honors & AP®

Force Applied at an Angle

Unit: Forces in Multiple Dimensions

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 3.A.2.1, 3.B.1.1, 3.B.1.2, 3.B.1.3

Mastery Objective(s): (Students will be able to…)

- Calculate forces applied at different angles, using trigonometry.

Success Criteria:

- Forces are split or combined correctly using the Pythagorean Theorem and trigonometry.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the concept of a component of a force.
- Explain why it is incorrect to just add together the vertical and horizontal components of a force.

Tier 2 Vocabulary: force

Labs, Activities & Demonstrations:

- Mass hanging from one or two scales. Change angle and observe changes in force.
- Fan cart with fan at an angle.
- For rope attached to heavy object, pull *vs.* anchor rope at both ends & push middle.

Notes:

An important property of vectors is that a vector has no effect on a second vector that is perpendicular to it. As we saw with projectiles, this means that the velocity of an object in the horizontal direction has no effect on the velocity of the same object in the vertical direction. This allowed us to solve for the horizontal and vertical velocities as separate problems.

The same is true for forces. If forces are perpendicular to each other, they act independently, and the two can be separated into separate, independent mathematical problems:

In the x-direction: $\vec{\boldsymbol{F}}_{net,x} = m\vec{\boldsymbol{a}}_x$

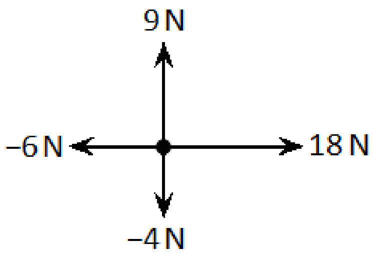
In the y-direction: $\vec{\boldsymbol{F}}_{net,y} = m\vec{\boldsymbol{a}}_y$

Note that the above is for linear situations. Two-dimensional rotational problems require calculus and are therefore outside the scope of this course.

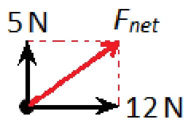
Use this space for summary and/or additional notes:

honors & AP®

For example, if we have the following forces acting on an object:



The net horizontal force (F_x) would be 18 N + (−6 N) = +12 N, and the net vertical force (F_y) would be 9 N + (−4 N) = +5 N. The total net force would be the resultant of the net horizontal and net vertical forces:



Using the Pythagorean Theorem:

$$a^2 + b^2 = c^2 \qquad 169 = F_{net}^2$$

$$5^2 + 12^2 = F_{net}^2 \qquad \sqrt{169} = F_{net} = 13\text{ N}$$

We can get the angle from trigonometry:

$$\tan\theta = \frac{opposite}{adjacent} = \frac{5}{12} = 0.417$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.417) = 22.6°$$

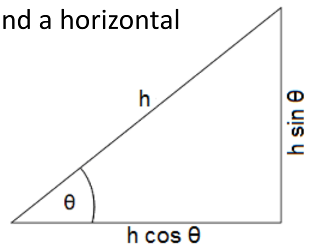
(Of course, because you have just figured out the length of the hypotenuse, you could get the same answer by using $\sin^{-1}$ or $\cos^{-1}$.)

Use this space for summary and/or additional notes:

honors & AP®

If we have one or more forces that is neither vertical nor horizontal, we can use trigonometry to split the force into a vertical component and a horizontal component.

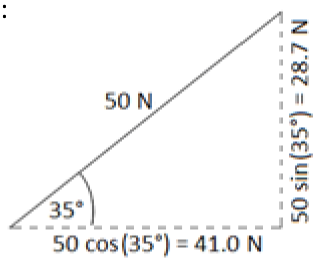
Recall the following relationships from trigonometry:



Suppose we have a force of 50 N at a direction of 35° above the horizontal. In the above diagram, this would mean that *h* = 50 N and *θ* = 35°:

The horizontal force is $\vec{F}_x = h\cos(\theta) = 50\cos(35°) = 41.0\,\text{N}$

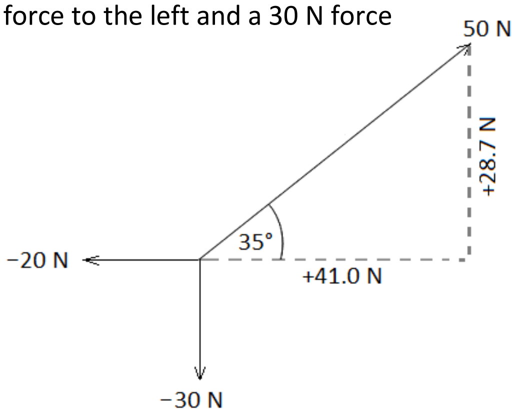
The vertical force is $\vec{F}_y = h\sin(\theta) = 50\sin(35°) = 28.7\,\text{N}$



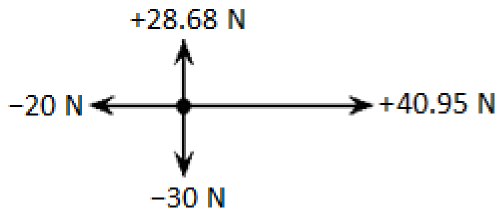
Now, suppose that same object was subjected to the same 50 N force at an angle of 35° above the horizontal, but also a 20 N force to the left and a 30 N force downward.

The net horizontal force would therefore be 41 + (−20) = 21 N to the right.

The net vertical force would therefore be 28.7 + (−30) = −1.3 N upwards (which equals 1.3 N downwards).



Once you have calculated the net vertical and horizontal forces, you can resolve them into a single net force, as in the previous example. (Because the vertical component of the net force is so small, an extra digit is necessary in order to see the difference between the total net force and its horizontal component.)



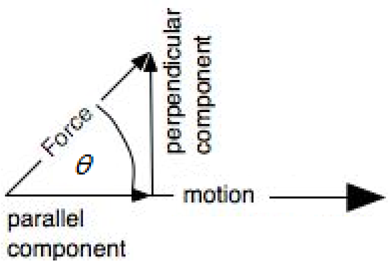
To find the angle of the resultant force, $\tan\theta = \left(\frac{-1.32}{20.95}\right) = (-0.630)$, which means

$\theta = \tan^{-1}(-0.630) = -3.6°$.

Use this space for summary and/or additional notes:

honors & AP®

In some physics problems, a force is applied at an angle but the object can move in only one direction. A common problem is a force applied at an angle to an object resting on a flat surface, which causes the object to move horizontally:



In this situation, only the horizontal (parallel) component of the applied force $(F_{||})$ actually causes the object to move. If magnitude of the total force is *F*, then the horizontal component of the force is given by:

$$F_x = F_{||} = F\cos\theta$$

If the object accelerates horizontally, that means ***only*** the horizontal component is causing the acceleration, which means the net force must be $F_{||} = F\cos\theta$ and we can ignore the vertical component.

For example, suppose the worker in the diagram at the right pushes on the hand truck with a force of 200 N at an angle of 60°.

The force in the direction of motion (horizontally) would be:

$$F_{||} = F\cos\theta = 200\cos(60°)$$
$$= (200)(0.5) = 100\text{ N}$$

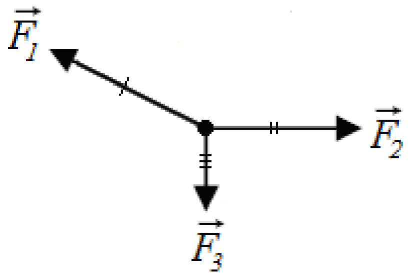
In other words, if the worker applies 200 N of force at an angle of 60°, the resulting ***horizontal*** force will be 100 N.

Use this space for summary and/or additional notes:

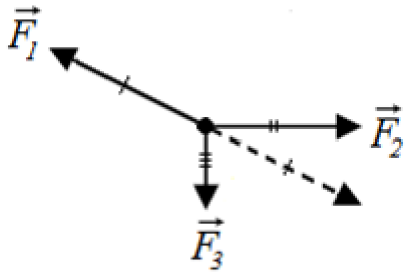
honors & AP®

Static Problems Involving Forces at an Angle

Many problems involving forces at an angle are based on an object with no net force (either a stationary object or an object moving at constant velocity) that has three or more forces acting at different angles. In the following diagram, the forces are $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$.



