CP1 & honors (not AP®)

Significant Figures

Unit: Laboratory & Measurement

NGSS Standards/MA Curriculum Frameworks (2016): SP4

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): SP2.B

Mastery Objective(s): (Students will be able to…)

- Identify the significant figures in a number.
- Perform calculations and round the answer to the appropriate number of significant figures

Success Criteria:

- Be able to identify which digits in a number are significant.
- Be able to count the number of significant figures in a number.
- Be able to determine which places values will be significant in the answer when adding or subtracting.
- Be able to determine which digits will be significant in the answer when multiplying or dividing.
- Be able to round a calculated answer to the appropriate number of significant figures.

Language Objectives:

- Explain the concepts of significant figures and rounding.

Tier 2 Vocabulary: significant, round

Notes:

Because it would be tedious to calculate the uncertainty for every calculation in physics, we can use significant figures (or significant digits) as a simple way to estimate and represent the uncertainty.

Significant figures are based on the following approximations:

- All stated values are rounded off so that the uncertainty is only in the last unrounded digit.
- Assume that the uncertainty in the last unrounded digit is ±1.
- The results of calculations are rounded so that the uncertainty of the result is only in the last unrounded digit, and is assumed to be ±1.

While these assumptions are often (though not always) the right order of magnitude, they rarely give a close enough approximation of the uncertainty to be useful. For this reason, ***significant figures are used as a convenience, and are used only when the uncertainty does not actually matter***.

If you need to express the uncertainty of a measured or calculated value, you must express the uncertainty separately from the measurement, as described in the previous section.

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Therefore, when you take measurements and perform calculations in the laboratory, you will specifically state the measurements and their uncertainties. ***Never use significant figures in lab experiments!***

For homework problems and written tests, you will not be graded on your use of significant figures, but you may use them as a simple way to keep track of the approximate effects of uncertainty on your answers, if you wish.

The *only* reasons that significant figures are presented in these notes are:

1. If you are taking the AP® exam, you are expected to round your answers to an appropriate number of significant figures.
2. After a year of surviving the emotional trauma of significant figures in chemistry class, students expect to be required to use significant figures in physics and every science course afterwards. It is kinder to just say "[*sigh*] Yes, please do your best to round to the correct number of significant figures." than it is to say "Nobody actually uses significant figures. All that trauma was for nothing."

Every time you perform a calculation, you need to express your answer to enough digits that you're not introducing additional uncertainty. However, as long as that is true, feel free to round your answer off in order to omit digits that are one or more orders of magnitude smaller than the uncertainty.

In the example on page 63, we rounded the number 1 285.74 off to the tens place, resulting in the value of 1 290, because we couldn't show more precision than we actually had.

In the number 1 290, we would say that the first three digits are "significant", meaning that they are the part of the number that is not rounded off. The zero in the ones place is "insignificant," because the digit that was there was lost when we rounded.

<u>significant figures</u> (significant digits): the digits in a measured value or calculated result that are not rounded off. (Note that the terms "significant figures" and "significant digits" are used interchangeably.)

<u>insignificant figures</u>: the digits in a measured value or calculated result that were "lost" (became zeroes before a decimal point or were cut off after a decimal point) due to rounding.

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Identifying the Significant Digits in a Number

The first significant digit is where the "measured" part of the number begins—the first digit that is not zero.

The last significant digit is the last "measured" digit—the last digit whose true value is known.

- If the number doesn't have a decimal point, the last significant digit will be the last digit that is not zero. (Anything after that has been rounded off.)

  Example: If we round the number 234 567 to the thousands place, we would get 235 000. (Note that because the digit after the "4" in the thousands place was 5 or greater, so we had to "round up".) In the rounded-off number, the first three digits (the 2, 3, and 5) are the significant digits, and the last three digits (the zeroes at the end) are the insignificant digits.

- If the number has a decimal point, the last significant digit will be the last digit shown. (Anything rounded after the decimal point gets chopped off.)

  Example: If we round the number 11.223 344 to the hundredths place, it would become 11.22. When we rounded the number off, we "chopped off" the extra digits.

- If the number is in scientific notation, it has a decimal point. Therefore, the above rules tell us (correctly) that all of the digits before the "times" sign are significant.

In the following numbers, the significant figures have been underlined:

- <u>13</u> 000
- 0.0<u>275</u>
- 0.0<u>150</u>
- <u>6 804.305 00</u>
- <u>6.0</u> $\times 10^{23}$
- <u>3400.</u> (note the decimal point at the end)

Digits that are not underlined are insignificant. Notice that only zeroes can ever be insignificant.

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Mathematical Operations with Significant Figures

Addition & Subtraction

When adding or subtracting, calculate the total normally. Then identify the smallest place value where nothing is rounded. Round your answer to that place.

For example, consider the following problem.

problem:		“sig figs” equivalent:
123 000	± 1000	123 ???.????
0.0075	± 0.0001	0.0075
+ 1 650	± 10	+ 1 65?.????
124 650.0075	± 1010.0001	124 ???.????

(Check this digit for rounding)

In the first number (123 000), the hundreds, tens, and ones digit are zeros, presumably because the number was rounded to the nearest 1000. The second number (0.0075) is presumably rounded to the ten-thousandths place, and the number 1650 is presumably rounded to the tens place.

The first number has the largest uncertainty, so we need to round our answer to the thousands place to match, giving 125 000 ± 1 000.

A silly (but pedantically correct) example of addition with significant digits is:

$$100 + 37 = 100$$

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Multiplication and Division

When multiplying or dividing, calculate the result normally. Then count the total *number* of significant digits in the values that you used in the calculation. Round your answer so that it has the same number of significant digits as the value that had the *fewest*.

Consider the problem:

$$34.52 \times 1.4$$

The answer (without taking significant digits into account) is $34.52 \times 1.4 = 48.328$

The number 1.4 has the fewest significant digits (2). Remember that 1.4 really means 1.4 ± 0.1, which means the actual value, if we had more precision, could be anything between 1.3 and 1.5. Using "crank three times," the actual answer could therefore be anything between 34.52 × 1.3 = 44.876 and 34.52 × 1.5 = 51.780.

To get from the answer of 48.328 to the largest and smallest answers we would get from "crank three times," we would have to add or subtract approximately 3.5. (Notice that this agrees with the number we found previously for this same problem by propagating the relative error.) If the uncertainty is in the ones digit (greater than or equal to 1, but less than 10), this means that the ones digit is approximate, and everything beyond it is unknown. Therefore, using the rules of significant figures, we would report the number as 48.

In this problem, notice that the least significant term in the problem (1.4) had 2 significant digits, and the answer (48) also has 2 significant digits. This is where the rule comes from.

A silly (but pedantically correct) example of multiplication with significant digits is:

$$141 \times 1 = 100$$

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Mixed Operations

For mixed operations, keep all of the digits until you're finished (so round-off errors don't accumulate), but keep track of the last significant digit in each step by putting a line over it (even if it's not a zero). Once you have your final answer, round it to the correct number of significant digits. Don't forget to use the correct order of operations (PEMDAS)!

For example:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7\,\overline{1}44.8 + 2\overline{1}2.4 = 7\,\overline{3}57.2 = 7\,400$$

Note that in the above example, ***we kept all of the digits and didn't round until the end***. This is to avoid introducing small rounding errors at each step, which can add up to enough to change the final answer. Notice how, if we had rounded off the numbers at each step, we would have gotten the wrong answer:

$$137.4 \times 52 + 120 \times 1.77$$
$$(137.4 \times 52) + (120 \times 1.77)$$
$$7\,\overline{1}00 + 2\overline{1}0 = 7\,\overline{3}10 = 7\,300 \longleftarrow ☹$$

However, if we had done actual error propagation (remembering to add absolute errors for addition/subtraction and relative errors for multiplication/division), we would get the following:

$$137.4 \times 52 = 7144.8;\ \text{R.E.} = \frac{0.1}{137.4} + \frac{1}{52} = 0.01996$$
$$\text{partial answer} = 7144.8 \pm 142.6$$
$$120 \times 1.77 = 212.4;\ \text{R.E.} = \frac{1}{120} + \frac{0.01}{1.77} = 0.01398$$
$$\text{partial answer} = 212.4 \pm 2.97$$

The total absolute error is therefore 142.6 + 2.97 = 145.6

The best answer is therefore 7357.2 ± 145.6. *I.e.,* the actual value lies between approximately 7200 and 7500.

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What to Do When Rounding Doesn't Give the Correct Number of Significant Figures

If you have a different number of significant digits from what the rounding shows, you can place a line over the last significant digit, or you can place the whole number in scientific notation. Both of the following have four significant digits, and both are equivalent to writing 13 000 ± 10

- $13\,\bar{0}00$
- 1.300×10^{4}

When Not to Use Significant Figures

Significant figure rules only apply in situations where the numbers you are working with have a limited precision. This is usually the case when the numbers represent measurements. Exact numbers have infinite precision, and therefore have an infinite number of significant figures. Some examples of exact numbers are:

- Pure numbers, such as the ones you encounter in math class.
- Anything you can count. (*E.g.,* there are 24 people in the room. That means exactly 24 people, not 24.0 ± 0.1 people.)
- Whole-number exponents in formulas. (*E.g.,* the area of a circle is πr^2. The exponent "2" is a pure number.)
- Exact values. (*E.g.,* in the International System of Units, the speed of light is defined to be ***exactly*** $2.997\,924\,58\times10^{8}\ \frac{\text{m}}{\text{s}}$.)

You should also avoid significant figures any time the uncertainty is likely to be substantially different from what would be implied by the rules for significant figures, or any time you need to quantify the uncertainty more exactly.

Summary

Significant figures are a source of ongoing stress among physics students. To make matters simple, realize that few formulas in physics involve addition or subtraction, so you can usually just apply the rules for multiplication and division: look at each of the numbers you were given in the problem. Find the one that has the fewest significant figures, and round your final answer to the same number of significant figures.

If you have absolutely no clue what else to do, ***round to three significant figures and stop worrying***. You would have to measure quite carefully to have more than three significant figures in your original data, and three is usually enough significant figures to avoid unintended loss of precision, at least in a high school physics course. ☺

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Homework Problems

1. **(M)** For each of the following, Underline the significant figures in the number and Write the assumed uncertainty as ± the appropriate quantity.

   57 300 ± 100 ← Sample problem with correct answer.

   a. 13 500
   b. 26.0012
   c. 01902
   d. 0.000 000 025
   e. 320.
   f. 6.0×10^{-7}
   g. 150.00
   h. 10
   i. 0.005 3100

2. **(M)** Round off each of the following numbers as indicated and indicate the last significant digit if necessary.

   a. 13 500 to the nearest 1000
   b. 26.0012 to the nearest 0.1
   c. 1902 to the nearest 10
   d. 0.000 025 to the nearest 0.000 01
   e. 320. to the nearest 10
   f. 6.0×10^{-7} to the nearest 10^{-6}
   g. 150.00 to the nearest 100
   h. 10 to the nearest 100

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CP1 & honors (not AP®)

3. Solve the following math problems and round your answer to the appropriate number of significant figures.

   a. **(M)** $3\,521 \times 220$

   b. **(S)** $13\,580.160 \div 113$

   c. **(M)** $2.71828 + 22.4 - 8.31 - 62.4$

   d. **(A)** $23.5 + 0.87 \times 6.02 - 105$ (Remember PEMDAS!)

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