Universal Gravitation

Unit: Gravitation

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.6.A, 2.6.A.1, 2.6.A.1.i, 2.6.A.1.ii, 2.6.A.1.iii, 2.6.A.2, 2.6.A.2.i, 2.6.A.2.ii, 2.6.A.3

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving Newton's Law of Universal Gravitation.
- Assess the effect on the force of gravity of changing one of the parameters in Newton's Law of Universal Gravitation.

Success Criteria:

- All variables are identified and substituted correctly.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how changing each of the parameters in Newton's Law of Universal Gravitation affects the result.

Tier 2 Vocabulary: gravity

Notes:

Approximately 150 years after Copernicus published *On the Revolutions of the Heavenly Spheres*, and approximately 70 years after Kepler published his laws, the English polymath* Sir Isaac Newton realized that a planet is orbiting the sun is an example of circular motion. Due to its inertia, a planet should move in a straight line at a constant velocity. He concluded that there must therefore be a centripetal force that is constantly pulling planets toward the sun and pulling the moon toward the Earth.

It takes the moon 27.3 days to orbit the Earth. (The time from one full moon to the next—the "synodic month"—is 29.5 days, because the moon must also "catch up" with the distance that Earth rotates in that time.) From that plus Kepler's third law, $\frac{T^2}{r^3_{ave.}}$, Newton was able to determine the radius of the moon's orbit, which turns out to be about 60 times the radius of the Earth. From the radius, Newton could calculate the circumference of the moon's orbit $(C = 2\pi r)$ and therefore the average velocity of the moon as it orbits the Earth $\left(v_{ave.} = \frac{d}{t}\right)$.

* A polymath is a person whose knowledge spans many different subjects, known to draw on complex bodies of knowledge to solve specific problems. Newton was a mathematician, theoretical physicist, astronomer, alchemist, theologian, and author, and one of the inventors of calculus. Newton and Benjamin Franklin are two famous polymaths.

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From this velocity and the equation for centripetal acceleration, $a_c = \frac{v^2}{r}$, Newton could calculate the centripetal acceleration of the moon. The mass of the moon must be constant, so based on his own second law, $F_{net} = ma$, Newton concluded that the force attracting the moon to the Earth must therefore be inversely proportional to the square of the distance between the Earth and the moon:

$$F_g \propto \frac{1}{r^2}$$

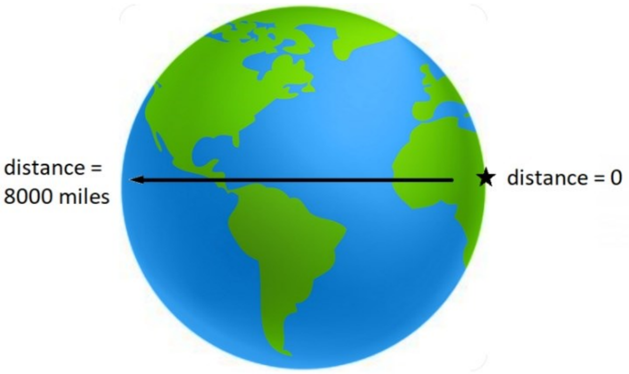
Newton further reasoned (also from his second law) that this attraction must be proportional to the mass. This would therefore mean that *every object that has mass* must attract *every other object that has mass*, and that the more mass an object has, the more attractive it must therefore be.

Therefore, if two planets (or any other two objects with mass) attract each other, the force would be directly proportional to the product of the masses. Therefore:

$$F_g \propto \frac{m_1 \cdot m_2}{r^2}$$

Sir Isaac Newton first published this equation in *Philosophiæ Naturalis Principia Mathematica* in 1687.

The gravitational force at every point on the surface of the Earth is approximately the same. (There are slight differences because the Earth is not a perfect sphere and because its density is not completely uniform.) If we are on the surface of the Earth, our distance from the part of the Earth that we are standing on is zero, but our distance from a point on the opposite side of the Earth would be the diameter of the Earth, which is about 8 000 miles.



This means that our average distance from any point on Earth must be the distance to the *center of mass of the Earth* (which is approximately at the center of the Earth), and therefore, the force of gravity on the surface of the Earth must be proportional to the square of the Earth's average radius, which is 6.37×10^6 m (a little less than 4 000 miles).

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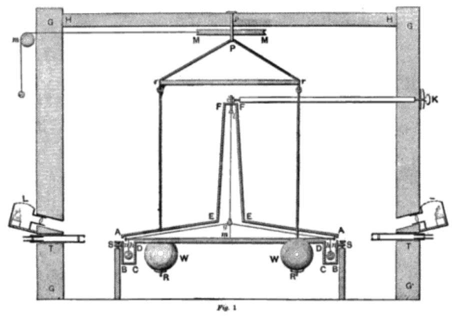

In 1797–1798, English scientist Henry Cavendish concluded that if all masses attract one another, it should be possible to measure this attraction with very sensitive equipment. Cavendish built a large torsional balance, which contained two small lead spheres (about 2 inches in diameter and a mass of 0.73 kg ≈ 1.6 lbs.). When he placed two much larger lead spheres (12 inches in diameter and a mass of 158 kg ≈ 348 lbs.) near them, he was able to measure the torsional force applied to the wire, which turned out to be quite small: 1.74×10^{-7} N. From Cavendish's experiment, it was possible to determine the value of the constant, *G*, that would turn Newton's proportion into an equation. Cavendish calculated the value of G to be $6.74\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ *, which is very close to the currently-accepted value of $6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$. Thus, the universal gravitation equation becomes:

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})m_1m_2}{r^2}$$

This relationship is the universal gravitation equation, which we saw earlier in the section on the *Gravitational Force*, starting on page 278.

Discovery of Neptune

In the 1820s, irregularities were discovered in the orbit of Uranus. In 1845, the French mathematician and astronomer Urbain Le Verrier theorized that the gravitational force from another undiscovered planet must be causing Uranus' unusual behavior. Based on calculations using Kepler's and Newton's laws, Le Verrier predicted the existence and location of this new planet and sent his calculations to astronomer Johann Galle at the Berlin Observatory. Based on Le Verrier's work, Galle found the new planet on the night that he received Le Verrier's letter—September 23–24, 1846—within one hour of starting to look, and within 1° of its predicted position. Le Verrier's feat—predicting the existence and location of Neptune using only mathematics, was one of the most remarkable scientific achievements of the 19th century and a dramatic validation of celestial mechanics.

* The units are simply the ones needed to cancel the m^2 and kg^2 from the formula and give newtons, which is the desired unit.

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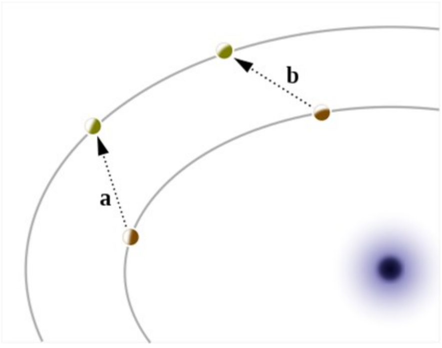

This diagram shows the orbits of Uranus (inner arc) and Neptune (outer arc). The planets are both orbiting from the top right to the bottom left.

At position *b*, the gravitational force from Neptune pulls ahead of its predicted location. At position *a*, the gravitational force pulls back on Uranus, leaving it behind its predicted location.

Diagram by R.J. Hall. Used with permission.

Relationship between *G* and *g*

As we saw in the section on the *Gravitational Force, t*he strength of the gravitational field anyplace in the universe can be calculated from the universal gravitation equation.

If m_1 is the mass of the planet (moon, star, *etc.*) that we happen to be standing on and m_2 is the object that is being attracted by it, we can divide the universal gravitation equation by m_2, which gives us:

$$\frac{F_g}{m_2} = \frac{Gm_1 \cancel{m_2}}{r^2 \cancel{m_2}} = \frac{Gm_1}{r^2}$$

as we saw previously.

Therefore, $g = \frac{Gm_1}{r^2}$ where m_1 is the mass of the planet in question and r is its radius.