$\vec{F}_1$ needs to cancel the resultant of $\vec{F}_2$ and $\vec{F}_3$:



Of course, $\vec{F}_2$ will also cancel the resultant of $\vec{F}_1$ and $\vec{F}_3$, and $\vec{F}_3$ will also cancel the resultant of $\vec{F}_1$ and $\vec{F}_2$.

Strategy

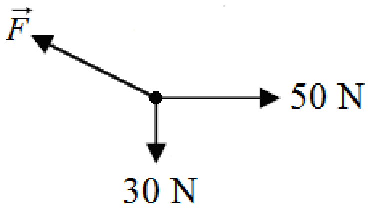
1. Resolve all known forces into their horizontal and vertical components.
2. Add the horizontal and vertical components separately.
3. Use the Pythagorean Theorem to find the magnitude of forces that are neither horizontal nor vertical.
4. Because you know the vertical and horizontal components of the resultant force, use arcsine ($\sin^{-1}$), arccosine ($\cos^{-1}$) or arctangent ($\tan^{-1}$) to find the angle.

Use this space for summary and/or additional notes:

honors & AP®

Sample Problems:

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitude and direction of $\vec{F}$?

A: $\vec{F}$ is equal and opposite to the resultant of the other two vectors. The magnitude of the resultant is:

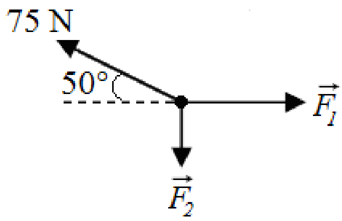
$$\|\vec{F}'\| = \sqrt{30^2 + 50^2} = \sqrt{3400} = 58.3\,\text{N}$$

The direction is:

$$\tan\theta = \frac{30}{50} = 0.6$$

$$\theta = \tan^{-1}(\tan\theta) = \tan^{-1}(0.6) = 31.0^\circ \text{ up from the left (horizontal)}$$

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitudes of $\vec{F}_1$ and $\vec{F}_2$?

A: $\vec{F}_1$ and $\vec{F}_2$ are equal and opposite to the vertical and horizontal components of the 75 N force, which we can find using trigonometry:

$$\|\vec{F}_1\| = \text{horizontal} = 75\cos(50^\circ) = (75)(0.643) = 48.2\,\text{N}$$

$$\|\vec{F}_2\| = \text{vertical} = 75\sin(50^\circ) = (75)(0.766) = 57.5\,\text{N}$$

Use this space for summary and/or additional notes:

honors & AP®

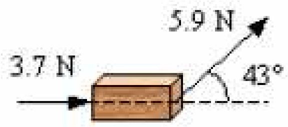
Homework Problems

1. **(M – honors; A – CP1)** An object has three forces acting on it, a 15 N force pushing to the right, a 10. N force pushing to the right, and a 20. N force pushing to the left.

   a. **(M – honors; A – CP1)** Draw a free-body diagram for the object showing each of the forces that acts on the object (including a legend showing which direction is positive).

   b. **(M – honors; A – CP1)** Calculate the magnitude of the net force on the object.

2. **(M – honors; A – CP1)** A force of 3.7 N horizontally and a force of 5.9 N at an angle of 43° act on a 4.5-kg block that is resting on a frictionless surface, as shown in the following diagram:



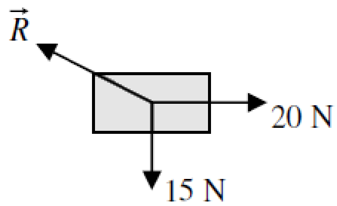
   What is the magnitude of the horizontal acceleration of the block?

