AP®

Degrees, Radians and Revolutions

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): SP2.A

Knowledge/Understanding:

- Express angles and arc length in degrees, radians, and full revolutions.

Skills:

- Convert between degrees, radians and revolutions.

Language Objectives:

- Understand and correctly use the terms "degree," "radian," and "revolution".

Tier 2 Vocabulary: degree, revolution

Notes:

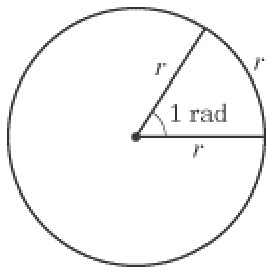
degree: an angle equal to $^{1}\!/_{360}$ of a full circle. A full circle is therefore 360°.

revolution: a rotation of exactly one full circle (360°) by an object.

radian: the angle that results in an arc length equal to the radius of a circle. *I.e.*, one "radius" of the way around the circle. Because the distance all the way around the circle is 2π times the radius, a full circle (or one rotation) is therefore 2π radians.

This means that $1\,\text{radian} = \frac{1}{2\pi}$ of a circle $= \left(\frac{1}{2\pi}\right)(360°) \approx 57.3°$

We are used to measuring angles in degrees. However, trigonometry functions are often more convenient if we express the angle in radians:



This is often convenient because if we express the angle in radians, the angle is equal to the arc length (distance traveled around the circle) times the radius, which makes much easier to switch back and forth between the two quantities.

Note that radians are a *dimensionless* unit, because the unit for the radius is the same as the unit for the arc length, and they cancel.

Use this space for summary and/or additional notes:

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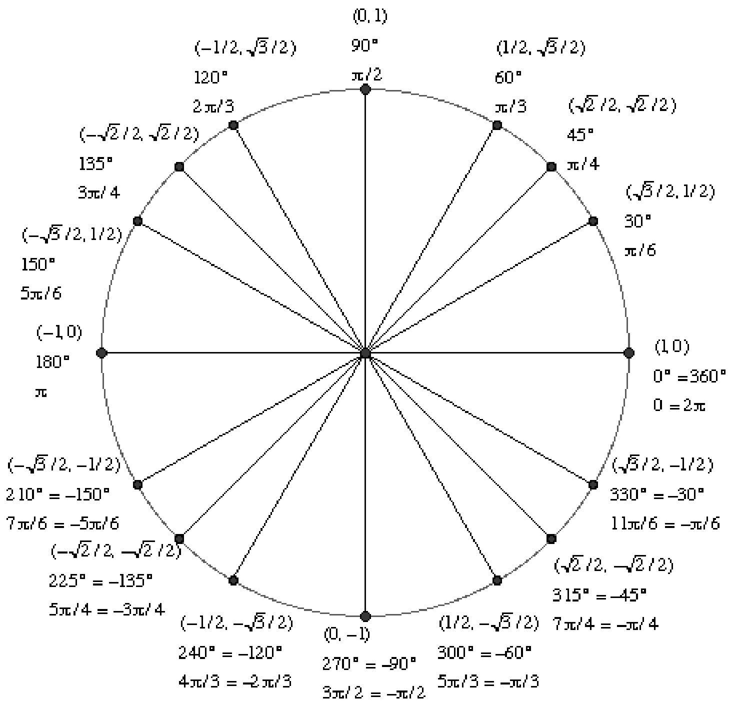
On the following unit circle (a circle with a radius of 1), several of the key angles around the circle are marked in radians, degrees, and the (*x,y*) coördinates of the corresponding point around the circle.



In each case, the angle in radians is equal to the distance traveled around the circle, starting from the point (1,0).

It is particularly useful to memorize the following:

Degrees	0°	90°	180°	270°	360°
Rotations	0	¼	½	¾	1
Radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

In general, you can convert between degrees and radians using the conversion factor $360° = 2\pi \text{ rad}$. For example, to convert 225° to radians, we would do:

$$\frac{225°}{1} \times \frac{2\pi}{360°} = 1.25\pi \text{ radians}$$

Note that because a radian is arc length divided by radius (distance divided by distance), it is a dimensionless quantity, *i.e.,* a quantity that has no unit. This is convenient because it means you never have to convert radians from one unit to another.

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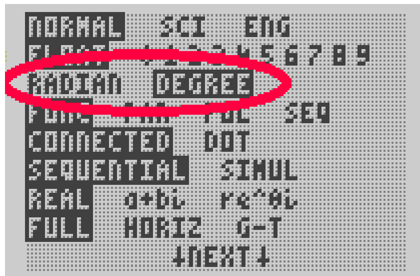
Precalculus classes often emphasize learning to convert between degrees and radians. However, in practice, these conversions are rarely, if ever necessary. Expressing angles in radians is useful in rotational problems in physics because it combines all of the quantities that depend on radius into a single variable, and avoids the need to use degrees at all. If a conversion is necessary,

In physics, you will usually use degrees for linear (Cartesian) problems, and radians for rotational problems. For this reason, when using trigonometry functions it is important to make sure your calculator mode is set correctly for degrees or radians, as appropriate to each problem:

TI-30 scientific calculator



TI-83 or later graphing calculator

If you switch your calculator between degrees and radians, don't forget that this will affect math class as well as physics!

Use this space for summary and/or additional notes: