Motion Graphs*

Unit: Kinematics (Motion) in One Dimension

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-10(MA)

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 1.3.A.4, 1.3.A.4.i, 1.3.A.4.ii, 1.3.A.4.iii, 1.3.A.4.iv, 1.3.A.4.v

Mastery Objective(s): (Students will be able to…)

- Determine velocity, position and displacement from a position *vs.* time graph.
- Determine velocity, acceleration and displacement from a velocity *vs.* time graph.

Success Criteria:

- The correct aspect of the graph (slope or area) is used in the calculation.
- The magnitude (amount) and direction (sign, *i.e.,* + or −) is correct.

Language Objectives:

- Recall terms relating to graphs from algebra 1, such as "rise," "run," and "slope" and relate them to physics phenomena.

Tier 2 Vocabulary: position, velocity, acceleration, direction

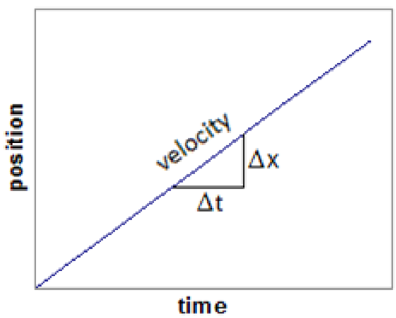
Lab Activities & Demonstrations:

- Have one student plot a position *vs.* time graph and have another student act it out.

Notes:

Position *vs.* Time Graphs

Suppose you were to plot a graph of position (x) *vs.* time (t) for an object that is moving at a constant velocity.



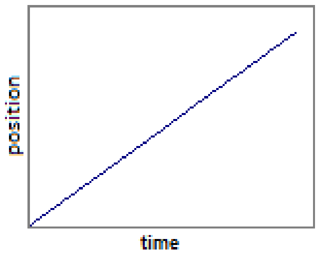
Note that $\frac{\Delta x}{\Delta t}$ is the slope of the graph. Because $\frac{\Delta x}{\Delta t} = v$, this means that the *slope* of a graph of position *vs.* time is equal to the velocity.

* Most physics texts present motion graphs before Newton's equations of motion. In this text, the order has been reversed because many students are more comfortable with equations than with graphs. This allows students to use a concept that is easier for them to help them understand one that is more challenging.

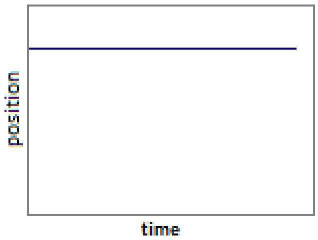
Use this space for summary and/or additional notes:

In fact, on any graph, the quantity you get when you divide the quantity on the *x*-axis by the quantity on the *y*-axis is, by definition, the slope. *I.e.,* the slope is $\frac{\Delta y}{\Delta x}$, which means the physics quantity defined by $\frac{\Delta y\text{-axis}}{\Delta x\text{-axis}}$ will always be the slope.

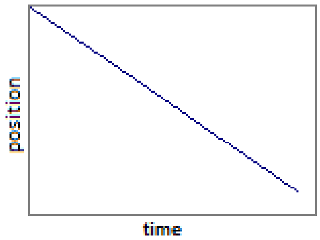
Recall that velocity is a vector, which means it can be positive, negative, or zero.



positive velocity (moving in the positive direction)

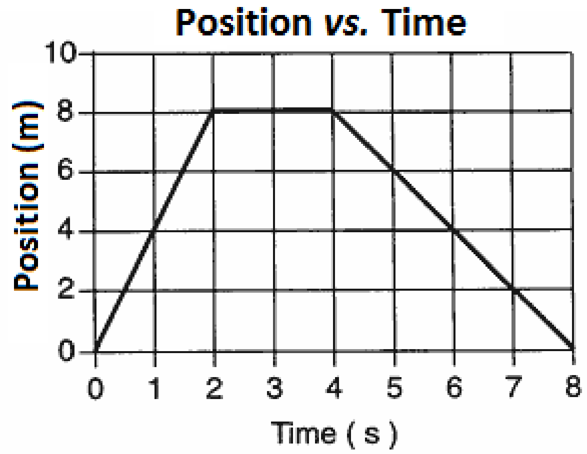


velocity = zero (not moving)



negative velocity (moving in the negative direction)

On the graph below, the velocity is $+4\,\frac{\text{m}}{\text{s}}$ from 0 s to 2 s, zero from 2 s to 4 s, and $-2\,\frac{\text{m}}{\text{s}}$ from 4 s to 8 s.