   Answer: $1.8\,\frac{m}{s^2}$

Use this space for summary and/or additional notes:

honors & AP®

3. **(S – honors; A – CP1)** A stationary block has three forces acting on it: a 20. N force to the right, a 15 N force downwards, and a third force, $\vec{R}$ of unknown magnitude and direction, as shown in the diagram to the right:



   a. **(S – honors; A – CP1)** What are the horizontal and vertical components of $\vec{R}$?

   b. **(S – honors; A – CP1)** What is the magnitude of $\vec{R}$?

   Answer: 25 N

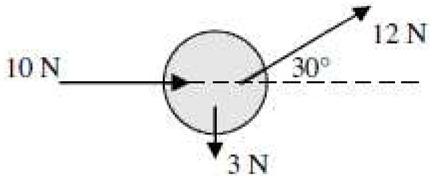
   c. **(S – honors; A – CP1)** What is the direction (angle up from the horizontal) of $\vec{R}$?

   Answer: 36.9°

Use this space for summary and/or additional notes:

honors & AP®

4. **(S – honors; A – CP1)** Three forces act on an object. One force is 10. N to the right, one force is 3.0N downwards, and one force is 12 N at an angle of 30.° above the horizontal, as shown in the diagram below.



a. **(S – honors; A – CP1)** What are the net vertical and horizontal forces on the object?

Answer: positive directions are up and to the right.
vertical: +3.0 N; horizontal: +20.4 N

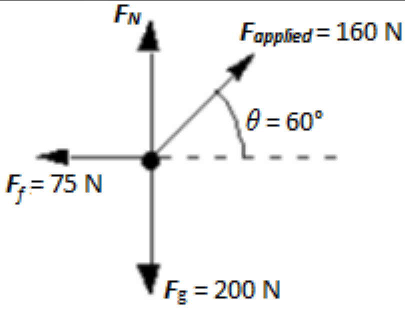
b. **(S – honors; A – CP1)** What is the net force (magnitude and direction) on the object?

Answer: 20.6 N at an angle of +8.4°

Use this space for summary and/or additional notes:

honors & AP®

5. **(M – AP®; S – honors; A – CP1)** An applied force of 160 N ($\vec{F}_{applied}$) pulls at an angle of 60° (θ) on a crate that is sitting on a rough surface. The weight of the crate ($\vec{F}_g$) is 200 N. The force of friction on the crate ($\vec{F}_f$) is 75 N. These forces are shown in the diagram to the right.



Using the variables but not the quantities from the diagram, derive an expression for the magnitude of the normal force ($\vec{F}_N$) on the crate, in terms of the given quantities $\vec{F}_{applied}$, $\vec{F}_g$, $\vec{F}_f$, θ, and natural constants (such as $\vec{g}$).

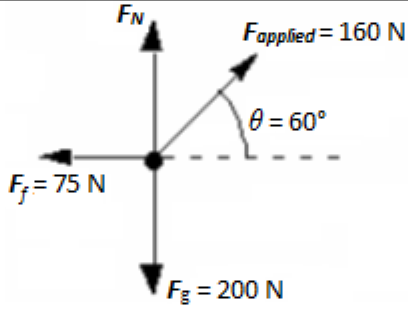
(*If you are not sure how to solve this problem, do #6 below and use the steps to guide your algebra.*)

Answer: $\vec{F}_N = \vec{F}_g - \vec{F}_{applied} \sin\theta$

Use this space for summary and/or additional notes:

honors & AP®

6. **(M – honors; A – CP1)** An applied force of 160 N ($\vec{F}_{applied}$) pulls at an angle of 60° (θ) on a crate that is sitting on a rough surface. The weight of the crate ($\vec{F}_g$) is 200 N. The force of friction on the crate ($\vec{F}_f$) is 75 N. These forces are shown in the diagram to the right.



(*<u>You must start with the equations in your</u>* <u>Physics Reference Tables</u> *<u>and show all of the steps of GUESS.</u> You may only use the answer to question #5 above as a starting point if you have already solved that problem.*)

a. What is the magnitude of the normal force ($\vec{F}_N$) on the crate?

Answer: 61 N

b. What is the acceleration of the crate?

Answer: $0.25\,\frac{m}{s^2}$

Use this space for summary and/or additional notes: