AP®

Angular Motion, Speed and Velocity

Unit: Kinematics (Motion) in Multiple Dimensions

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 5.1.A, 5.1.A.1, 5.1.A.1.i, 5.1.A.1.i, 5.1.A.1.i, 5.1.A.2, 5.1.A.4

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve angular position and velocity.

Success Criteria:

- Correct quantities are chosen in each dimension (r, ω & θ).
- Positive direction is chosen for each dimension and vector quantities in each dimension have the appropriate sign (+ or −).
- Time (scalar) is correct, positive, and the same in both dimensions.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: rotation, angular

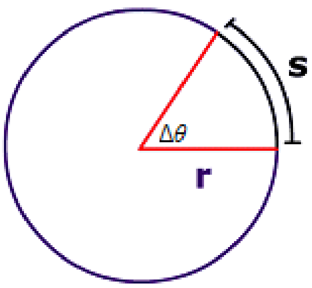
Labs, Activities & Demonstrations:

- Swing an object on a string.

Notes:

If an object is rotating (traveling in a circle), then its position at any given time can be described using polar coördinates by its distance from the center of the circle (r) and its angle (θ) relative to some reference angle (which we will call $\theta = 0$).

arc length (s): the length of an arc; the distance traveled around part of a circle.



$$s = r\Delta\theta$$

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angular velocity (ω): the rotational velocity of an object as it travels around a circle, *i.e.,* its change in angle per unit of time. (For purposes of comparison, the definition of angular velocity is presented along with its linear counterpart.)

$$\vec{\boldsymbol{v}} = \frac{\vec{\boldsymbol{d}}}{t} = \frac{\Delta \vec{\boldsymbol{x}}}{\Delta t} = \frac{\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_o}{t} \qquad \vec{\boldsymbol{\omega}} = \frac{\Delta \vec{\boldsymbol{\theta}}}{\Delta t} = \frac{\vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o}{t}$$

linear **angular**

In general, physicists use Greek letters for angular variables. The variable for angular velocity is the lower-case Greek letter omega (ω). Be careful to distinguish in your writing between the Greek letter "ω" and the Roman letter "*w*".

tangential velocity: the linear velocity of a point on a rigid, rotating body. The term tangential velocity is used because the instantaneous direction of the velocity is tangential to the direction of rotation.

To find the tangential velocity of a point on a rotating (rigid) body, the point travels an arc length of *s* in time *t*. If angle θ is in radians, then $s = r\Delta\theta$. This means:

$$\vec{\boldsymbol{v}}_{T,ave.} = \frac{\Delta \vec{\boldsymbol{s}}}{\Delta t} = \frac{r\Delta \vec{\boldsymbol{\theta}}}{\Delta t} = r\vec{\boldsymbol{\omega}}_{ave.} \quad \text{and therefore} \quad \vec{\boldsymbol{v}}_T = r\vec{\boldsymbol{\omega}}$$

Sample Problems:

Q: What is the angular velocity ($\frac{\text{rad}}{\text{s}}$) in of a car engine that is spinning at 2400 rpm?

A: 2400 rpm means 2400 revolutions per minute.

$$\left(\frac{2400\ \text{rev}}{1\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = \frac{4800\pi}{60} = 80\pi\,\frac{\text{rad}}{\text{s}} = 251\,\frac{\text{rad}}{\text{s}}$$

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Q: Find the total distance traveled in 10 s by a penny sitting on a spinning disc with a radius of 0.25 m that is rotating at a rate of 1 revolution per 2 s.

A: We are looking for the distance around the circle, which is the arc length. (This means we need to work in radians.)

We know that:

$$\vec{\boldsymbol{s}} = r\Delta\vec{\boldsymbol{\theta}}$$

and we know:

$$\Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\omega}}t$$

Substituting the second equation into the first gives:

$$\vec{\boldsymbol{s}} = r\Delta\vec{\boldsymbol{\theta}} = r\vec{\boldsymbol{\omega}}t$$

We need to convert $\vec{\boldsymbol{\omega}}$ to $\frac{\text{rad}}{\text{s}}$:

1 revolution per 2 s means $\vec{\boldsymbol{\omega}} = \left(\frac{1\,\cancel{\text{rev}}}{2\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{1\,\cancel{\text{rev}}}\right) = \frac{2\pi}{2} = \pi\,\frac{\text{rad}}{\text{s}}$

Now we can substitute and solve:

$$\vec{\boldsymbol{s}} = r\vec{\boldsymbol{\omega}}t = (0.25)(\pi)(10) = 2.5\pi = (2.5)(3.14) = 7.85\,\text{m}$$

Extension

Just as jerk is the rate of change of linear acceleration, angular jerk is the rate of change of angular acceleration. $\vec{\boldsymbol{\zeta}} = \frac{\Delta\vec{\boldsymbol{\alpha}}}{\Delta t}$. (ζ is the Greek letter "zeta". Many college professors cannot draw it correctly and just call it "squiggle".) Angular jerk has not been seen on AP® Physics exams.

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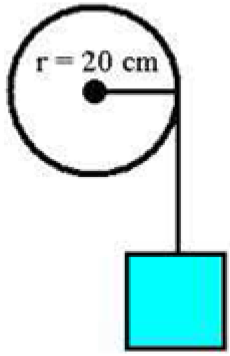
Homework Problems

1. **(M – AP®; A – honors & CP1)** Through what angle must the wheel shown at the right turn in order to unwind 40 cm of string?



Answer: 2 rad

2. **(M – AP®; A – honors & CP1)** Find the average angular velocity of a softball pitcher's arm (in $\frac{\text{rad}}{\text{s}}$) if, in throwing the ball, her arm rotates one-third of a revolution in 0.1 s.

Answer: $20.9\,\frac{\text{rad}}{\text{s}}$

3. **(M – AP®; A – honors & CP1)** A golfer swings a nine iron (radius = 1.1 m for the combination of the club and his arms) with an average angular velocity of $5.0\,\frac{\text{rad}}{\text{s}}$. Find the tangential velocity of the club head.

Answer: $5.5\,\frac{\text{m}}{\text{s}}$

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