Centripetal Motion

Unit: Kinematics (Motion) in Multiple Dimensions

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.9.A, 2.9.A.1, 2.9.A.1.i, 2.9.A.1.ii, 2.9.A.2, 2.9.A.2.i, 2.9.A.3, 2.9.A.4, 2.9.A.5, 2.9.A.5.i, 2.9.A.5.ii, 2.9.A.5.iii

Mastery Objective(s): (Students will be able to…)

- Calculate the tangential and angular velocity and acceleration of an object moving in a circle.

Success Criteria:

- Correct quantities are chosen in each dimension ($r, \omega, \omega_o, \alpha, a$ and/or ϑ).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why an object moving in a circle must be accelerating toward the center.
- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: centripetal, centrifugal

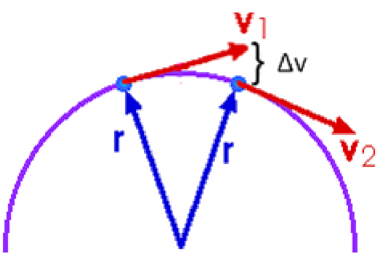
Labs, Activities & Demonstrations:

- Have students swing an object and let it go at the right time to try to hit something. (Be sure to observe the trajectory.)
- Swing a bucket of water in a circle.

Notes:

If an object is moving at a constant speed around a circle, its speed is constant, its direction keeps changing as it goes around. Because *velocity* is a vector (speed and direction), this means its velocity is constantly changing. (To be precise, the magnitude is staying the same, but the direction is changing.)

Because a change in velocity over time is acceleration, this means the object is constantly accelerating. This continuous change in velocity is toward the center of the circle, which means *there is continuous acceleration toward the center of the circle.*



Use this space for summary and/or additional notes:

centripetal acceleration (a_c): the constant acceleration of an object toward the center of rotation that keeps it rotating around the center at a fixed distance.

The equation* for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

(The derivation of this equation requires calculus, so it will not be presented here.)

Sample Problem:

Q: A weight is swung from the end of a string that is 0.65 m long at a rate of rotation of 10 revolutions in 6.5 s. What is the centripetal acceleration of the weight? How many "g's" is that? (*I.e.,* how many times the acceleration due to gravity is the centripetal acceleration?)

A: There are two ways to solve this problem.

Without using angular velocity:

In each revolution, the object travels a distance of $2\pi r$:

$$s_{rev} = 2\pi r = (2)(3.14)(0.65) = 4.08\,\text{m}$$

The total distance for 10 revolutions is therefore: $s = (4.08)(10) = 40.8\,\text{m}$

The velocity is the distance divided by the time: $v = \frac{d}{t} = \frac{40.8}{6.5} = 6.28\,\frac{\text{m}}{\text{s}}$

Finally, $a_c = \frac{v^2}{r} = \frac{(6.28)^2}{0.65} = 60.7\,\frac{\text{m}}{\text{s}^2}$

This is $\frac{60.7}{10} = 6.07$ times the acceleration due to gravity.

AP®

Using angular velocity:

The angular velocity is:

$$\left(\frac{10\ \cancel{\text{rev}}}{6.5\,\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right) = \frac{20\pi}{6.5} = 9.67\,\frac{\text{rad}}{\text{s}}$$

The centripetal acceleration is therefore:

$$a_c = r\omega^2$$

$$a_c = (0.65)(9.67)^2 = (0.65)(93.44) = 60.7\,\frac{\text{m}}{\text{s}^2}$$

This is $\frac{60.7}{10} = 6.07$ times the acceleration due to gravity.

* Centripetal motion relates to angular motion (which is studied in AP® Physics but not in the CP1 or honors courses). Equations or portions of equations with angular velocity (ω) and angular acceleration (α) apply only to the AP® course.

Use this space for summary and/or additional notes:

Centripetal motion is a form of simple harmonic motion (repetitive motion), and can be described using time period (*T*) and frequency (*f*).

(time) period (*T*, unit = s): The amount of time that it takes for an object to complete one complete cycle of periodic (repetitive) motion. In the case of centripetal motion, the period is the amount of time it takes for the object to make one complete revolution.

frequency (*f*, unit = Hz = $^{1}/_{s}$): The number of cycles of repetive motion per unit of time. Frequency and period are reciprocals of each other, *i.e.,* $f = \frac{1}{T}$ and $T = \frac{1}{f}$

Because $v_{avg} = \frac{d}{t}$ and the distance around the circle is the circumference, $C = 2\pi r$, this means the period is equal to $T = \frac{2\pi r}{v}$.

We will revisit these quantities and relationships further in the *Introduction: Simple* Harmonic Motion unit, starting on page 523.

Use this space for summary and/or additional notes:

Introduction: Simple Harmonic Motion unit, starting on page 523.

Use this space for summary and/or additional notes:

Homework Problem

1. One of the demonstrations we saw in class was swinging a bucket of water in a vertical circle without spilling any of the water.

   a. **(M)** Explain why the water stayed in the bucket.

   b. **(M)** If the combined length of your arm and the bucket is 0.90 m, what is the minimum tangential velocity that the bucket must have in order to not spill any water?

   Answer: $3.0\,\frac{m}{s}$

Use this space for summary and/or additional notes: