Rotational Inertia

Unit: Rotational Statics & Dynamics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 5.4.A, 5.4.A.1, 5.4.A.2, 5.4.A.3, 5.4.B, 5.4.B.1, 5.4.B.2

Mastery Objective(s): (Students will be able to…)

- Calculate the moment of (rotational) inertia of a system that includes one or more masses at different radiï from the center of rotation.

Success Criteria:

- Correct formula for moment of inertia of each basic shape is correctly selected.
- Variables are correctly identified and substituted correctly into the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how an object's moment of inertia affects its rotation.

Tier 2 Vocabulary: moment

Labs, Activities & Demonstrations:

- Try to stop a bicycle wheel with different amounts of mass attached to it.

Notes:

inertia: the tendency for an object to continue to do what it is doing (remain at rest or remain in motion).

rotational inertia (or angular inertia): the tendency for a rotating object to continue rotating.

moment of inertia (I): a quantitative measure of the rotational inertia of an object. Moment of inertia is measured in units of $kg{\cdot}m^2$.

Inertia in linear systems is a fairly easy concept to understand. The more mass an object has, the more it tends to remain at rest or in motion, and the more force is required to change its motion. *I.e.,* in a linear system, inertia depends only on mass.

center of mass: the point where all of an object's mass could be placed without changing the results of any forces acting on the object. (See *Center of Mass*, starting on page 268.)

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Rotational Inertia

Rotational inertia is somewhat more complicated than the inertia in a non-rotating system. Suppose we have a mass that is being rotated at the end of a string. (Let's imagine that we're doing this in space, so we can neglect the effects of gravity.) The mass's inertia keeps it moving around in a circle at the same speed. If you suddenly shorten the string, the mass continues moving at the *same speed through the air*, but because the radius is shorter, the mass makes more revolutions around the circle in a given amount of time.

In other words, the object has the same linear speed (*not* the same *velocity* because its *direction* is constantly changing), but its angular velocity (degrees per second) has increased.

This must mean that an object's moment of inertia (its tendency to continue moving at a constant angular velocity) must depend on its distance from the center of rotation as well as its mass.

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The formula for moment of inertia is:

$$I = \sum_i m_i r_i^2$$

I.e., for each object or component (designated by a subscript), first multiply mr^2 for the object and then add up the rotational inertias for each of the objects to get the total.

For a point mass (a simplification that assumes that the entire mass exists at a single point):

$$I = mr^2$$

This means the rotational inertia of the point-mass is the same as the rotational inertia of the object.

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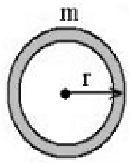
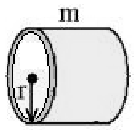
Calculating the moment of inertia for an arbitrary shape requires calculus. However, for solid, regular objects with well-defined shapes, their moments of inertia can be reduced to simple formulas:

Point Mass at a Distance:
$I = mr^2$

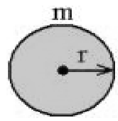
Hollow Sphere:
$I = \frac{2}{3}mr^2$



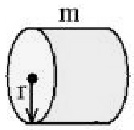
Hollow Cylinder:
$I = mr^2$



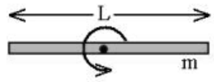
Solid Sphere:
$I = \frac{2}{5}mr^2$



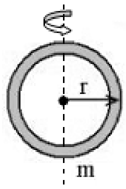
Solid Cylinder:
$I = \frac{1}{2}mr^2$



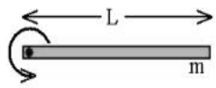
Rod about the Middle:
$I = \frac{1}{12}mL^2$



Hoop about Diameter:
$I = \frac{1}{2}mr^2$



Rod about the End:
$I = \frac{1}{3}mL^2$



In the above table, note that a rod can have a cross-section of any shape; for example a door hanging from its hinges is considered a rod rotated about the end for the purpose of determining its moment of inertia.

Sample Problem:

Q: A solid brass cylinder has a density of $8\,500\,\frac{\text{kg}}{\text{m}^3}$, a radius of 0.10 m and a height of 0.20 m and is rotated about its center. What is its moment of inertia?

A: In order to find the mass of the cylinder, we need to use the volume and the density.

$$V = \pi r^2 h = (3.14)(0.1)^2(0.2)$$
$$V = 0.00628\,\text{m}^3$$

$$\rho = \frac{m}{V}$$
$$8\,500 = \frac{m}{0.006\,28}$$
$$m = 53.4\,\text{kg}$$

Now that we have its mass, we can find the moment of inertia of the cylinder:

$$I = \frac{1}{2}mr^2$$
$$I = \frac{1}{2}(53.4)(0.1)^2 = 0.534\,\text{kg}\cdot\text{m}^2$$

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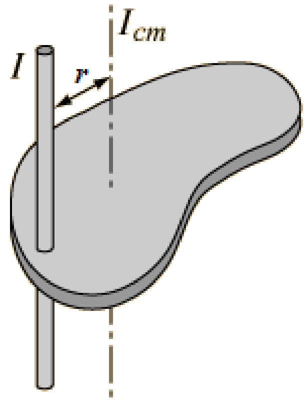
Parallel Axis Theorem

The moment of inertia of any object about an axis through its center of mass is always the minimum moment of inertia for any axis in that direction in space.

The moment of inertia about another axis that is parallel to the axis through the center of mass, at a distance $\vec{r}$ from the object's center of mass, is given by the equation:

$$I_{\text{parallel axis}} = I_{\text{cm}} + mr^2$$

You would use the parallel axis theorem if you have a mass that is forced to rotate around some axis other than its center of mass, such as the following example:



This can be demonstrated by spinning a "bicycle wheel" with handles, then attaching a 0.5-kg mass to the outside of the wheel and spinning it again. The new center of mass of the system is no longer where the handles are; when the wheel is spun, it requires a significant amount of force (*i.e.,* more than students are capable of applying) to keep it from wobbling. This is why car wheels or washing machines that are "out of balance" wobble in ways that can cause significant damage.

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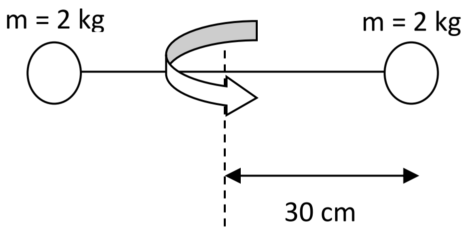
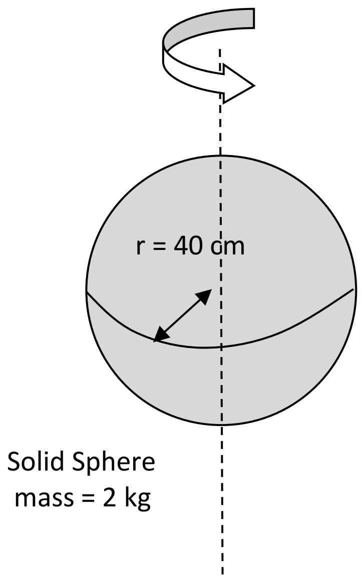
Homework Problems

Find the moment of inertia of each of the following objects. (Note that you will need to convert distances to meters.)

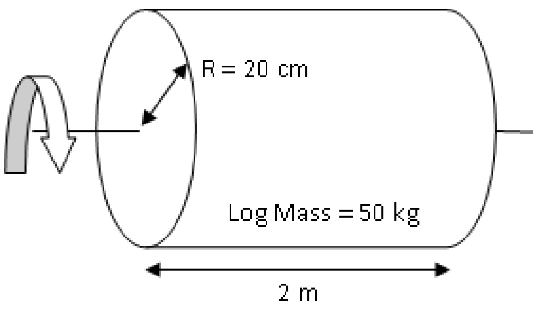
1. **(M – honors & AP®; A – CP1)** Answer: $0.36\,\text{kg}\cdot\text{m}^2$



2. **(M – honors & AP®; A – CP1)** Answer: $0.128\,\text{kg}\cdot\text{m}^2$



3. **(M – honors & AP®; A – CP1)** Answer: $1\,\text{kg}\cdot\text{m}^2$



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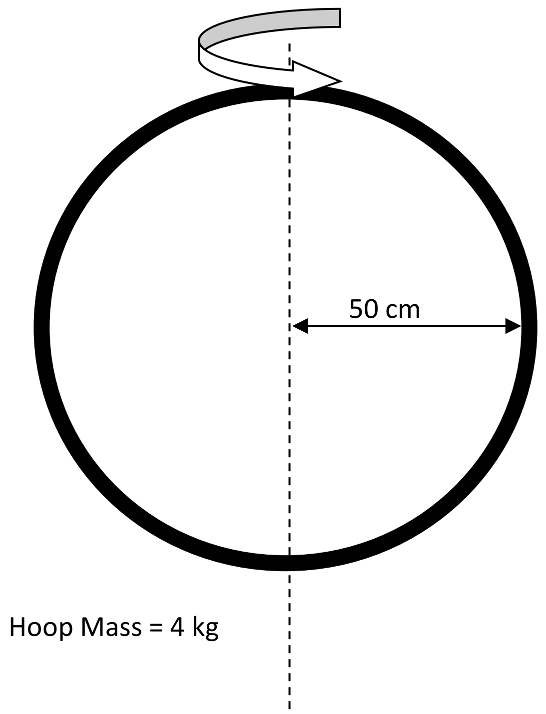
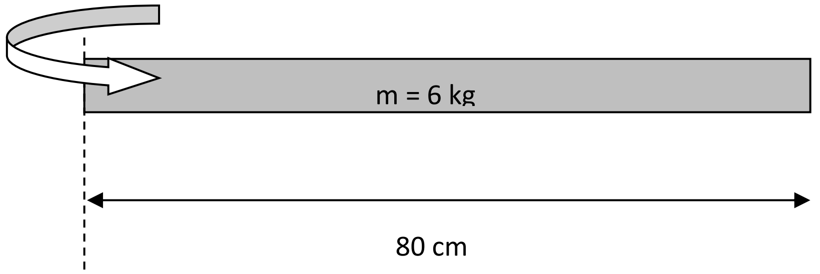
4. **(M – honors & AP®; A – CP1)** Answer: $0.5\,\text{kg}\cdot\text{m}^2$



5. **(M – honors & AP®; A – CP1)** Answer: $1.28\,\text{kg}\cdot\text{m}^2$



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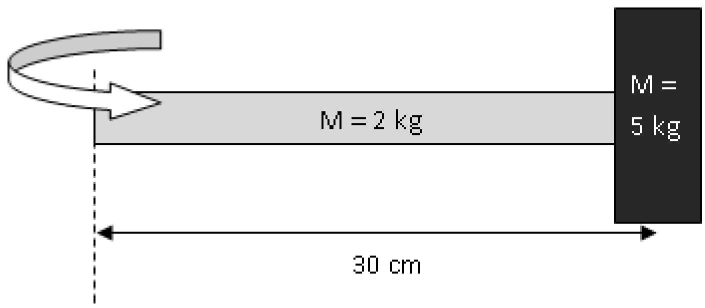
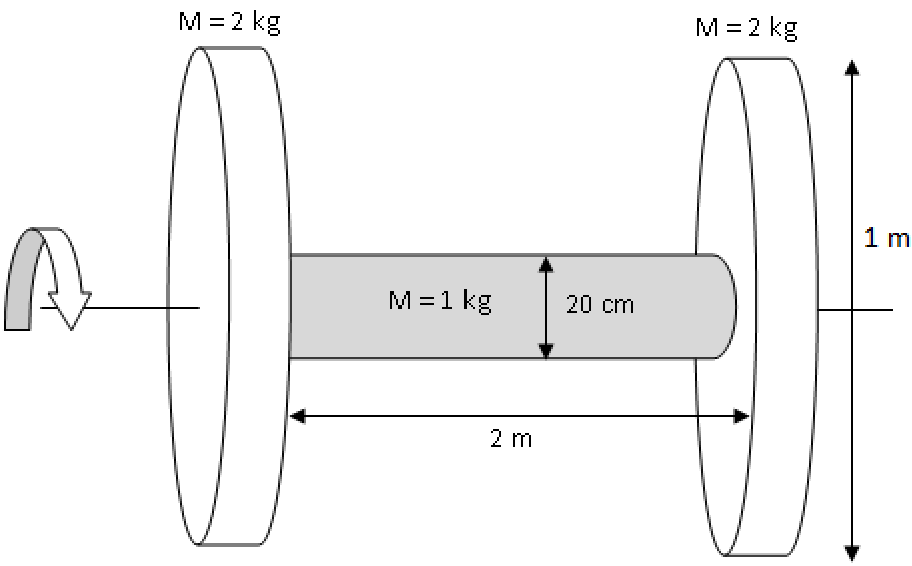
Find the moment of inertia of each of the following compound objects. (Be careful to note when the diagram gives a diameter instead of a radius.)

6. **(M – honors & AP®; A – CP1)** Sledge hammer: Answer: $0.51\,\text{kg}\cdot\text{m}^2$



7. **(M – honors & AP®; A – CP1)** Wheels and axle: Answer: $0.505\,\text{kg}\cdot\text{m}^2$



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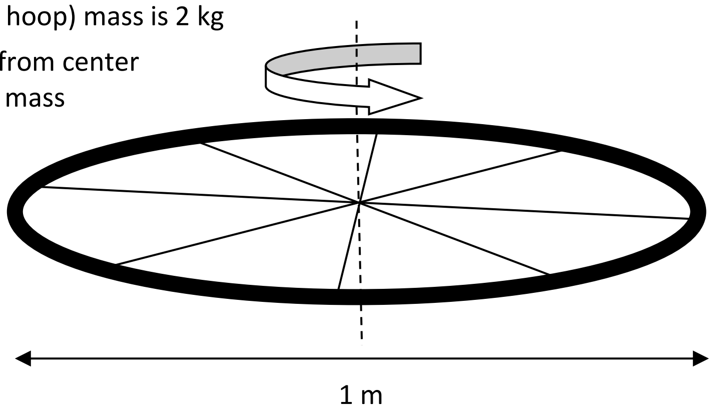
8. **(S – honors & AP®; A – CP1)** Wheel: Answer: $0.8\overline{3}\ \text{kg}\cdot\text{m}^2$

- Rim (outside hoop) mass is 2 kg
- Each spoke (from center to rim) has a mass of 0.5 kg



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