Centripetal Force

Unit: Rotational Statics & Dynamics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.9.A, 2.9.A.1, 2.9.A.1.i, 2.9.A.1.ii, 2.9.A.2, 2.9.A.2.i, 2.9.A.2.ii, 2.9.A.2.iii, 2.9.A.3, 2.9.A.4, 2.9.A.5, 2.9.A.5.i, 2.9.A.5.ii, 2.9.A.5.iii

Mastery Objective(s): (Students will be able to…)

- Explain qualitatively the forces involved in circular motion.
- Describe the path of an object when it is released from circular motion.
- Calculate the velocity and centripetal force of an object that is in uniform circular motion.

Success Criteria:

- Explanations account for constant change in direction.
- Variables are correctly identified and substituted correctly into the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why centripetal force is always toward the center of the circle.

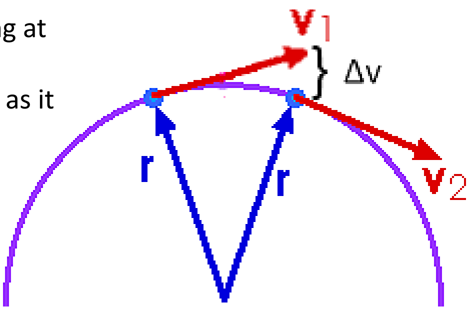
Tier 2 Vocabulary: centripetal, centrifugal

Labs, Activities & Demonstrations:

- Swing a bucket of water in a circle.
- Golf ball loop-the-loop.
- Spin a weight on a string and have the weight pull up on a mass or spring scale.

Notes:

As we saw previously, when an object is moving at a constant speed around a circle, its direction keeps changing toward the center of the circle as it goes around, which means *there is continuous acceleration toward the center of the circle.*



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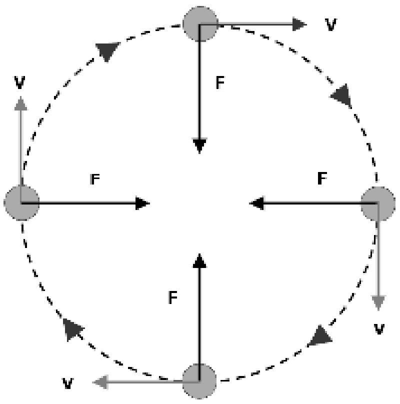
Because acceleration is caused by a net force (Newton's second law of motion), if there is continuous acceleration toward the center of the circle, then there must be a continuous force toward the center of the circle.



This force is called "centripetal force".

centripetal force: the inward force that keeps an object moving in a circle. If the centripetal force were removed, the object would fly away from the circle in a straight line that starts from a point tangent to the circle.

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Recall that the equation* for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r} = r\omega^2$$

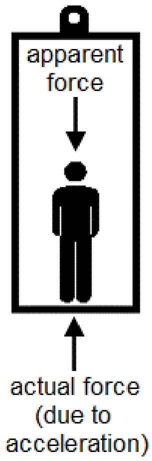
Given that $F = ma$, the equation for centripetal force is therefore:

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$

If you are in the reference frame of the object that is moving in a circle, you are being accelerated toward the center of the circle. You feel a force that appears to be pushing or pulling you away from the center of the circle. This is called "centrifugal force".

centrifugal force: the outward force felt by an object that is moving in a circle.

Centrifugal force is called a "fictitious force" because it does not exist in an inertial reference frame. However, centrifugal force does exist in a rotating reference frame; it is the inertia of objects resisting acceleration as they are continuously pulled toward the center of a circle by centripetal acceleration.



This is the same as the feeling of increased weight that you feel when you are in an elevator and it starts to move upwards (which is also a moving reference frame). An increase in the normal force from the floor because of the upward acceleration of the elevator feels the same as an increase in the downward force of gravity.

* Recall that centripetal motion and centripetal force relates to angular/rotational motion and forces (which are studied in AP® Physics but not in the CP1 or honors courses). Equations or portions of equations with angular velocity (ω) and angular acceleration (α) apply only to the AP® course.