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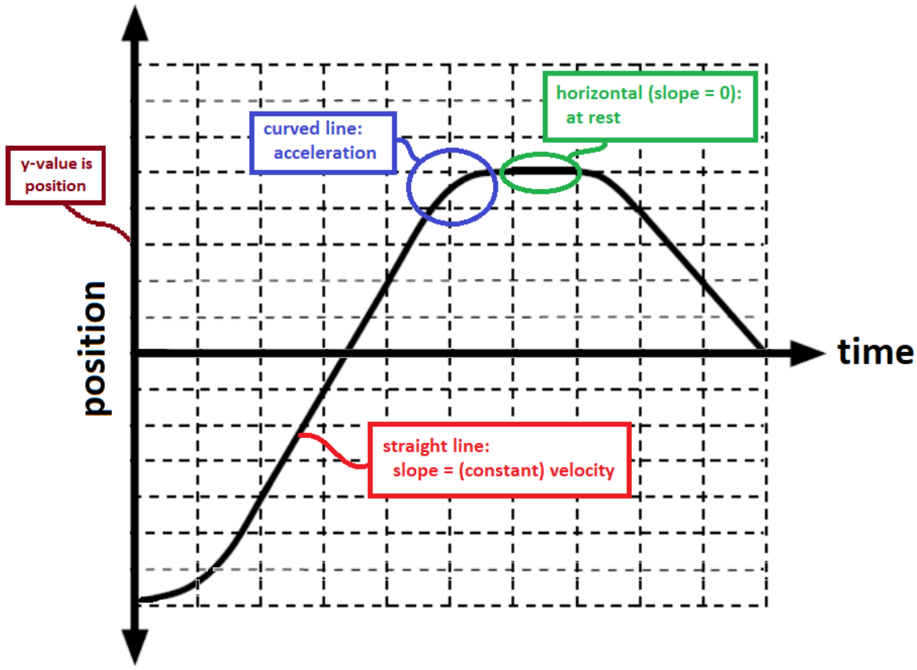
Features of Position *vs.* Time Graphs

The following diagrams show important features of position *vs.* time and velocity *vs.* time graphs.



On a position *vs.* time graph, note the following:

- The *y*-value is the position (location) of the object.
- A straight line indicates constant velocity.
- A curved line indicates acceleration.
- A horizontal line indicates a velocity of zero. (The object is at rest.)
- The slope of the graph is the velocity. A positive slope indicates positive velocity (moving in the positive direction). A negative slope indicates negative velocity (moving in the negative direction).

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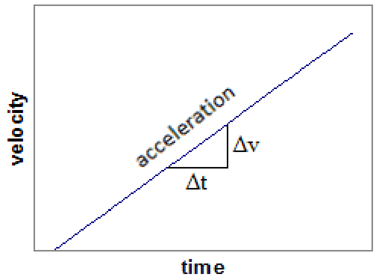
Velocity *vs.* Time Graphs

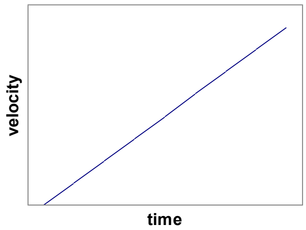
Suppose now that you were to plot a graph of *velocity* *vs.* time.

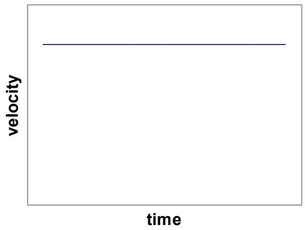


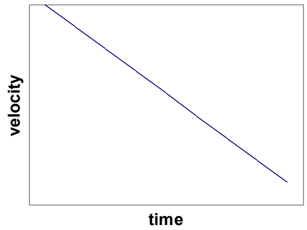
$\frac{\Delta v}{\Delta t}$ is the slope of a graph of velocity (v) *vs.* time (t). Because $\frac{\Delta v}{\Delta t} = a$, this means that acceleration is the *slope* of a graph of velocity *vs.* time.

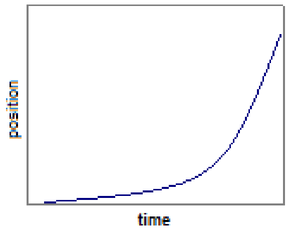
Note the relationship between velocity-time graphs and the corresponding position-time graphs.

positive acceleration | acceleration = zero | negative acceleration



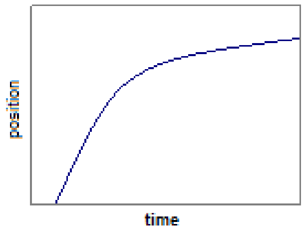




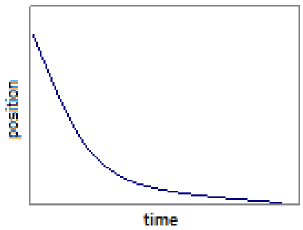


positive velocity, increasing speed

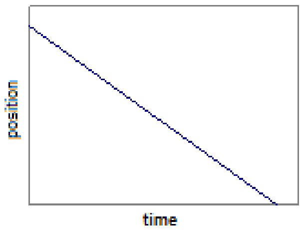
constant positive velocity



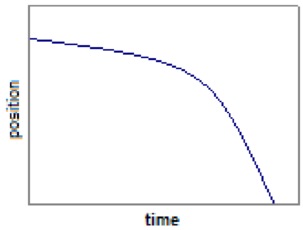
positive velocity, decreasing speed



negative velocity, decreasing speed



constant negative velocity

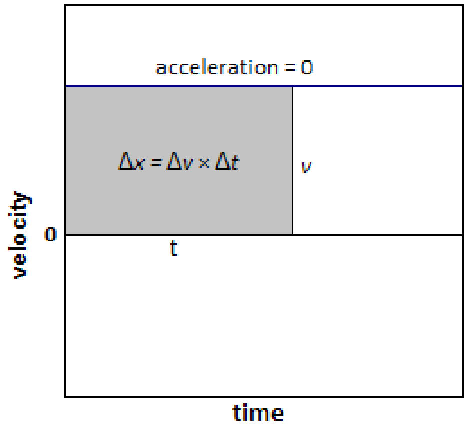


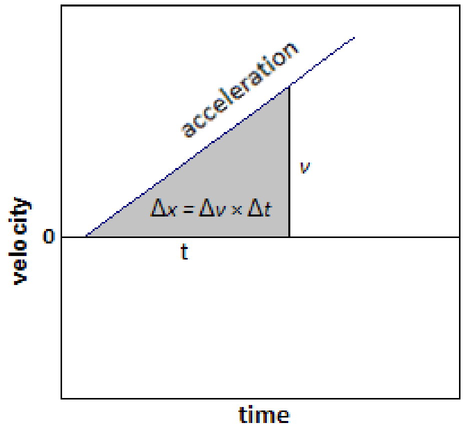
negative velocity, increasing speed

concave up | linear | concave down

Use this space for summary and/or additional notes:

Note also that $v_{ave.}t$ is the area under the graph (*i.e.,* the area between the curve and the x-axis) of velocity (v) *vs.* time (t). From the equations of motion, we know that $(v_{ave.})(t) = d$. Therefore, the ***area*** between a graph of velocity *vs.* time and the *x*-axis is the displacement. Note that this works both for constant velocity (the graph on the left) and changing velocity (as shown in the graph on the right).



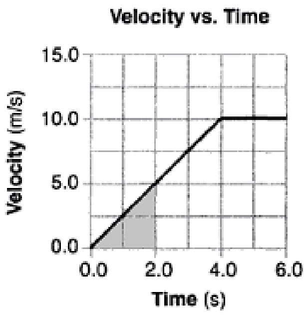


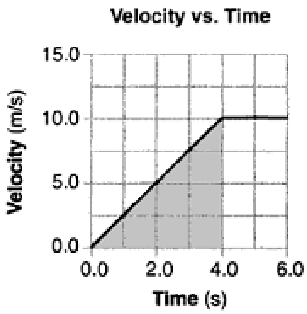
In fact, on any graph, the quantity you get when you multiply the quantities on the *x*- and *y*-axes is, by definition, the area under the graph.

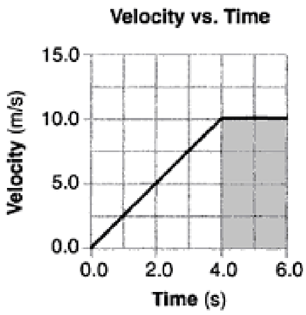
Use this space for summary and/or additional notes:

In the graphs below, between 0 s and 4 s, the slope of the graph is 2.5, which means the object is accelerating at a rate of $+2.5\,\frac{\text{m}}{\text{s}^2}$.

Between 4 s and 6 s the slope is zero, which indicates that object is moving at a constant velocity (of $+10\,\frac{\text{m}}{\text{s}}$) and the acceleration is zero.







Between 0 and 2 s	Between 0 and 4 s	Between 4 s and 6 s
$a = 2.5\,\frac{\text{m}}{\text{s}^2}$	$a = 2.5\,\frac{\text{m}}{\text{s}^2}$	$a = 0$
$\text{area} = \frac{1}{2}bh = \frac{1}{2}(2)(5) = 5\,\text{m}$	$\text{area} = \frac{1}{2}bh = \frac{1}{2}(4)(10) = 20\,\text{m}$	$\text{area} = bh = (2)(10) = 20\,\text{m}$

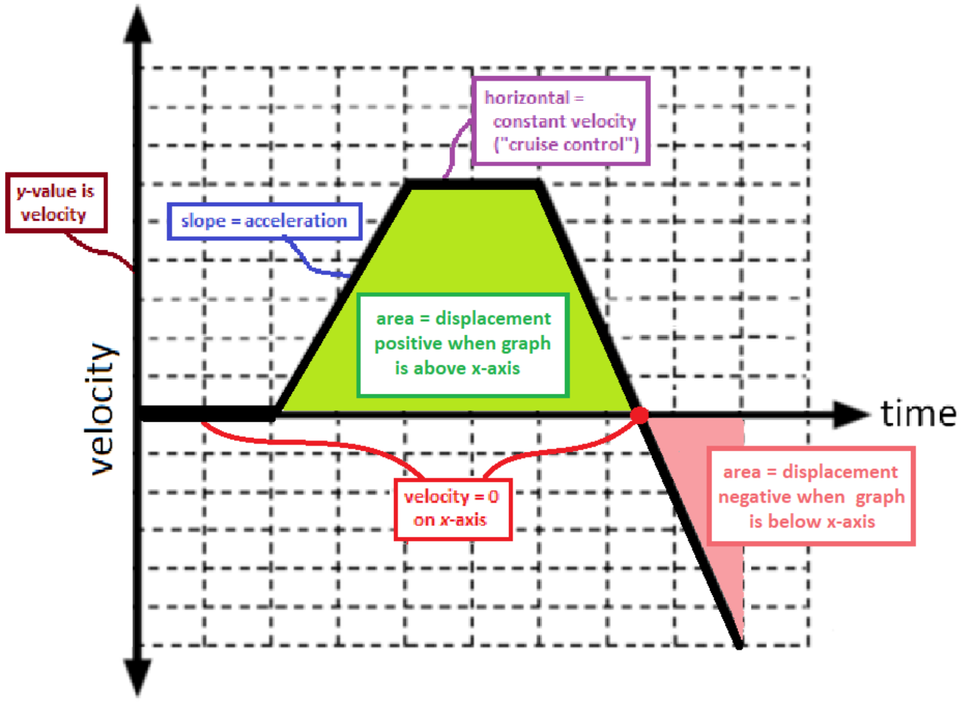
In each case, the area under the velocity-time graph equals the total distance traveled.

