Hoff factor of 1 was dissolved in 15.0 g of benzene. The freezing point of the solution was found to be 0.240 °C lower than the freezing point of pure benzene. If K_f for benzene is $K_f = 5.12\frac{^\circ\text{C}}{\text{m}}$, what is the molar mass of the compound?

   Answer: $776\,\frac{\text{g}}{\text{mol}}$

Use this space for summary and/or additional notes: