Solving Word Problems Systematically

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): SP1, SP5

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): SP 2.A

Mastery Objective(s): (Students will be able to…)

- Assign (declare) variables in a word problem according to the conventions used in physics.
- Substitute values for variables in an equation.

Success Criteria:

- Variables match the quantities given and match the units.
- Quantities are substituted for the correct variables in the equation.

Language Objectives:

- Describe the quantities used in physics, list their variables, and explain why that particular variable might have been chosen for the quantity.

Tier 2 Vocabulary: equation, variable

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word	meaning	word	meaning	word	meaning
and, more than (but not "is more than")	+	percent ("per" + "cent")	÷ 100	**is** at least	≥
less than (but not "is less than")	−	change in *x*, difference in *x*	Δx	**is** more than	>
of	×	is	=	**is** at most	≤
per, out of	÷			**is** less than	<

Use this space for summary and/or additional notes:

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable	quantity	S.I. unit	variable
mass	kg	*m*	work	J	*W*
distance, length	m	*d* , *L*	power	W	*P**
height	m	*h*	pressure	Pa	*P**
area	m^2	*A*	momentum	N·s	*p**
acceleration	m/s^2	*a*	density	kg/m^3	*ρ**
volume	m^3	*V*	moles	mol	*n*
velocity (speed)	m/s	*v*	temperature	K	*T*
time	s	*t*	heat	J	*Q*
force	N	*F*	electric charge	C	*q* , *Q*

*Note the subtle differences between uppercase "*P*", lowercase "*p*", and the Greek letter *ρ* ("rho").

Any time you see a number in a word problem that has a unit that you recognize (such as one listed in this table), notice which quantity the unit is measuring, and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

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Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$$\rho^{*} = \frac{m}{V} \quad \text{and you're given:} \quad m = 12.3\text{ g} \quad \text{and} \quad V = 2.8\text{ cm}^3$$

simply substitute 12.3 g for *m*, and 2.8 cm^3 for *V*, giving:

$$\rho = \frac{12.3\text{ g}}{2.8\text{ cm}^3} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often safer to leave the units out when you substitute numbers for variables and then add them back in at the end:[†]

$$\rho = \frac{12.3}{2.8} = 4.4\,\tfrac{\text{g}}{\text{cm}^3}$$

[*] Physicists use the Greek letter ρ ("rho") for density. Note that the Greek letter ρ is different from the Roman letter "p".

[†] Many physics teachers disagree with this approach and insist on having students include the units with the number throughout the calculation. However, this can lead to confusion about which symbols are variables and which are units. For example, if a device applies a power of 150 W for a duration of 30 s and we wanted to find out the amount of work done, we would have:

$$P = \frac{W}{t}$$

$$150\text{ W} = \frac{W}{30\text{ s}} \quad vs. \quad 150 = \frac{W}{30}$$

In the left equation, the student would need to realize that the W on the left side is the unit "watts", and the *W* on the right side of the equation is the variable *W*, which stands for "work".

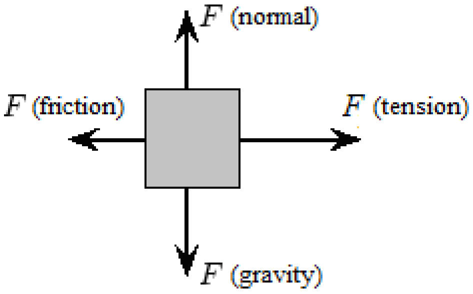
Use this space for summary and/or additional notes:

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

1. The force of gravity, pulling downward.
2. The "normal" force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

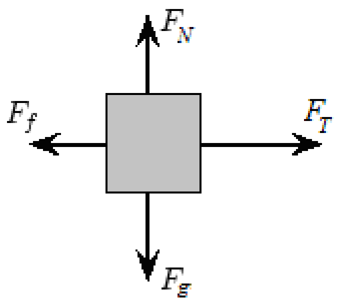
The variable for force is "*F*". There are four types of forces, which means "*F*" means four different things in this problem:



To make the diagram easier to read, we add subscripts to the variable "*F*". Note that in most cases, the subscript is the first letter of the word that describes the particular instance of the variable:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.

This results in the following free-body diagram:



We use these same subscripts in the equations that relate to the problem. For example:

$$F_g = mg \quad \text{and} \quad F_f = \mu F_N$$

Use this space for summary and/or additional notes:

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—***it needs to be smaller than the other letters and lowered slightly***. For example, when we write F_g, we add the subscript $_g$ (which stands for "gravity") to the variable F (force). ***Note that the subscript is part of the variable***; the variable is no longer F, but F_g.

An example is the following equation:

$$F_g = mg \qquad \leftarrow \qquad \text{right ☺}$$

It is important that the subscript $_g$ on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$Fg = mg \qquad \leftarrow \qquad \text{wrong! ☹}$$

$$F\cancel{g} = m\cancel{g}$$

$$F = m$$

A common use of subscripts is the subscript "o" to mean "initial". (Imagine that the word problem or "story problem" is shown as a video. When the slider is at the beginning of the video, the time is 0, and the values of all of the variables at that time are shown with a subscript of o.)

For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity "v_o*" where the subscript "o" means "at time zero", *i.e.,* at the beginning of the problem. The final velocity is simply "v" without the zero.

* pronounced "v-sub-zero", "v-zero" or "v-naught"

Use this space for summary and/or additional notes:

The Problem-Solving Process: "*GUESS*"

The following is an overview of the problem-solving process. The acronym "GUESS" may be helpful to remember it.

1. **Given**: Identify the given quantities in the problem, based on the units and any other information in the problem.
    - Assign the appropriate variables to those quantities.
2. **Unknown**: Identify the quantity that the question is asking for.
    - Assign the appropriate variable to the quantity.
3. **Equation**: Find an equation that contains the Unknown and one or more of the Given quantities.
    - The best choice is an equation in which *every quantity in the equation* is either the Unknown or one of the Givens.
    - If there is no equation in which every quantity is the unknown or one of the givens, choose the one that comes closest. However, *the equation must contain the unknown* or you won't be able to solve for it!
4. **Solve**: Use algebra to rearrange the equation to Solve it for the variable you're looking for. (Move all of the other quantities to the other side by "undoing PEMDAS.")* This process is explained in more detail in the previous section, *Solving Equations Symbolically*, starting on page 118.
5. **Substitute**: Replace the Given variables with their values and calculate the answer.
    - If you can't calculate the answer because you still need a variable, go back to step 2 above. The variable you need is your new unknown. Complete steps 2–5 above to find the value of that variable, then continue with the original equation.
6. Apply the appropriate unit(s) to the result.

* For CP1 physics, if students do not have strong algebra skills you may need to switch the order of steps 4 & 5, having students first substitute values into the equation, and then rearrange the equation when there is only one variable.

Use this space for summary and/or additional notes:

Sample Problem

A net force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object? (*mechanics/forces*)

1. **Given**: Identify the Given quantities in the problem and assign variables to them. We can use *Table C. Quantities, Variables and Units* on page 571 of your Physics Reference Tables:

   - 30 N uses the unit N (newtons). Newtons are used for force, and the variable for force is $\vec{F}$.
   - 1.5 kg uses the unit kg (kilograms). Kilograms are used for mass, and the variable for mass is *m*.

   A net force of 30 N ($\vec{F}$) acts on an object with a mass of 1.5 kg (m). What is the acceleration of the object?

2. **Unknown**: Identify the quantity that the question is asking for and assign a variable to it.

   - The unknown quantity is acceleration. From *Table C. Quantities, Variables and Units* on page 571, acceleration uses the variable $\vec{a}$, and the units $\frac{m}{s^2}$ (which we will need later for the answer).

   A net force of 30 N ($\vec{F}$) acts on an object with a mass of 1.5 kg (m). What is the acceleration ($\vec{a}$) of the object?

3. **Equation**: Find an equation that includes the Unknown and one or more of the Given quantities:

   $$\vec{F}_{net} = m\vec{a}$$

4. **Solve**: Use algebra ("undo PEMDAS") to rearrange the equation.

   We need to get $\vec{a}$ by itself. In the equation, *m* is attached to $\vec{a}$ by multiplication, so we need to get rid of *m* by ***undoing multiplication***, which means we ***divide*** by *m* on both sides.

   $$\frac{\vec{F}_{net}}{m} = \frac{\cancel{m}\vec{a}}{\cancel{m}}$$

   $$\frac{\vec{F}_{net}}{m} = \vec{a}$$

5. **Substitute**: Replace the Given quantities with their values and calculate the answer. (*Remember to add the units!*)

   $$\frac{\vec{F}_{net}}{m} = \vec{a} \rightarrow \frac{30}{1.5} = \vec{a} \rightarrow \boxed{20\,\tfrac{m}{s^2}} = \vec{a}$$

Use this space for summary and/or additional notes:

Homework Problems

To solve these problems, refer to your Physics Reference Tables starting on page 567. To make the equations easier to find, *the table and section of the table in your Physics Reference Tables where the equation can be found is given in parentheses*.

Note that this is probably one of the most frustrating assignments in this course. The process is unfamiliar, the problem set feels more like a scavenger hunt than a problem set, and the problems intentionally contain pesky details that you will encounter throughout the year that you will learn about here by struggling with them. Please be advised that this is meant to be a *productive* struggle!

For problems #1–3 below, ***identify the variables*** that correspond with the Given and Unknown quantities in the following problems. (*You do not need to find the equation or solve the problem.*)

1. **(M = Must Do)** What is the average velocity of a car that travels 90. m in 4.5 s? (*mechanics/kinematics*)

2. **(M = Must Do)** If a net force of 100. N acts on a mass of 5.0 kg, what is its acceleration? (*mechanics/forces*)

3. **(S = Should Do)** A 25 Ω resistor is placed in an electrical circuit with a voltage of 110 V. How much current flows through the resistor? (*electricity/circuits*)

For problems #4–6 below, ***identify the variables*** (as above) and ***find the equation*** that relates those variables. (*You do not need to rearrange the equation or solve the problem.*)

4. **(M)** What is the potential energy due to gravity of a 95 kg anvil that is about to fall off a 150 m cliff onto Wile E. Coyote's head?
   Note: "fall" means gravity is involved and will appear in the equation.
   (*mechanics/energy, work & power*)

Use this space for summary and/or additional notes:

5. **(S – honors & AP®; M – CP1)** If the momentum of a block is 18 N·s and its velocity is $3\,\frac{m}{s}$, what is the mass of the block?

   (*mechanics/momentum*)

honors & AP®

6. **(M – honors & AP®; A – CP1)** If the momentum of a block is *p* and its velocity is *v*, derive an expression for the mass of the block.

   (*mechanics/momentum*) (If you're not sure how to solve this, #5 is the same problem, but with numbers.

For the remaining problems (#7–20 below), use the *GUESS* method to ***identify the variables***, ***find the equation***, and ***solve*** the problems. (Answers are given so you can check your work; <u>*credit will be given only if all steps of GUESS are shown*</u>.)

7. **(M)** What is the <u>frequency</u> of a wave that is traveling at a velocity of <u>$300.\,\frac{m}{s}$</u> and has a wavelength of <u>10. m</u>?
   (*waves/waves*)

   Answer: 30. Hz

8. **(S)** What is the <u>energy of a photon</u> that has a frequency of <u>6×10^{15} Hz</u>?
   Note: the equation includes Planck's constant, which you need to look up.
   (*atomic, particle, and nuclear physics/energy*)

   Answer: 3.96×10^{-18} J

Use this space for summary and/or additional notes:

9. **(S)** A piston with an area of $2.0\,\text{m}^2$ is compressed by a force of 10 000 N. What is the pressure applied by the piston?
(*fluids/pressure*)

Answer: 5 000 Pa

honors & AP®

10. **(M – honors & AP®; A – CP1)** Derive an expression for the acceleration (a) of a car whose velocity changes from v_o to v in time t.
(*If you are not sure how to do this problem, do #11 below and use the steps to guide your algebra.*)
(*mechanics/kinematics*)

Answer: $a = \frac{v - v_o}{t}$

11. **(M)** What is the acceleration of a car whose velocity changes from $60.\,\frac{\text{m}}{\text{s}}$ to $80.\,\frac{\text{m}}{\text{s}}$ over a period of 5.0 s?
Hint: v_o is the initial velocity and v is the final velocity.
(*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #10 above as a starting point if you have already solved that problem.*)
(*mechanics/kinematics*)

Answer: $4.0\,\frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes:

12. **(S)** If the normal force on an object is 100. N and the coëfficient of kinetic friction between the object and the surface it is sliding on is 0.35, what is the force of friction on the object as it slides along the surface?
Note: the coefficient of kinetic friction is a material-specific constant whose value is given in the problem.
(*mechanics/forces*)

Answer: 35 N

13. **(M)** A 1200 W hair dryer is plugged into a electrical circuit with a voltage of 110 V. How much electric current flows through the hair dryer?
(*electricity/circuits*)

Answer: 10.9 A

honors & AP®

14. **(S – honors & AP®; A – CP1)** A car has mass *m* and kinetic energy *K*. Derive an expression for its velocity (*v*).
(*If you are not sure how to do this problem, do #15 below and use the steps to guide your algebra.*)
(*mechanics/energy*)

Answer: $v = \sqrt{\frac{2K}{m}}$

15. **(S)** A car has a mass of 1 200 kg and kinetic energy of 240 000 J. What is its velocity?
(<u>*You must start with the equations in your* Physics Reference Tables and show all of the steps of GUESS.</u> *You may only use the answer to question #14 above as a starting point if you have already solved that problem.*)
(*mechanics/energy*)

Answer: $20.\,\frac{m}{s}$

Use this space for summary and/or additional notes:

16. **(S)** What is the velocity of a photon (wave of light) as it passes through a block of clear plastic that has an index of refraction of 1.40?
Hint: The index of refraction is a material-specific constant whose value is given in the problem.
(*waves/reflection & refraction*)

Answer: $2.14\times10^{8}\ \frac{\text{m}}{\text{s}}$

17. **(M)** If a pressure of 100 000 Pa is applied to a gas and the volume decreases by $0.05\,\text{m}^3$, how much work was done on the gas?
Note: ΔV is two symbols, but it is a single variable that represents the change in volume. Pay attention to whether ΔV is positive or negative.
(*heat/thermodynamics*)

Answer: 5 000 J

honors & AP®

18. **(S – honors & AP®; A – CP1)** If the distance from a mirror to an object is s_o and the distance from the mirror to the image is s_i, derive an expression for the distance from the lens to the focus (*f*).
(*If you are not sure how to do this problem, do #19 below and use the steps to guide your algebra.*)
(*waves/mirrors & lenses*)

Answer: $f=\frac{s_i s_o}{s_i+s_o}$

Use this space for summary and/or additional notes:

19. **(S)** If the distance from a mirror to an object is 0.8 m and the distance from the mirror to the image is 0.6 m, what is the distance from the mirror to the focus?

    (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #18 above as a starting point if you have already solved that problem.*)

    (*waves/mirrors & lenses*)

    Answer: 0.343 m

20. **(S)** What is the momentum of a photon that has a wavelength of 400 nm?
    Hint: you will need to convert nanometers to meters.
    (*atomic, Particle, and Nuclear physics/energy*)

    Answer: $1.65\times10^{-27}\ \text{N}\cdot\text{s}$

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