AP®

Polar, Cylindrical & Spherical Coördinates

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): SP2.A

Knowledge/Understanding:

- Express a position in Cartesian, polar, cylindrical, or spherical coördinates.

Skills:

- Convert between Cartesian coördinates and polar, cylindrical and/or spherical coördinates.

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Tier 2 Vocabulary: polar

Notes:

In your math classes so far, you have expressed the location of a point using Cartesian coördinates—either (x, y) in two dimensions or (x, y, z) in three dimensions.

Cartesian coördinates: (or rectangular coördinates): a two- or three-dimensional coordinate system that specifies locations by separate distances from each of two or three axes (lines). These axes are labeled x, y, and z, and a point is specified using its distance from each axis, in the form (x, y) or (x, y, z).

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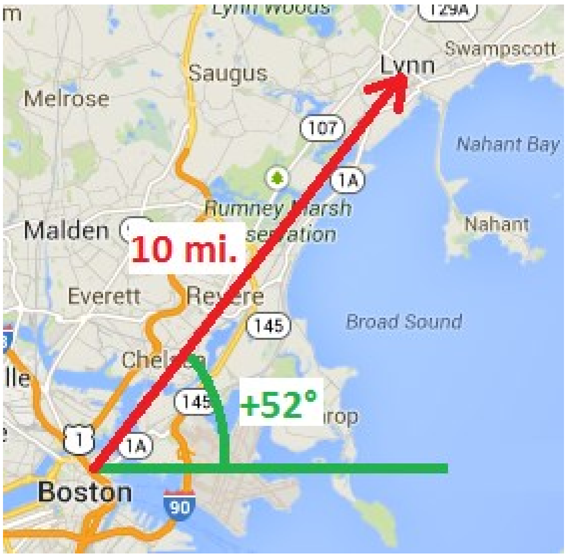
polar coördinates: a two-dimensional coördinate system that specifies locations by their distance from the origin (radius) and angle from some reference direction. The radius is *r*, and the angle is θ (the Greek letter "theta"). A point is specified using the distance and angle, in the form (r, θ).

For example, if we say that the city of Lynn, Massachusetts is 10 miles from Boston, at heading of 52° north of due east, we are using polar coördinates:



(Note: cardinal or "compass" direction is traditionally specified with North at 0° and 360°, and clockwise as the positive direction, meaning that East is 90°, South is 180°, West is 270°. This means that the compass heading from Boston to Lynn would be 38° to the East of true North. However, in this class we will specify angles as mathematicians do, with 0° indicating the direction of the positive *x*-axis.)

cylindrical coördinates: a three-dimensional coördinate system that specifies locations by distance from the origin (radius), angle from some reference direction, and height above the origin. The radius is *r*, the angle is θ, and the height is *z*. A point is specified using the distance and angle, and height in the form (r, θ, z).

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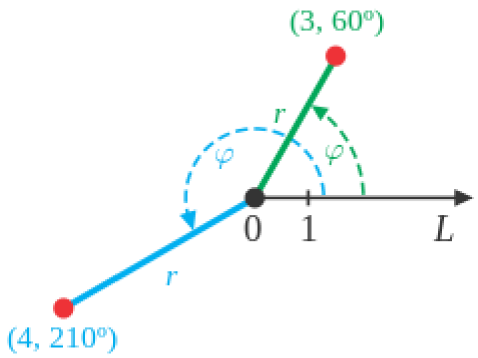
spherical coördinates: a three-dimensional coördinate system that specifies locations by distance from the origin (radius), and two separate angles, one from some horizontal reference direction and the other from some vertical reference direction. The radius is labeled *r*, the horizontal angle is θ, and the vertical angle is ϕ (the Greek letter "phi"). A point is specified using the distance and angle, and height in the form (r, θ, ϕ).

When we specify a point on the Earth using longitude and latitude, we are using spherical coördinates. The distance is assumed to be the radius of the Earth (because the interesting points are on the surface*), the longitude is θ, and the latitude is ϕ. (Note, however, that latitude on the Earth is measured up from the equator. In physics, we generally use the convention that ϕ = 0° is straight upward, meaning ϕ will indicate the angle *downward* from the "North pole".)

In AP® Physics 1, the problems we will see are one- or two-dimensional. For each problem, we will use the simplest coördinate system that applies to the problem: Cartesian (x, y) coördinates for linear problems and polar (r, θ) coördinates for problems that involve rotation.

Note that while mathematicians almost always prefer to express angles in radians, physicists typically usually use degrees for linear problems and radians for rotational problems.

The following example shows the locations of the points (3, 60°) and (4, 210°) using polar coördinates:



* The "surface" of the Earth is generally taken to mean the surface of its solid & liquid parts. The radius of the Earth at this surface is approximately 6.38×10^{6} m. However, the entire Earth also includes its atmosphere, which extends between 10^{4} and 10^{7} m above the surface.

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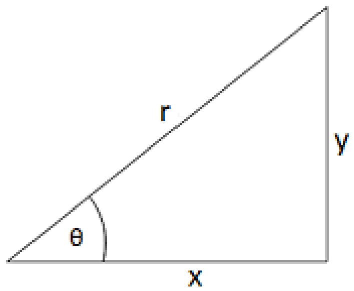
Converting Between Cartesian and Polar Coördinates

If vectors make sense to you, you can simply think of polar coördinates as the magnitude (r) and direction (θ) of a vector.

Converting from Cartesian to Polar Coördinates

If you know the *x*- and *y*-coördinates of a point, the radius (r) is simply the distance from the origin to the point. You can calculate r from x and y using the distance formula:

$$r=\sqrt{x^2+y^2}$$



The angle comes from trigonometry:

$\tan\theta=\frac{y}{x}$, which means $\theta=\tan^{-1}\left(\frac{y}{x}\right)$

Sample Problem:

Q: Convert the point (5,12) to polar coördinates.

A: $r=\sqrt{x^2+y^2}$

$r=\sqrt{5^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$

$\theta=\tan^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{12}{5}\right)=\tan^{-1}(2.4)=67.4°=1.18\text{ rad}$

(13, 67.4°) or (13, 1.18 rad)

Converting from Polar to Cartesian Coördinates

As we saw in our review of trigonometry, if you know r and θ, then $x=r\cos\theta$ and $y=r\sin\theta$.

Sample Problem:

Q: Convert the point (8, 25°) to Cartesian coördinates.

A: $x=8\cos(25°)=(8)(0.906)=7.25$

$y=8\sin(25°)=(8)(0.423)=3.38$

(7.25, 3.38)

In practice, you will rarely need to convert between the two coördinate systems. The reason for using polar coördinates in a rotating system is because the quantities of interest are based on the rotational angle and the distance from the center of rotation. Using polar coördinates for these problems *avoids* the need to use trigonometry to convert between systems.

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