AP®

Angular Acceleration

Unit: Kinematics (Motion) in Multiple Dimensions

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 5.1.A, 5.1.A.2, 5.1.A.3, 5.1.A.4, 5.1.A.4.i, 5.1.A.4.ii, 5.2.A, 5.2.A.2, 5.2.A.3

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve angular acceleration.

Success Criteria:

- Correct quantities are chosen in each dimension (r, ω, ω_o, α and θ).
- Positive direction is chosen for each dimension and vector quantities in each dimension have the appropriate sign (+ or −).
- Time (scalar) is correct, positive, and the same in both dimensions.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: rotation, angular

Labs, Activities & Demonstrations:

- Swing an object on a string and then change its angular velocity.

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Notes:

If a rotating object starts rotating faster or slower, this means its rotational velocity is changing.

angular acceleration (α): the change in angular velocity with respect to time. (Again, the definition is presented with the linear equation for comparison.)

$$\vec{\boldsymbol{a}} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_o}{t} \qquad \vec{\boldsymbol{\alpha}} = \frac{\Delta\vec{\boldsymbol{\omega}}}{\Delta t} = \frac{\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o}{t}$$

linear **angular**

As before, be careful to distinguish between the lower case Greek letter "α" and the lower case Roman letter "a".

As with linear acceleration, if the object has angular velocity and then accelerates, the position equation looks like this:

$$\vec{\boldsymbol{x}} - \vec{\boldsymbol{x}}_o = \vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2 \qquad \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o = \Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\omega}}_o t + \tfrac{1}{2}\vec{\boldsymbol{\alpha}}t^2$$

linear **angular**

tangential acceleration: the linear acceleration of a point on a rigid, rotating body. The term tangential acceleration is used because the instantaneous direction of the acceleration is tangential to the direction of rotation.

The tangential acceleration of a point on a rigid, rotating body is:

$$\vec{\boldsymbol{a}}_T = r\vec{\boldsymbol{\alpha}}$$

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Sample Problem:

Q: A bicyclist is riding at an initial (linear) velocity of $7.5\,\frac{\text{m}}{\text{s}}$, and accelerates to a velocity of $10.0\,\frac{\text{m}}{\text{s}}$ over a duration of 5.0 s. If the wheels on the bicycle have a radius of 0.343 m, what is the angular acceleration of the bicycle wheels?

A: First we need to find the initial and final angular velocities of the bike wheel. We can do this from the tangential velocity, which equals the velocity of the bicycle.

$$\vec{v}_{o,T} = r\vec{\omega}_o$$
$$\frac{\vec{v}_{o,T}}{r} = \vec{\omega}_o$$
$$\frac{7.5}{0.343} = \vec{\omega}_o = 21.87\,\frac{\text{rad}}{\text{s}}$$

$$\vec{v}_T = r\vec{\omega}$$
$$\frac{\vec{v}_T}{r} = \vec{\omega}$$
$$\frac{10.0}{0.343} = \vec{\omega} = 29.15\,\frac{\text{rad}}{\text{s}}$$

Then we can use the equation:

$$\vec{\omega} - \vec{\omega}_o = \vec{\alpha}t$$
$$\frac{\vec{\omega} - \vec{\omega}_o}{t} = \vec{\alpha}$$
$$\frac{29.15 - 21.87}{5.0} = \vec{\alpha} = 1.46\,\frac{\text{rad}}{\text{s}^2}$$

An alternative method is to solve the equation by finding the linear acceleration first:

$$\vec{v} - \vec{v}_o = \vec{a}t$$
$$\frac{\vec{v} - \vec{v}_o}{t} = \vec{a}$$
$$\frac{10.0 - 7.5}{5} = \vec{a} = 0.5\,\frac{\text{m}}{\text{s}^2}$$

Then we can use the relationship between tangential acceleration and angular acceleration:

$$\vec{a}_T = r\vec{\alpha}$$
$$\frac{\vec{a}_T}{r} = \vec{\alpha}$$
$$\frac{0.5}{0.343} = \vec{\alpha} = 1.46\,\frac{\text{rad}}{\text{s}^2}$$

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Homework Problems

1. **(M – AP®; A – honors & CP1)** A turntable rotating with an angular velocity of ω_o is shut off. It slows down at a constant rate and coasts to a stop in time t. What is its angular acceleration, α?
(*If you are not sure how to do this problem, do #2 below and use the steps to guide your algebra.*)

Answer: $\alpha = \frac{-\omega_o}{t}$

2. **(S – AP®; A – honors & CP1)** A turntable rotating at 33⅓ RPM is shut off. It slows down at a constant rate and coasts to a stop in 26 s. What is its angular acceleration?
(*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #1 above as a starting point if you have already solved that problem.*)

Answer: $-0.135 \ \frac{\text{rad}}{\text{s}^2}$

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