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Right-Angle Trigonometry

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): SP2.B

Mastery Objective(s): (Students will be able to…)

- Use the Pythagorean theorem to find one side of a right triangle, given the other two sides.
- Use the trigonometry functions sine, cosine and tangent to find one side of a right triangle, given one of the non-right angles and one other side.
- Use the inverse trigonometry functions arcsine ($\sin^{-1}$), arccosine ($\cos^{-1}$), and arctangent ($\tan^{-1}$) to find one of the non-right angles of a right triangle, given any two sides.

Success Criteria:

- Sides and angles are correctly identified (opposite, adjacent, hypotenuse).
  - Correct function/equation is chosen based on the relationship between the sides and angles.

Language Objectives:

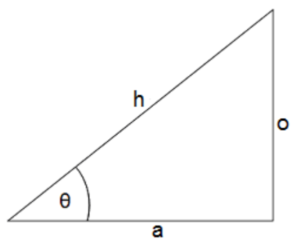
- Describe the relationships between the sides and angles of a right triangle.

Tier 2 Vocabulary: opposite, adjacent

Notes:

The word trigonometry comes from "trigon*" = "triangle" and "ometry" = "measurement", and is the study of relationships among the sides and angles of triangles.

If we have a right triangle, such as the one shown to the right:



- side "h" (the longest side, opposite the right angle) is the ***hypotenuse***.
- side "o" is the side of the triangle that is ***opposite*** (across from) angle θ.
- side "a" is the side of the triangle that is ***adjacent*** to (connected to) angle θ (and is not the hypotenuse).

*"trigon" is another word for a 3-sided polygon (triangle), just as "octagon" is an 8-sided polygon.

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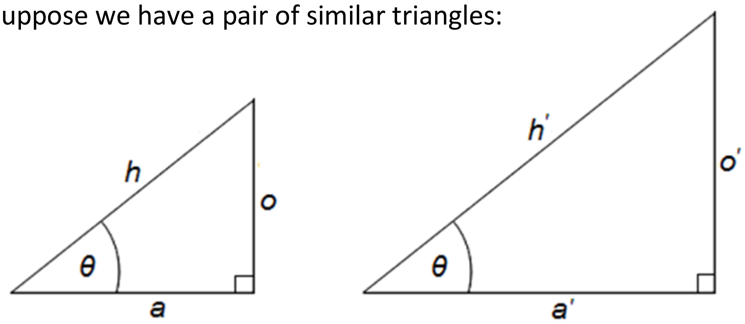
Now, suppose we have a pair of similar triangles:



Because the triangles are similar, the corresponding angles and the ratios of corresponding pairs of sides must be equal. For example, the ratio of the opposite side to the hypotenuse would be $\frac{o}{h}=\frac{o'}{h'}$. ***This ratio must be the same for every triangle that is similar to the ones above***, *i.e.,* ***for every right triangle that has an angle equal to θ***. This means that if we know the angle, θ, then we know the ratio of the opposite side to the hypotenuse.

We define this ratio as the sine of the angle, *i.e.,* $\text{sine}\,(\theta)=\sin\theta=\frac{o}{h}=\frac{o'}{h'}$.

We define similar quantities for ratios of other sides:

$$\text{cosine}\,(\theta) = \cos\theta = \frac{a}{h} = \frac{a'}{h'}$$

$$\text{tangent}\,(\theta) = \tan\theta = \frac{o}{a} = \frac{o'}{a'} = \frac{\sin\theta}{\cos\theta}$$

We can create a table of these ratios (sines, cosines and tangents) for different values of the angle θ, as shown in *Table II. Values of Trigonometric Functions* on page 574 of your Physics Reference Tables. The **sin**, **cos** and **tan** buttons on your calculator calculate this ratio for any value of θ. (Just make sure your calculator is correctly set for degrees or radians, depending on how θ is expressed.)*

There are a lot of stupid mnemonics for remembering which sides are involved in which functions. (You may been taught SOH CAH TOA.) My favorite of these is "**O**h **h**eck, **a**nother **h**our **o**f **a**lgebra!"

* In physics, problems that use Cartesian coördinates use degrees, and problems involving rotation (which is studied in AP® Physics 1, but not the CP1 or honors course) use polar coördinates and radians. This means that if you are taking CP1 or honors Physics 1, angles will always be expressed in degrees. If you are taking AP® Physics 1, you will need to use degrees for some problems and radians for others.

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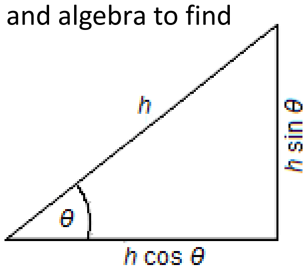
The most common use of trigonometry functions in physics is to decompose a vector into its components in the x- and y-directions. If we know the angle of inclination of the vector quantity, we can use trigonometry and algebra to find the components of the vector in the x- and y-directions:

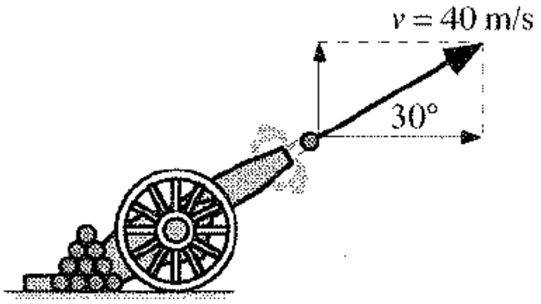
$$\cos\theta = \frac{a}{h} \rightarrow h\cdot\cos\theta = \frac{a}{h}\cdot h \rightarrow \boxed{a = h\cos\theta}$$

$$\sin\theta = \frac{o}{h} \rightarrow h\cdot\sin\theta = \frac{o}{h}\cdot h \rightarrow \boxed{o = h\sin\theta}$$



Memorize these relationships! It will save you a lot of time throughout the rest of the year.

For example, consider the following situation:



The horizontal velocity of the cannon ball is:

$$v_x = h\cos\theta = (40\,\tfrac{m}{s})\cos(30°) = (40)(0.866) = 34.6\,\tfrac{m}{s}$$

The initial vertical velocity of the cannon ball is:

$$v_{o,y} = h\sin\theta = (40\,\tfrac{m}{s})\sin(30°) = (40)(0.5) = 20\,\tfrac{m}{s}$$

You can look up the sine and cosine of 30° on a trigonometry table similar to the one in *Table II. Values of Trigonometric Functions* on page 574 of your Physics Reference Tables. You can, of course, also use the sin, cos and tan functions on your calculator.

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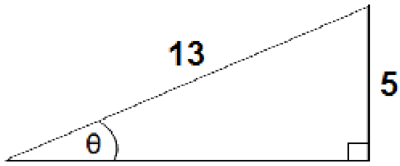
Finding Angles (Inverse Trigonometry Functions)

If you know the sides of a triangle and you need to find an angle, you can use the inverse of a trigonometric function. For example, suppose we had the following triangle:



We don't know what angle θ is, but we know that $\sin\theta = \frac{5}{13} = 0.385$.

If we look for a number that is close to 0.385 in the sine column of *Table II. Values of Trigonometric Functions* on page 574 of your Physics Reference Tables, we see that 0.385 would be somewhere between 22° and 23°, a little closer to 23°. (By inspection, we might guess 22.6° or 22.7°.)*

To perform the same function on a calculator, we use the inverse of the sine function (which means to go from the sine of an angle to the angle itself, instead of the other way around). The inverse sine (the proper name of the function is actually "arcsine") is usually labeled $\sin^{-1}$ on calculators. Doing this, we see that:

$$\theta = \sin^{-1}(0.385) = 22.64^\circ$$

which is between 22.6° and 22.7°, as expected.

Summary

- If you know two sides of a right triangle, a and b, you can find the third side from the Pythagorean theorem: $c^2 = a^2 + b^2$
- If you know one of the acute angles, θ, of a right triangle, the other acute angle is $90^\circ - \theta$.
- If you know one side of a right triangle and one acute angle (*e.g.,* a problem involving a force or velocity at an angle), you can find the remaining sides using sine, cosine, or tangent. (Especially remember $x = h\cos\theta$ and $y = h\sin\theta$.)
- If you know two sides of a right triangle and you need an angle, use one of the inverse trigonometric functions, *i.e.,* $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

* Although it may be tempting to just teach students to use the inverse trig functions on their calculators, they will gain a more intuitive understanding of what an inverse trig function is if they start with trig tables.

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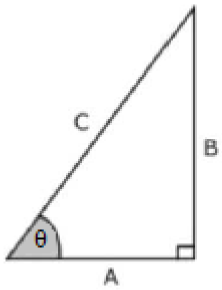
Homework Problems

Questions 1–5 are based on the following right triangle, with sides *A*, *B*, and *C*, and angle θ between *A* and *C*.



Note that the drawing is not to scale, and that angle *x* and the lengths of *A*, *B* and *C* will be different for each problem.

Some problems may also require use of the fact that the angles of a triangle add up to 180◦.

1. **(S)** If $A = 5$ and $C = 13$, what is *B*?

2. **(M)** If $A = 5$ and $C = 13$, what is $\sin\theta$?

3. **(M)** If $C = 20$ and $\theta = 50°$, what are *A* and *B*?

4. **(M)** If $A = 100$ and $C = 150$, what is θ?

5. **(S)** If $B = 100$ and $C = 150$, what is θ?

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6. **(M)** You are a golfer, and your ball is in a sand trap with a hill next to it. You need to hit your ball so that it goes over the hill to the green. If your ball is 10. m away from the side of the hill and the hill is 2.5 m high, what is the minimum angle above the horizontal that you need to hit the ball in order to just get it over the hill? (*Hint: draw a sketch.*)

7. **(M)** If a force of 80 N is applied at an angle of 40° above the horizontal, how much of that force is applied in the horizontal direction?

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