As we will see in the next section, the equation for displacement as a function of velocity and time is $d = v_o t + \frac{1}{2}at^2$, which becomes $d = \frac{1}{2}at^2$ for an object starting at rest. If we apply this equation to each of these situations, we would get the same numbers that we got from the area under the graph:

Between 0 and 2 s	Between 0 and 4 s	Between 4 s and 6 s
$a = 2.5\,\frac{\text{m}}{\text{s}^2}$	$a = 2.5\,\frac{\text{m}}{\text{s}^2}$	$a = 0$
$d = \frac{1}{2}(2.5)(2^2) = 5\,\text{m}$	$d = \frac{1}{2}(2.5)(4^2) = 20\,\text{m}$	$d = v_{ave.}t = (10)(2) = 20\,\text{m}$

Use this space for summary and/or additional notes:

Features of Velocity *vs.* Time Graphs



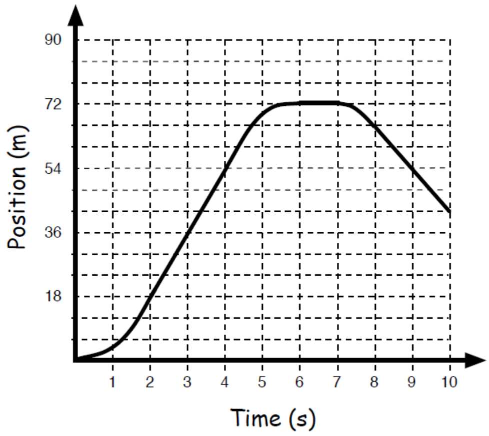
On a velocity *vs.* time graph, note the following:

- The velocity of the object is the *y*-value on the graph. When the *y*-value is positive, the object is moving in the positive direction. When the *y*-value is negative, the object is moving in the negative direction.
- A horizontal line indicates constant velocity. ***The object is at rest only when the graph is <u>on</u> the x-axis.***
- The ***slope*** of the graph is the ***acceleration***. A positive slope indicates positive acceleration, and a negative slope indicates negative acceleration.
- The ***area*** between a velocity *vs.* time graph and the *x*-axis is the ***displacement***. Areas above the x-axis indicate positive displacement, and areas below the *x*-axis indicate negative displacement.
- Note that an object cannot be moving with a nonzero velocity in the *x*- and *y*-direction at the same time. This means ***at any given time, the area can be either above the x-axis or below it, but never both***.
- The position of an object cannot be determined from a velocity *vs.* time graph.

Use this space for summary and/or additional notes:

Homework Problems: Motion Graphs

1. **(M)** An object's motion is described by the following graph of position *vs.* time:

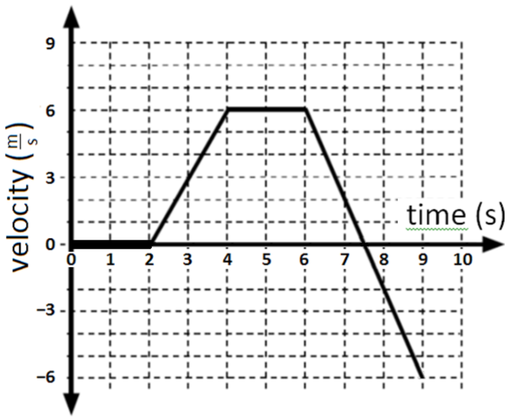


   a. What is the object doing between 2 s and 4 s? What is its velocity during that interval?

   b. What is the object doing between 6 s and 7 s? What is its velocity during that interval?

   c. During which time interval(s) is the object at rest? During which time interval(s) is it moving at a constant velocity? During which time interval(s) is it accelerating?

2. **(M)** An object's motion is described by the following graph of velocity *vs.* time:



   a. What is the object doing between 2 s and 4 s? What is its acceleration during that interval?

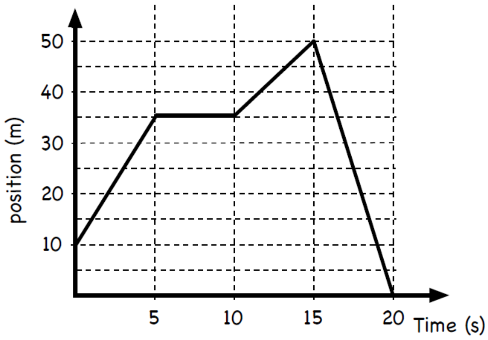
   b. What is the object doing between 4 s and 6 s? What are its velocity and acceleration during that interval?

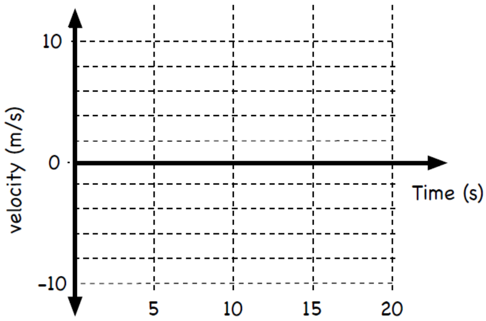
   c. What is the object's displacement between 2 s and 4 s? What is its displacement between 6 s and 9 s? (*Hint: you will need to split the areas into the area above the x-axis and the area below it.*)

Use this space for summary and/or additional notes:

3. **(M)** The graph on the left below shows the position of an object *vs.* time.

   a. Sketch a graph of velocity *vs.* time for the same object on a graph similar to the one on the right.



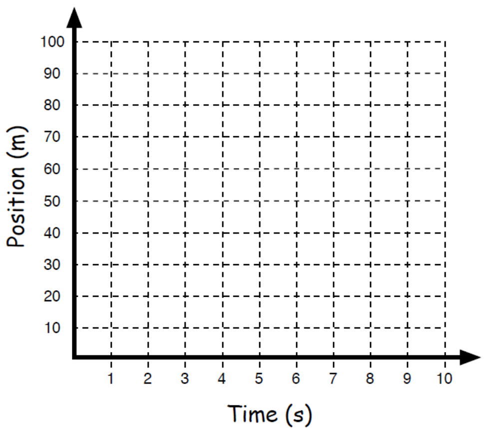


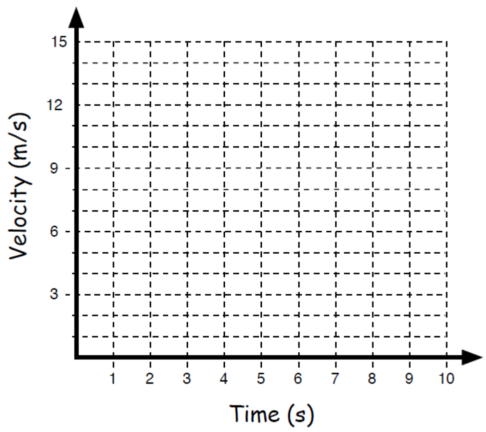
   b. Using your velocity *vs.* time graph, calculate the displacement for each 5-second segment. Use your position *vs.* time graph to check your answers.

4. **(M – AP®; S – honors & CP1)** In 1991, Carl Lewis became the first sprinter to break the 10-second barrier for the 100 m dash, completing the event in 9.86 s. The chart below shows his time for each 10 m interval.

position (m)	0	10	20	30	40	50	60	70	80	90	100
time (s)	0	1.88	2.96	3.88	4.77	5.61	6.45	7.29	8.12	8.97	9.86

Plot Lewis's position *vs.* time on the graph on the left. Draw a best-fit line (freehand). Then use the slope (rise over run) from the position *vs.* time graph to get the *y*-values for the velocity *vs.* time graph on the right.

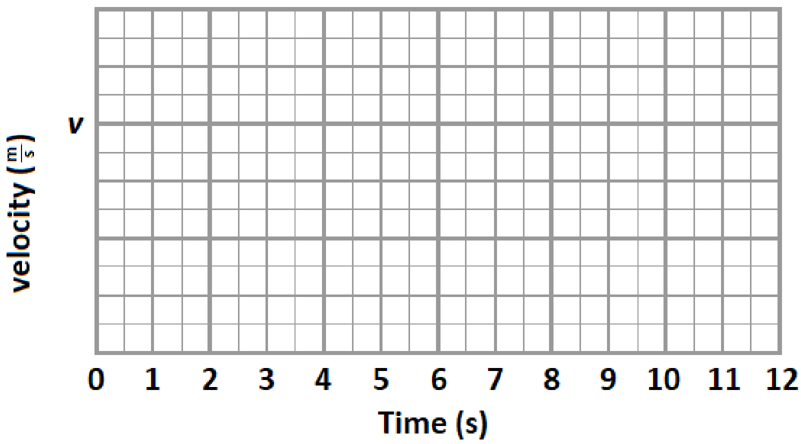




Use this space for summary and/or additional notes:

5. **(M – honors & AP®; S – CP1)** An elevator travels 4.0 m as it moves from the first floor of a building to the second floor. The elevator starts from rest and accelerates upward at a constant rate for 0.5 s, then travels at a constant (unknown) velocity *v* for 9.5 s, then decelerates at a constant rate for 0.5 s until it comes to rest on the second floor.

   a. **(M – honors & AP®; S – CP1)** Plot a graph of the elevator's motion on the graph below.



   b. **(M – honors & AP®; S – CP1)** Using your graph, determine the (constant) velocity, *v*, of the elevator during the interval from *t* = 0.5 s to *t* = 10 s.

   Answer: $0.4\,\frac{m}{s}$

   c. **(M – honors & AP®; S – CP1)** Determine the acceleration of the elevator during the interval from *t* = 0 to *t* = 0.5 s.

   Answer: $0.8\,\frac{m}{s^2}$

Use this space for summary and/or additional notes:

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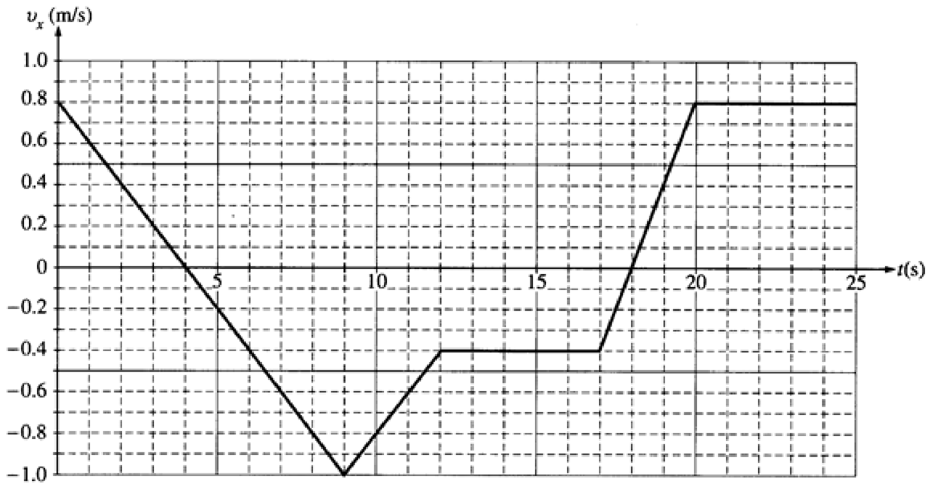
What an AP Motion Graph Problem Looks Like

AP motion problems almost always involve either graphs or projectiles. Free-response problems will often ask you to compare two graphs, such as a position-time graph *vs.* a velocity-time graph, or a velocity-time graph *vs.* an acceleration-time graph.

Here is an example of a free-response question involving motion graphs:

Q: A 0.50 kg cart moves on a straight horizontal track. The graph of velocity *v* versus time *t* for the cart is given below.



a. Indicate every time *t* for which the cart is at rest.

The cart is at rest whenever the velocity is zero. Velocity is the *y*-axis, so we simply need to find the places where *y* = 0. These are at t = 4 s and t = 18 s.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

For the velocity vector, we use positive and negative to indicate direction. Therefore, the magnitude is the absolute value. The magnitude of the velocity is increasing whenever the graph is moving away from the x-axis, which happens in the intervals 4–9 s *and* 18–20 s.

The most likely mistake would be to give the times when the acceleration is positive. Positive acceleration can mean that the speed is increasing in the positive direction, but it can also mean that it is decreasing in the negative direction.

Use this space for summary and/or additional notes:

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c. Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.

Position is the area under a velocity-time graph. Therefore, if we add the positive and subtract the negative areas from t = 0 *to t* = 9.0 s*, the result is the position at t* = 9.0 s.

The area of the triangular region from 0–4 s *is* $(\frac{1}{2})(4)(0.8) = 1.6\,\text{m}$.

