

Center of Mass

Unit: Forces in One Dimension

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.1.B, 2.1.B.1, 2.1.B.2, 2.1.B.3

Mastery Objective(s): (Students will be able to...)

- Find the center of mass of an object.

Success Criteria:

- Object balances at its center of mass.

Language Objectives:

- Explain why an object balances at its center of mass.

Tier 2 Vocabulary: center

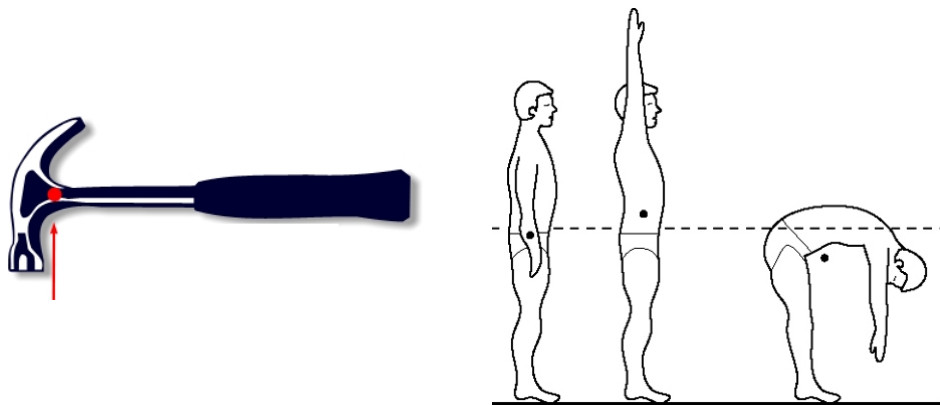
Labs, Activities & Demonstrations:

- Spin an object (*e.g.*, a hammer or drill team rifle) with its center of mass marked.

Notes:

center of mass: the point where all of an object's mass could be placed without changing the results of any forces acting on the object.

Objects have nonzero volumes. For any object, its mass is distributed in some way throughout its volume. In most of the problems that you will see in this course, we can simplify the calculations by pretending that all of the mass of the object is at a single point.



You can find the location of the center of mass of an object from the following formula:

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

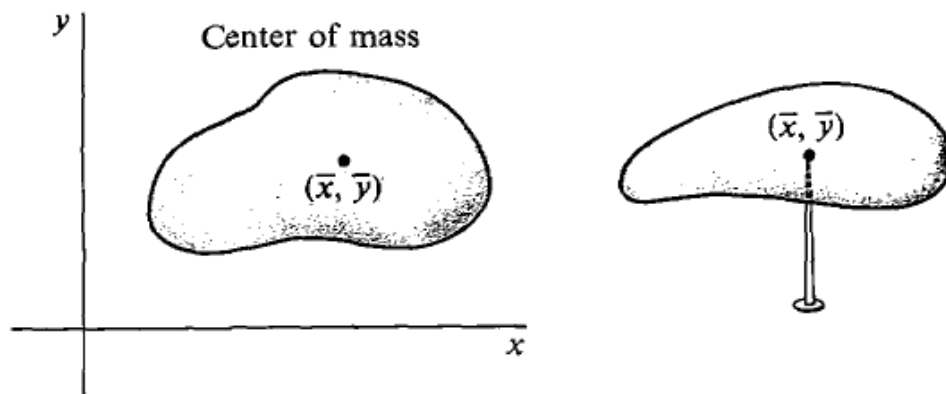
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In this equation, the symbol Σ means “summation.” When this symbol appears in a math equation, calculate the equation to the right of the symbol for each set of values, then add them up.

In this case, for each object (designated by a subscript), first multiply the mass (m) of each part of the system by its distance from some reference point (\vec{r}). Add all of those individual $m_i\vec{r}_i$ pieces and divide by the total mass. The resulting value of \vec{r} is the distance from that reference point.

If each of the individual values of \vec{r}_i has coordinates (\vec{x}_i, \vec{y}_i) , then the coordinates of the \vec{r} that we calculate are the coordinates of the center of mass.



On the AP® Physics 1 exam, you will only need to perform this calculation in one dimension, which means the above equation becomes:

$$\bar{x}_{cm} = \frac{\sum_i m_i \bar{x}_i}{\sum_i m_i}$$

Because an object at rest remains at rest, this means that the object’s center of mass is also the point at which the object will balance on a point. (Actually, because gravity is involved, the object balances because the torques around the center of mass cancel. This is discussed in detail in the *Torque* section, starting on page 364.)

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*honors & AP®***Sample Problem:**

Q: Two people sit at the ends of a massless 3.5 m long seesaw. One person has a mass of 59 kg, and the other has a mass of 71 kg. Where is their center of mass?

A: (Yes, there's no such thing as a massless seesaw. This is an idealization to make the problem easy to solve.)

In order to make this problem simple, let us place the 59-kg person at a distance of zero.

$$r_{cm} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$
$$r_{cm} = \frac{(59)(0) + (71)(3.5)}{(59 + 71)}$$
$$r_{cm} = \frac{248.5}{130} = 1.91 \text{ m}$$

Their center of mass is 1.91 m away from the 59-kg person.

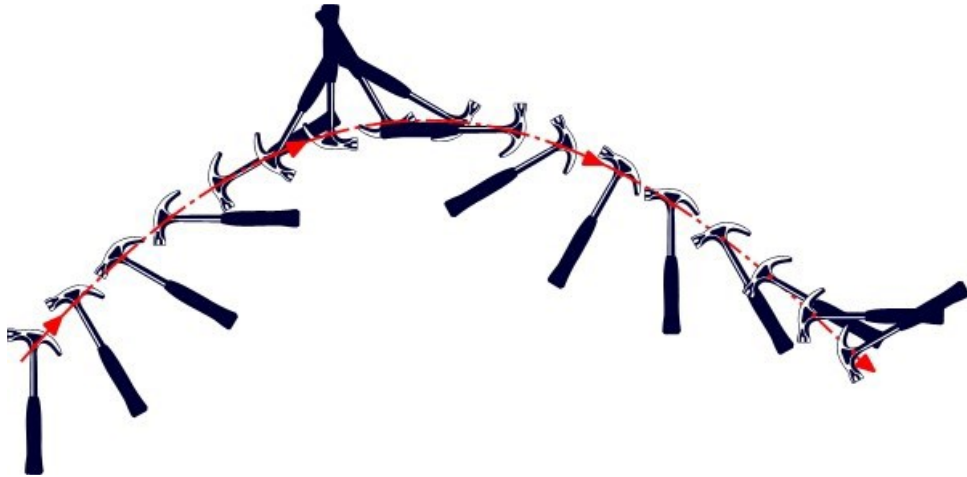
We can apply the concept of center of mass to Newton's Laws and systems:

- If Newton's First Law applies (all of the forces on the system are balanced and there is no net force), then the velocity of the center of mass of the system does not change, regardless of what is happening inside the system.
- If Newton's Second Law applies (there is at least one unbalanced force on the system, which means there is a net force), then the velocity of the center of mass changes, regardless of what is happening inside the system.
- Because of Newton's Third Law, that forces that exist entirely within the system do not affect the motion of the center of mass of the system, because the action force and the reaction force both act within the system.

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In order to illustrate the concept that “whatever is happening inside the system doesn’t affect the motion of the center of mass”, consider object that is rotating freely in space. The object will rotate about its center of mass.

If we throw a spinning hammer, its center of mass will move in the same manner as if we had thrown a ball, showing that the motion of the the center of mass is not affected by the rotation of the object.



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