AP®

Solving Linear & Rotational Motion Problems

Unit: Kinematics (Motion) in Multiple Dimensions

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 5.1.A, 5.1.A.1, 5.1.A.1.i, 5.1.A.1.ii, 5.1.A.1.iii, 5.1.A.2, 5.1.A.3, 5.1.A.4, 5.1.A.4.i, 5.1.A.4.ii, 5.2.A, 5.2.A.2, 5.2.A.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving any combination of linear and/or angular motion.

Success Criteria:

- Correct quantities are identified, and correct variables are chosen for each dimension.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Correctly identify quantities with respect to type of quantity and direction in word problems.
- Assign variables correctly in word problems.

Tier 2 Vocabulary: N/A

Notes:

The following is a summary of the variables used for motion problems. Note the correspondence between the linear and angular quantities.

Linear			Angular		
Var.	**Unit**	**Description**	**Var.**	**Unit**	**Description**
x	m	position	θ	rad (—)	angle; angular position
$\vec{d}, \Delta x$	m	displacement	$\Delta\theta$	rad (—)	angular displacement
$\vec{v}$	$\frac{\text{m}}{\text{s}}$	velocity	$\vec{\omega}$	$\frac{\text{rad}}{\text{s}} \left(\frac{1}{\text{s}}\right)$	angular velocity
$\vec{a}$	$\frac{\text{m}}{\text{s}^2}$	acceleration	$\vec{\alpha}$	$\frac{\text{rad}}{\text{s}^2} \left(\frac{1}{\text{s}^2}\right)$	angular acceleration
t	s	time	t	s	time

Notice that each of the linear variables has an angular counterpart.

Note also that "radian" is a dimensionless quantity. A radian is a ratio that describes an angle as the ratio of the arc length to the radius of the circle. This ratio is dimensionless (has no unit), because the units cancel—an angle is the same regardless of the distance units used.

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Of course, the same would be true if we measured angles in degrees (or gradians* or anything else), but using radians makes many of the calculations particularly convenient.

We have learned the following equations for solving motion problems. Again, note the correspondence between the linear and angular equations.

Linear Equation	Angular Equation	Relationship	Comments
$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$	$\Delta\vec{\theta} = \vec{\theta} - \vec{\theta}_o$	$s = r\Delta\theta$	Definition of displacement.
$\vec{v}_{ave.} = \frac{\vec{d}}{t} = \frac{\Delta\vec{x}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	$\vec{\omega}_{ave.} = \frac{\Delta\vec{\theta}}{t} = \frac{\vec{\omega}_o + \vec{\omega}}{2}$	$v_T = r\omega$	Definition of *<u>average</u>* velocity. Note that you can't use $\vec{v}_{ave.}$ or $\vec{\omega}_{ave.}$ if there is acceleration.
$\vec{a} = \frac{\Delta\vec{v}}{t} = \frac{\vec{v} - \vec{v}_o}{t}$	$\vec{\alpha} = \frac{\Delta\vec{\omega}}{t} = \frac{\vec{\omega} - \vec{\omega}_o}{t}$	$a_T = r\alpha$	Definition of acceleration.
$\vec{x} - \vec{x}_o = \vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$	$\vec{\theta} - \vec{\theta}_o = \Delta\vec{\theta} = \vec{\omega}_o t + \frac{1}{2}\vec{\alpha}t^2$		Position/ displacement formula.
$\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}$ $\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}(\Delta\vec{x})$	$\vec{\omega}^2 - \vec{\omega}_o^2 = 2\vec{\alpha}\Delta\vec{\theta}$		Relates velocities, acceleration and distance. Useful if time is not known.
$a_c = \frac{v^2}{r}$	$a_c = r\omega^2$		Centripetal acceleration (toward the center of a circle.)

Note that vector quantities can be positive or negative, depending on direction.

Note that $\vec{r}$, $\vec{\omega}$ and $\vec{\alpha}$ are vector quantities. However, the equations that relate linear and angular motion and the centripetal acceleration equations apply to magnitudes only, because of the differences in coordinate systems and changing frames of reference.

Note that the relationship $s = r\Delta\theta$ is not listed on the AP® Physics exam sheet (even though it appears explicitly in the Course & Exam Description), so ***you need to memorize it***!

* A gradian is $\frac{9}{10}$ of a degree, which means a right angle measures 100 gradians. It is sometimes called a "metric degree" because it was introduced as part of the metric system in France in the 1790s.

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Selecting the Right Equation

(This is the same as the list from page 188, with the addition of angular velocity.)

When you are faced with a problem, choose an equation based on the following criteria:

- The equation *must* contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.

Linear	**Angular**
• If an object starts at rest (not moving), then $\vec{v}_o = 0$.	• If an object's rotation starts from rest (not rotating), then $\vec{\omega}_o = 0$.
• If an object comes to a stop, then $\vec{v} = 0$.	• If an object stops rotating, then $\vec{\omega} = 0$.
• If an object is moving at a constant velocity, then $\vec{v} = \text{constant} = \vec{v}_{ave.}$ and $\vec{a} = 0$.	• If an object is rotating at a constant rate (angular velocity), then $\vec{\omega} = \text{constant} = \vec{\omega}_{ave.}$ and $\vec{\alpha} = 0$.
• If an object is in free fall, then $\vec{a} = \vec{g} \approx 10\,\frac{\text{m}}{\text{s}^2}$.	

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

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