If we wanted to calculate the value of g on Earth, m_1 would be the mass of the Earth $(5.97\times10^{24}\text{ kg})$ and r would be the radius of the Earth $(6.38\times10^{6}\text{ m})$. Substituting these numbers into the equation gives:

$$g = \frac{Gm_1}{r^2} = \frac{(6.67\times10^{-11})(5.97\times10^{24})}{(6.38\times10^{6})^2} = 9.81\,\frac{\text{N}}{\text{kg}}\ ^*$$

* In most places in this book, we round g to $10\,\frac{\text{N}}{\text{kg}}$ to simplify calculations. However, if we are using 3 significant figures for the terms in this equation, we should express g to 3 significant figures as well. Note, however, that the value of g varies because the Earth does not have a uniform density, and because the distance from any given point on Earth's surface to the center (of mass) of the Earth varies. The reason for the latter is that the Earth is a heterogeneous mixture, not a single solid object; the inertia of the particles as the Earth spins causes its equator to bulge ("equatorial bulge"), which takes mass from the poles ("polar flattening"). For example, the value of g in Boston, Massachusetts is approximately $9.80\,\frac{\text{N}}{\text{kg}}$.

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Sample Problems:

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is 5.97 × 10[24] kg, and its radius is 6.37 × 10[6] m.

A: $F_g = \frac{Gm_1m_2}{r^2}$

$$F_g = \frac{(6.67\times10^{-11})(5.97\times10^{24})(75)}{(6.38\times10^{6})^2}$$

$$F_g = 736\,\text{N}$$

This is the same number that we would get using $F_g = mg$, with $g = 9.81\,\frac{\text{N}}{\text{kg}}$.

Note that if we use the approximation $g = 10\,\frac{\text{N}}{\text{kg}}$ (which is about 2 % higher), we get $F_g = 750\,\text{N}$.

Q: Find the acceleration due to gravity on the moon.

A: $g_{moon} = \frac{Gm_{moon}}{r^2_{moon}}$

$$g_{moon} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2} = 1.62\,\frac{\text{N}}{\text{kg}} \equiv 1.62\,\frac{\text{m}}{\text{s}^2}$$

Q: If the distance between an object and the center of mass of a planet is tripled, what happens to the force of gravity between the planet and the object?

A: There are two ways to solve this problem.

Starting with $F_g = \frac{Gm_1m_2}{r^2}$, if we replace r with 3r, we would get:

$$F'_g = \frac{Gm_1m_2}{(3r)^2} = \frac{Gm_1m_2}{9r^2} = \frac{1}{9}\cdot\frac{Gm_1m_2}{r^2}$$

A useful shortcut for these kinds of problems is to set them up as “before and after” problems, using the number 1 for every quantity on the “before” side, and replacing the ones that change with their new values on the “after” side. This shortcut is often called “the rule of 1s”:

Before	**After**
$F_g = \frac{1\cdot1\cdot1}{1^2} = 1$	$F'_g = \frac{1\cdot1\cdot1}{3^2} = \frac{1}{9}$

Thus F'_g is $\frac{1}{9}$ of the original F_g.

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Homework Problems

You will need to use data from *Table T. Planetary Data* and *Table U. Sun & Moon Data* on page 580 of your Physics Reference Tables.

1. **(M)** Find the force of gravity between the earth and the sun.

   Answer: 3.52×10^{22} N

2. **(M)** Find the acceleration due to gravity (the value of *g*) on the planet Mars.

   Answer: $3.70\,\frac{m}{s^2}$

3. **(S)** A mystery planet in another part of the galaxy has an acceleration due to gravity of $5.0\,\frac{m}{s^2}$. If the radius of this planet is 2.0×10^{6} m, what is its mass?

   Answer: 3.0×10^{23} kg

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4. A person has a mass of 80. kg.

   a. **(S)** What is the weight of this person on the surface of the Earth?

      You may use $\vec{F}_g = m\vec{g}$ *for this problem, but use* $\vec{g} = 9.81\,\frac{\text{N}}{\text{kg}}$ *instead of* $\vec{g} = 10\,\frac{\text{N}}{\text{kg}}$ *so you will get the same answer as you would get with the universal gravitation equation.)*

      Answer: 785 N

   b. **(M – honors & AP®; S – CP1)** What is the weight of the same person when orbiting the Earth at a height of 4.0×10^6 m above its surface?

      (*Hint: Remember that Earth's gravity is calculated from the center of mass of the Earth. Therefore, the "radius" in this problem is the distance from the center of the Earth to the spaceship, which includes* <u>*both*</u> *the radius of the Earth* <u>*and*</u> *the distance from the Earth's surface to the spaceship. It may be helpful to draw a sketch.*)

      Answer: 296N

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