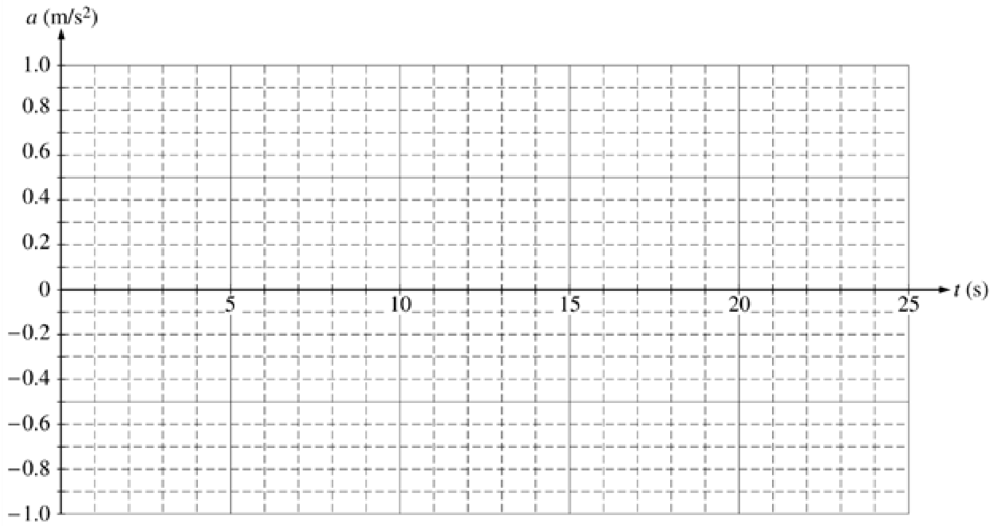
The area of the triangular region from 4–9 s *is* $(\frac{1}{2})(5)(-1.0) = -2.5\,\text{m}$.

The total displacement is therefore $\Delta x = 1.6 + (-2.5) = -0.9\,\text{m}$.

Because the cart's initial position was +2.0 m*, its final position is* 2.0 + (−0.9) = **+1.1 m**.

The most likely mistakes would be to add the areas regardless of whether they are negative or positive, and to forget to add the initial position after you have found the displacement.

d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from t = 0 to t = 25 s.



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Acceleration is the slope of a velocity-time graph. Because the graph is discontininuous, we need to split it at each point where the slope suddenly changes. Each of the regions is a straight line (constant slope), which means all of the accelerations are constant (horizontal lines on the graph).

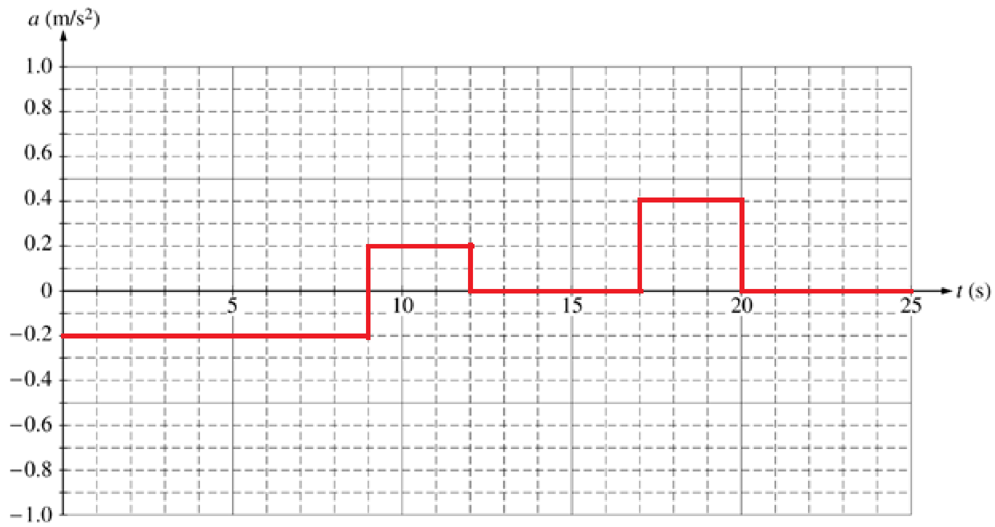
From 0–9 s, *the slope is* $\frac{\Delta y}{\Delta x} = \frac{-1.8}{9} = -0.2\,\frac{m}{s^2}$.

From 9–12 s, *the slope is* $\frac{\Delta y}{\Delta x} = \frac{+0.6}{3} = +0.2\,\frac{m}{s^2}$.

From 12–17 s *and from* 20–25 s, *the slope is zero.*

From 17–20 s, *the slope is* $\frac{\Delta y}{\Delta x} = \frac{+1.2}{3} = +0.4\,\frac{m}{s^2}$.

The graph therefore looks like the following:



e. The original problem also included a part (e), which was a simple projectile problem (discussed later).

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