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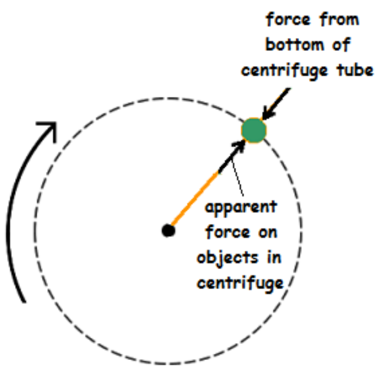
Similarly, a sample being spun in a centrifuge is subjected to the force *from the bottom of the centrifuge tube* as the tube is accelerated toward the center. The faster the rotation, the stronger the force. An increase in the normal force from the bottom of the centrifuge tube would feel like a downward force in the reference frame of the centrifuge tube.



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Sample Problems:

Q: A 300 kg roller coaster car reaches the bottom of a hill traveling at a speed of $20\,\frac{\text{m}}{\text{s}}$. If the track curves upwards with a radius of 50 m, what is the total force exerted by the track on the car?

A: The total force on the car is the normal force needed to resist the force of gravity on the car (equal to the weight of the car) plus the centripetal force exerted on the car as it moves in a circular path.

$$F_g = mg = (300)(10) = 3\,000\,\text{N}$$
$$F_c = \frac{mv^2}{r} = \frac{(300)(20)^2}{50} = 2\,400\,\text{N}$$
$$F_N = F_g + F_c = 3\,000 + 2\,400 = 5\,400\,\text{N}$$

Q: A 20 g ball attached to a 60 cm long string is swung in a horizontal circle 80 times per minute. Neglecting gravity, what is the tension in the string?

A: Converting to MKS units, the mass of the ball is 0.02 kg and the string is 0.6 m long.

We can solve this two ways: we can convert revolutions either to meters by multiplying by $2\pi r$, or to radians by multiplying by 2π:

$$\omega = \frac{80\,\cancel{\text{revolutions}}}{1\,\cancel{\text{min}}} \times \frac{(2\pi)(0.60\,\text{m})}{1\,\cancel{\text{revolution}}} \times \frac{1\,\cancel{\text{min}}}{60\,\text{s}} = \frac{96\pi\,\text{m}}{60\,\text{s}} = 5.03\,\frac{\text{m}}{\text{s}}$$
$$F_T = F_c = \frac{mv^2}{r} = \frac{(0.02)(5.03)^2}{0.6} = 0.842\,\text{N}$$

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$$\omega = \frac{80\,\cancel{\text{revolutions}}}{1\,\cancel{\text{min}}} \times \frac{2\pi\,\text{rad}}{1\,\cancel{\text{revolution}}} \times \frac{1\,\cancel{\text{min}}}{60\,\text{s}} = \frac{160\pi\,\text{rad}}{60\,\text{s}} = 8.38\,\frac{\text{rad}}{\text{s}}$$
$$F_T = F_c = mr\omega^2$$
$$F_T = F_c = (0.02)(0.6)(8.38)^2 = 0.842\,\text{N}$$

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Homework Problems

1. **(M – AP®; A – honors & CP1)** Find the force needed to keep a 0.5 kg ball traveling in a 0.70 m radius circle with an angular velocity of 15 revolutions every 10 s.

   Answer: 31.1 N

2. **(M – honors & AP®; A – CP1)** Find the force of friction needed to keep a 3 000 kg car traveling with a speed of $22\,\frac{\text{m}}{\text{s}}$ around a level highway exit ramp curve that has a radius of 100 m.

   Answer: 14 520 N

3. **(S – honors & AP®; A – CP1)** A passenger on an amusement park ride is cresting a hill in the ride at $15\,\frac{\text{m}}{\text{s}}$. If the top of the hill has a radius of 30 m, what force will a 50 kg passenger feel from the seat? What fraction of the passenger's weight is this?

   Answer: 125 N; $\frac{1}{4}$

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4. **(M – honors & AP®; A – CP1)** A roller coaster has a vertical loop with a 40 m radius. What speed at the top of the loop will make a 60 kg rider feel "weightless?"

   Answer: $20\,\frac{\text{m}}{\text{s}}$

5. **(S – AP®; A – honors & CP1)** A ride called "The Rotor" at Six Flags is a cylinder that spins at 56 RPM, which is enough to "stick" people to the walls. What force would a 90 kg rider feel from the walls of the ride, if the ride has a diameter of 6 m?

   Answer: 9 285 N

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