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Graphical Solutions & Linearization

Unit: Laboratory & Measurement

NGSS Standards/MA Curriculum Frameworks (2016): SP4, SP5

AP® Physics 1 Learning Objectines/Essential Knowledge (2024): 1.B, 2.A, 2.B, 2.D, 3.C

Mastery Objective(s): (Students will be able to…)

- Use a graph to calculate the relationship between two variables.

Success Criteria:

- Graph has the manipulated variable on the *x*-axis and the responding variable on the *y*-axis.
- Graph includes best-fit line that appears to minimize the total accumulated distance between the points and the line.
- Axes and best-fit line drawn with straightedge.
- Divisions on axes are evenly spaced.
- Slope of line determined correctly (rise/run).
- Slope used correctly in calculation of desired result.

Language Objectives:

- Explain why a best-fit line gives a better answer than calculating an average.
- Explain how the slope of the line relates to the desired quantity.

Tier 2 Vocabulary: plot, axes

Notes:

Most experiments in a high-school physics class involve finding a mathematical relationship between two quantities. While it is possible to simply measure each quantity once and calculate, or to measure several times and calculate the average, an approach that measures the relationship across a range of values will provide a better result.

A common way to achieve this is to manipulate equations and plot data such that the expected result is a straight line, and the experimental quantity is calculated by plotting a best-fit line and determining the slope. This means that you need to plot graphs ***accurately***, either on graph paper or using a computer or calculator. If you use graph paper:

- The data points need to be as close to their actual locations as you are capable of drawing.
- The best-fit line needs to be as close as you can practically get to its mathematically correct location.
- The best-fit line must be drawn with a straightedge.
- The slope needs to be calculated using the actual rise and run of points on the best-fit line.

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As mentioned in the previous section, a good rule of thumb for quantitative experiments is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Once you have your data points, arrange the equation into $y = mx + b$ form, such that the slope (or $^1/_{\text{slope}}$) is the quantity of interest. Then accurately plot your data and draw a best-fit line. The slope of this line will be the quantity of interest (or its reciprocal).

For example, suppose you wanted to calculate the spring constant of a spring by measuring the displacement caused by an applied force. You are given the following data:

Applied Force (N)	0.0	1.0	2.0	3.0	5.0
Displacement (m)	−0.01	0.05	0.16	0.20	0.34
Uncertainty (m)	± 0.06	± 0.06	± 0.06	± 0.06	± 0.06

The equation is $F_s = kx$, which is already in $y = mx + b$ form. However, we varied the force and measured the displacement, which means force is the manipulated variable (*x*-axis), and displacement is the responding variable (*y*-axis). Therefore, we need to rearrange the equation to:

$$\begin{array}{ccccc} y & = & m & x & +b \\ \downarrow & & \downarrow & \downarrow & \downarrow \\ x & = & \left(\frac{1}{k}\right) & F_s & +0 \end{array}$$

This means that if we plot a graph of all of our data points, a graph of F_s *vs.* x will have a slope of $\frac{1}{k}$.

You therefore need to:

1. Plot the data points, expressing the uncertainties as error bars.
2. Draw a best-fit line that passes through each error bar and minimizes the total accumulated distance away from each data point. (You can use linear regression, provided that the regression line actually passes through each error bar. If the line cannot pass through all of the error bars, you need to determine what the problem was with the outlier(s).) You may disregard a data point in your determination of the best-fit line *only* if you know *and can explain* the problem that caused it to be an outlier.

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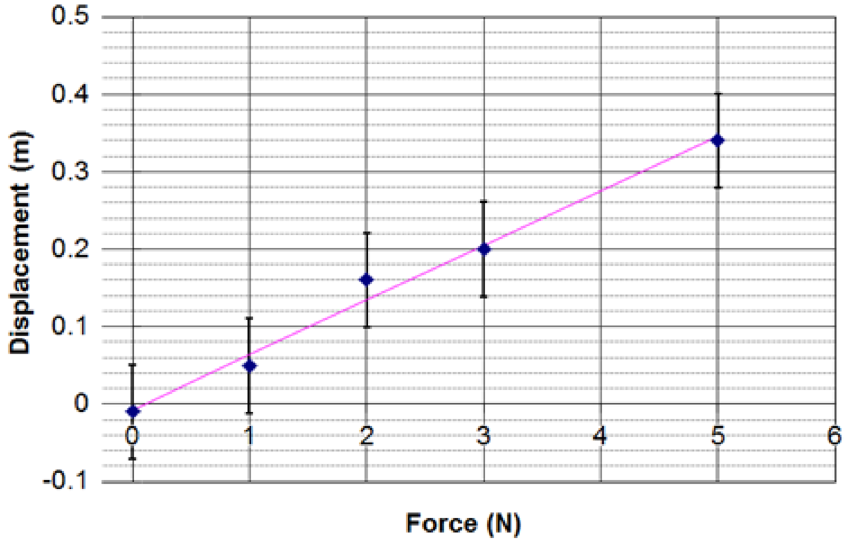
The plot looks like the following*:



We calculate the slope using the actual rise (Δy) and run (Δx) from the graph. The best-fit line goes through the points (0, 0) and (3.0, 0.21). From these points, we would calculate the slope as:

$$m = \frac{\Delta y}{\Delta x} = \frac{0.21 - 0}{3.0 - 0} = 0.07$$

Because the slope is $\frac{1}{k}$, the spring constant is the reciprocal of the slope of the above graph. $\frac{1}{0.07} = 14\,\frac{\text{N}}{\text{m}}$ (rounded to two significant figures).

* Note that graphs of Hooke's Law are almost always drawn with displacement on the abscissa (*x*-axis) and force on the ordinate (*y*-axis). The axes were reversed intentionally in this text for three reasons:

1. In most cases it is better to plot the responding variable on the ordinate, to show mathematically that the responding variable is a function of the manipulated variable, $y = f(x)$.
2. Plotting this way creates an example that shows what to do when the slope of the graph is the reciprocal of the quantity of interest.
3. To illustrate the fact that the graph is a valid representation of the data (and may therefore be used for calculations) regardless of which quantity is plotted on which axis.

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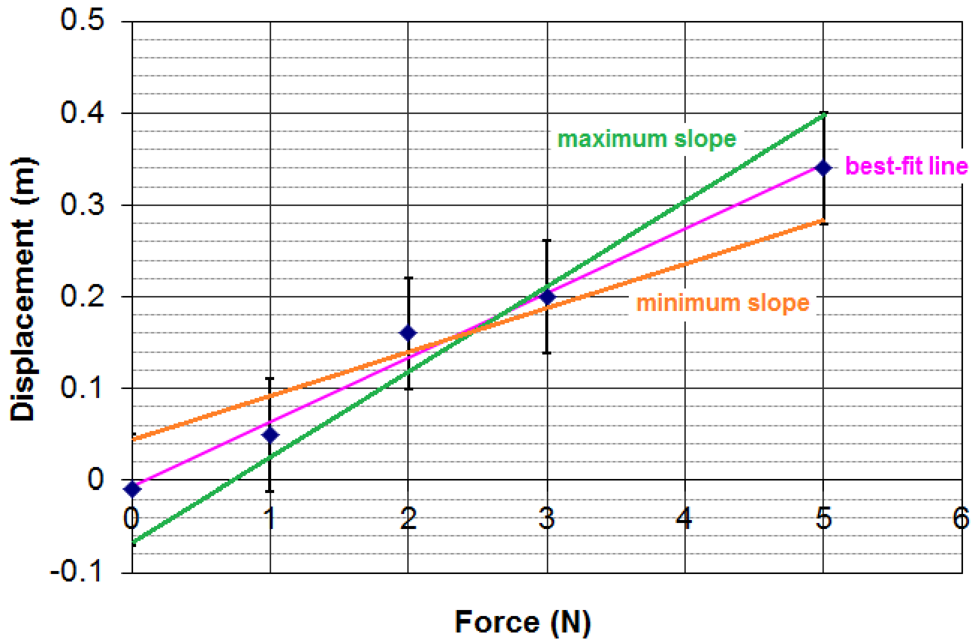
Uncertainty and Linearization

If you need to determine the uncertainty for a quantity that was calculated using the slope of a graph, you can draw "lines of worst fit" (which still need to go through the error bars) and then "crank three times" to find the maximum and minimum slope.



Note that calculating the uncertainty for quantities that are determined graphically is beyond the scope of this course.

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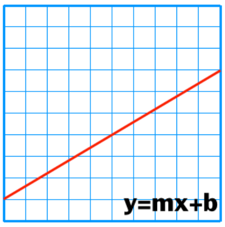
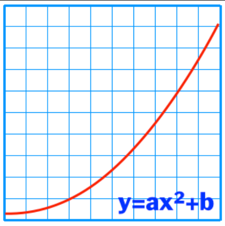
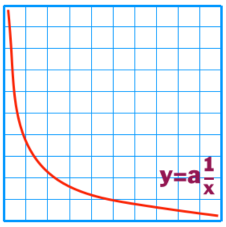
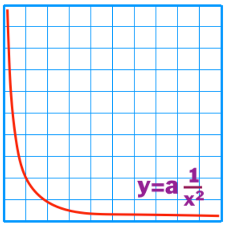
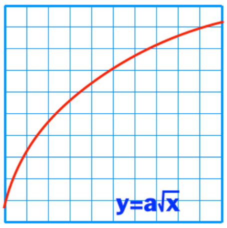
Recognizing Shapes of Graphs

Of course, not all graphs are linear. When you know the equation in advance, it is easy to rearrange the equation in order to linearize it. However, if you do not know the equation before looking at the data, it is useful to memorize the general shapes of these graphs so you can predict the type of relationship and the form of the equation.*

Plot of *y vs. x*	Equation	Linear Plot
	**Linear** $y = mx + b$ b = y-intercept	y vs. x slope = m
	**Power** $y = ax^2$ or $y = ax^2 + b$ b = minimum y-value	y vs. x^2 slope = a
	**Inverse** $y = \frac{a}{x}$ or $y = a \cdot \frac{1}{x}$ undefined (∞) at $x = 0$	y vs. $\frac{1}{x}$ slope = a
	**Inverse Square** $y = \frac{a}{x^2}$ or $y = a \cdot \frac{1}{x^2}$ undefined (∞) at $x = 0$	y vs. $\frac{1}{x^2}$ slope = a
	**Square Root** $y = a\sqrt{x}$	y vs. $\sqrt{x}$ slope = a

*Graphs by Tony Wayne. Used with permission.

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