Newton's Second Law

Unit: Forces in One Dimension

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-1, HS-PS2-10(MA)

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.5.A, 2.5.A.1, 2.5.A.2, 2.5.A.3

Mastery Objective(s): (Students will be able to…)

- Solve problems relating to Newton's second law $(\vec{F}_{net} = m\vec{a})$.
- Solve problems that combine kinematics (motion) and forces.

Success Criteria:

- Free-body diagram is correct.
- Vector quantities position, velocity, and acceleration are correct, including sign (direction).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Identify the quantities in a word problem and assign the correct variables to them.
- Select equations that relate the quantities given in the problem.

Tier 2 Vocabulary: force, free, body, displacement, acceleration

Labs, Activities & Demonstrations:

- Handstands in an elevator.

Notes:

Newton's Second Law: Forces cause acceleration (a change in velocity). "A net force, $\vec{F}$, acting on an object causes the object to accelerate in the direction of the net force."

If there is a net force, the object accelerates (its velocity changes). If there is no net force, the object's velocity remains the same.

If an object accelerates (its velocity changes), there was a net force on it. If an object's velocity remains the same, there was no net force on it.

Remember that forces are vectors. "No net force" can either mean that there are no forces at all, or it can mean that there are equal forces in opposite directions and their effects cancel.

static equilibrium: when all of the forces on an object cancel each other's effects (resulting in a net force of zero) and the object remains stationary.

dynamic equilibrium: when all of the forces on an object cancel each other's effects (resulting in a net force of zero) and the object remains in motion with constant velocity.

Use this space for summary and/or additional notes:

In equation form:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{\sum \vec{F}}{m} \quad \text{or} \quad \vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

The first form is preferred for teaching purposes, because acceleration is what results from a force applied to a mass. (*I.e.,* force and mass are the manipulated variables and acceleration is the responding variable. Forces cause acceleration, not the other way around.) However, the equation is more commonly written in the second form, which makes the typesetting and the algebra easier.

Sample Problems

Most of the physics problems involving forces require the application of Newton's Second Law, $\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$.

Q: A net force of 50 N in the positive direction is applied to a cart that has a mass of 35 kg. How fast does the cart accelerate?

A: Applying Newton's Second Law:

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{50}{35} = 1.43\,\tfrac{\text{m}}{\text{s}^2}$$

Q: Two children are fighting over a toy.

Jamie pulls to the left with a force of 40 N, and Edward pulls to the right with a force of 60 N. If the toy has a mass of 0.6 kg, what is the resulting acceleration of the toy?

A: The free-body diagram looks like this:

(We chose the positive direction to the right because it makes more intuitive sense for the positive direction to be the direction that the toy will move.)

$$\sum \vec{F} = m\vec{a}$$

$$-40 + 60 = (0.6)\vec{a}$$

$$\vec{a} = \frac{+20}{0.6} = +33.3\,\tfrac{\text{m}}{\text{s}^2} \text{ (to the right)}$$

Use this space for summary and/or additional notes:

Q: A person applies a net force of 100. N to cart full of books that has a mass of 75 kg. If the cart starts from rest, how far will the cart have moved by the time it gets to a speed of $4.0\,\frac{\text{m}}{\text{s}}$?

A: Using the GUESS system, we can see that only two of the quantities are known (initial velocity and final velocity). However, we can find acceleration from $\vec{F}_{net} = m\vec{a}$, at which point we have the quantities that we need to solve the motion problem. This means we need to add a second GUESS chart for Newton's second law. Because $\vec{a}$ appears in both equations, we connect it in the two charts.

Motion Equations

var.	dir.	value
$\vec{d}$	→	$\vec{d}$
$\vec{v}_o$	N/A	0
$\vec{v}$	→	$+4\,\frac{\text{m}}{\text{s}}$
$\vec{a}$	→	$\vec{a}$
t	–	–

$$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$$

$$\vec{v} - \vec{v}_o = \vec{a}t$$

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$\boxed{\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}}$$

Newton's Second Law

var.	dir.	value
$\vec{F}_{net}$	→	$\vec{F}_{net}$
m	N/A	5 kg
$\vec{a}$	→	$\vec{a}$

$$\vec{F}_{net} = m\vec{a}$$

Our strategy is therefore:

1. Find acceleration from $\vec{F}_{net} = m\vec{a}$:

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{100}{75} = 1.\overline{3}\,\frac{\text{m}}{\text{s}^2}$$

2. Now that we have $\vec{a}$ we can use the last motion equation to solve the problem:

$$\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}$$

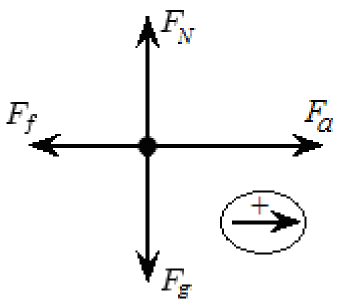
$$\frac{\vec{v}^2 - \vec{v}_o^{\,2}}{2a} = \vec{d}$$

$$\vec{d} = \frac{4^2 - 0^2}{(2)(1.\overline{3})} = \frac{16}{2.\overline{6}} = 6\,\text{m}$$

Use this space for summary and/or additional notes:

Q: A 5.0 kg block is resting on a horizontal, flat surface. How much force is needed to overcome a force of 2.0 N of friction and accelerate the block from rest to a velocity of $6.0\,\frac{m}{s}$ over a 1.5-second interval?

A: This is a combination of a Newton's second law problem, and a motion problem. There are multiple forces in the problem, so we should draw a free-body diagram so we can visualize what's going on.



We are trying to find the applied force, $\vec{F}_a$.

Again using the GUESS system, we now have three connected equations. Our strategy is to start with the equation that contains the quantity we need ($\vec{F}_a$). Each time we need a quantity that we don't have, we tack on an additional GUESS chart that enables us to calculate that quantity.

List of Forces

var.	dir.	value
$\vec{F}_{net}$	→	$\vec{F}_{net}$
$\vec{F}_a$	→	$\vec{F}_a$
$\vec{F}_f$	←	−2 N

$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_a + \vec{F}_f$$

Newton's Second Law

var.	dir.	value
$\vec{F}_{net}$	→	$\vec{F}_{net}$
m	N/A	5 kg
$\vec{a}$	→	$\vec{a}$

$$\vec{F}_{net} = ma$$

Motion Equations

var.	dir.	value
$\vec{d}$	–	–
$\vec{v}_o$	N/A	0
$\vec{v}$	→	$+6\,\frac{m}{s}$
$\vec{a}$	→	$\vec{a}$
t	N/A	1.5 s

$$\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$$

$$\boxed{\vec{v} - \vec{v}_o = \vec{a}t}$$

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 - \vec{v}_o^{\,2} = 2\vec{a}\vec{d}$$

Use this space for summary and/or additional notes:

Based on our GUESS charts, our strategy is therefore:

1. Use motion equations to find acceleration:

$$\vec{v} - \vec{v}_o = \vec{a}t$$
$$\frac{\vec{v} - \vec{v}_o}{t} = \vec{a}$$
$$\frac{6-0}{1.5} = \vec{a} = 4\,\frac{\text{m}}{\text{s}^2}$$

2. Use $\vec{F}_{net} = m\vec{a}$ to find $\vec{F}_{net}$:

$$\vec{F}_{net} = m\vec{a} = (5)(4) = 20\text{ N}$$

3. Use $\vec{F}_{net} = \sum \vec{F}$ to find $\vec{F}_a$. We need to remember that $\vec{F}_f$ is negative because it is in the negative direction.

$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_a + \vec{F}_f$$
$$20 = \vec{F}_a + (-2)$$
$$\vec{F}_a = 22\text{ N}$$

Use this space for summary and/or additional notes:

Homework Problems

1. **(S)** Two horizontal forces, 225 N and 165 N are exerted on a canoe. If these forces are both applied eastward, what is the net force on the canoe?

2. **(S)** Two horizontal forces are exerted on a canoe, 225 N westward and 165 N eastward. What is the net force on the canoe?

3. **(M)** Three confused sled dogs are trying to pull a sled across the snow in Alaska. Alutia pulls to the east with a force of 135 N. Seward pulls to the east with a force of 143 N. Kodiak pulls to the west with a force of 153 N.

   a. **(M)** What is the net force on the sled?

      Answer: 125 N east

   b. **(M)** If the sled has a mass of 150. kg and the driver has a mass of 100. kg, what is the acceleration of the sled? (Assume there is no friction between the runners of the sled and the snow.)

      Answer: $0.500\,\frac{m}{s^2}$

Use this space for summary and/or additional notes:

4. **(S)** When a net force of 10. N acts on a hockey puck, the puck accelerates at a rate of $50.\frac{m}{s^2}$. Determine the mass of the puck.

   Answer: 0.20 kg

5. **(S)** A 15 N net force is applied for 6.0 s to a 12 kg box initially at rest. What is the speed of the box at the end of the 6.0 s interval?

   Answer: $7.5\frac{m}{s}$

6. **(S)** A cart with a mass of 0.60 kg is propelled by a fan. The cart starts from rest, and travels 1.2 m in 4.0 s. What is the net force applied by the fan?

   Answer: 0.09 N

7. **(M)** A child with a mass of 44 kg stands on a scale that reads in newtons.
   a. **(M)** What is the child's weight?
   b. **(M)** The child now places one foot on each of two scales side-by-side. If the child distributes equal amounts of weight between the two scales, what is the reading on each scale?

Use this space for summary and/or additional notes:

8. **(S)** A 70.0 kg astronaut pushes on a spacecraft with a force $\vec{F}$ in space. The spacecraft has a total mass of 1.0×10^4 kg. The push causes the astronaut to accelerate to the right with an acceleration of $0.36\,\frac{m}{s^2}$. Determine the magnitude of the acceleration of the spacecraft.

Answer: $0.0025\,\frac{m}{s^2}$

honors & AP®

9. **(M – honors & AP®; A – CP1)** How much force will it take to accelerate a student with mass *m*, wearing special frictionless roller skates, across the ground from rest to velocity *v* in time *t*?
(*If you are not sure how to do this problem, do #10 below and use the steps to guide your algebra.*)

Answer: $F = \frac{mv}{t}$

10. **(S – honors & AP®; M – CP1)** How much force will it take to accelerate a 60 kg student, wearing special frictionless roller skates, across the ground from rest to $16\,\frac{m}{s}$ in 4 s?
(<u>You must start with the equations in your Physics Reference Tables *and show all of the steps of GUESS.*</u> *You may only use the answer to question #9 above as a starting point if you have already solved that problem.*)

Answer: 240 N

Use this space for summary and/or additional notes:

11. **(M)** How much force would it take to accelerate a 60 kg student upwards at $2\frac{m}{s^2}$?

    Hint: you need to account for gravity. Draw a free-body diagram.

    Answer: 720 N

12. **(S)** An air conditioner weighs 400 N on Earth. How much would the air conditioner weigh on the planet Mercury, where the value of $\vec{g}$ is only $3.6\frac{N}{kg}$?

    Answer: 144 N

13. **(M – honors & AP®; S – CP1)** A person pushes a 500 kg crate with a force of 1200 N and the crate accelerates at $0.5\frac{m}{s^2}$. What is the force of friction acting on the crate?
    Hint: draw the free-body diagram.

    Answer: 950N

Use this space for summary and/or additional notes: