Universal Gravitation

Unit: Gravitation

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 2.6.A, 2.6.A.1, 2.6.A.1.i, 2.6.A.1.ii, 2.6.A.1.iii, 2.6.A.2, 2.6.A.2.i, 2.6.A.2.ii, 2.6.A.3

Mastery Objective(s): (Students will be able to…)

- Set up and solve problems involving Newton's Law of Universal Gravitation.
- Assess the effect on the force of gravity of changing one of the parameters in Newton's Law of Universal Gravitation.

Success Criteria:

- All variables are identified and substituted correctly.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how changing each of the parameters in Newton's Law of Universal Gravitation affects the result.

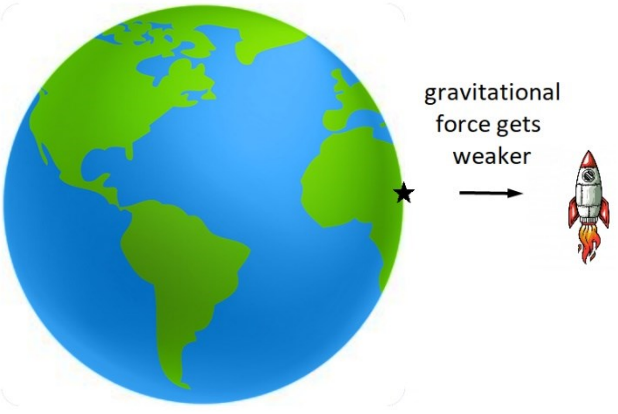
Tier 2 Vocabulary: gravity

Notes:

Gravity is a force of attraction between two objects because of their mass. The cause of this attraction is not currently known, though the most popular theory is that it is a force mediated by an elementary particle called a graviton.

An object with more mass causes a stronger gravitational force, which means "the more mass you have, the more attractive you are."

However, the force gets weaker as the object gets farther away.



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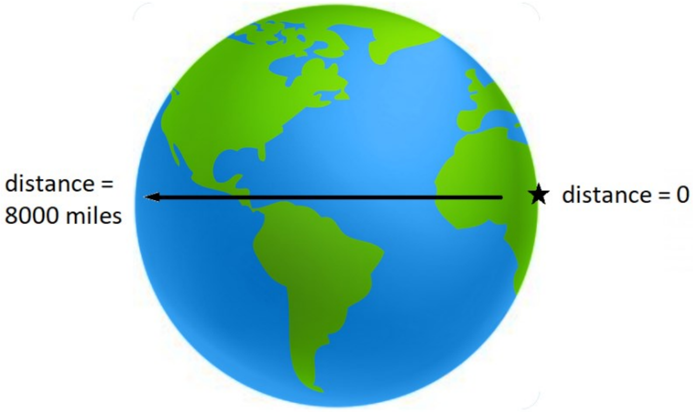
If we are on the Earth, our distance from the part of the Earth that we are standing on is zero, but our distance from the opposite side of the Earth would be the diameter of the Earth, which is about 8 000 miles.



This means that we need to measure distance from the *center of mass of the Earth*, which is approximately the center of the Earth. If we are on the surface of the Earth, this distance would be the radius of the Earth, which is 6.37×10^6 m (a little less than 4 000 miles).

If we represent the gravitational pull of the Earth as a fraction, it would be proportional to:

$$\frac{\text{mass of Earth}}{\text{distance from center of Earth}}$$

If we write this as an equation, using the mathematical symbol $\propto$, which "is proportional to", it would look like:

$$F_g \propto \frac{m}{r}$$

However, all objects with mass have gravity. If we have two objects, such as the Earth and the sun, they pull on each other. This means that the total gravitational pull between the Earth and the sun would be:

$$\left(\frac{\text{mass of sun}}{\text{distance from the sun}}\right) \cdot \left(\frac{\text{mass of Earth}}{\text{distance from the Earth}}\right)$$

If we call the sun #1 and the Earth #2, this would give us:

$$F_g \propto \frac{m_1}{r_1} \cdot \frac{m_2}{r_2}$$

However, because the distance from the sun to the Earth is the same as the distance from the Earth to the sun, $r_1 = r_2 = r$, which means:

$$F_g \propto \frac{m_1 m_2}{r_1 r_2} = \frac{m_1 m_2}{r \cdot r} = \frac{m_1 m_2}{r^2}$$

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However, we want an equation, not a proportion.

If we multiply the right side of the equation, using the masses in kilograms and the distance in meters, we would get a much larger number than the actual force (in newtons). This means we have to include the conversion factor, which is called the "universal gravitational constant". This constant turns out to be $6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$.

(The units are because they cancel the m^2 and kg^2 from the formula and give newtons, which is the desired unit.) The symbol used for this constant is *G*. Thus our formula becomes:

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})m_1m_2}{r^2}$$

This relationship is the universal gravitation equation, which we saw in the section on the *Gravitational Force*, starting on page 279. Sir Isaac Newton first published this equation in *Philosophiæ Naturalis Principia Mathematica* in 1687.

Discovery of Neptune

In the 1820s, irregularities were discovered in the orbit of Uranus. In 1845, the French mathematician and astronomer Urbain Le Verrier theorized that the gravitational force from another undiscovered planet must be causing Uranus' unusual behavior. Based on calculations using Kepler's and Newton's laws, Le Verrier predicted the existence and location of this new planet and sent his calculations to astronomer Johann Galle at the Berlin Observatory. Based on Le Verrier's work, Galle found the new planet on the night that he received Le Verrier's letter—September 23–24, 1846—within one hour of starting to look, and within 1° of its predicted position. Le Verrier's feat—predicting the existence and location of Neptune using only mathematics, was one of the most remarkable scientific achievements of the 19th century and a dramatic validation of celestial mechanics.

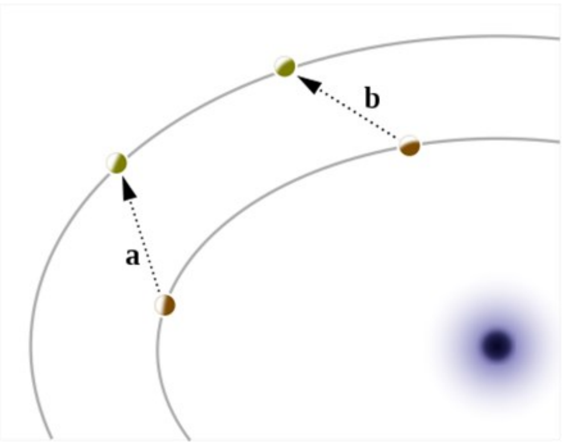

Diagram by R.J. Hall. Used with permission.

This diagram shows the orbits of Uranus (inner arc) and Neptune (outer arc). The planets are both orbiting from the top right to the bottom left.

At position *b*, the gravitational force from Neptune pulls Uranus ahead of its predicted location. At position *a*, the gravitational force pulls back on Uranus, leaving it behind its predicted location.

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Relationship between G and g

As we saw in the section on the *Gravitational Force,* the strength of the gravitational field anyplace in the universe can be calculated from the universal gravitation equation.

If m_1 is the mass of the planet (moon, star, *etc.*) that we happen to be standing on and m_2 is the object that is being attracted by it, we can divide the universal gravitation equation by m_2, which gives us:

$$\frac{F_g}{m_2} = \frac{Gm_1 \cancel{m_2}}{r^2 \cancel{m_2}} = \frac{Gm_1}{r^2}$$

as we saw previously.

Therefore, $g = \frac{Gm_1}{r^2}$ where m_1 is the mass of the planet in question and r is its radius.

If we wanted to calculate the value of g on Earth, m_1 would be the mass of the Earth $(5.97 \times 10^{24}\text{ kg})$ and r would be the radius of the Earth $(6.38 \times 10^{6}\text{ m})$. Substituting these numbers into the equation gives:

$$g = \frac{Gm_1}{r^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.38 \times 10^{6})^2} = 9.81\,\tfrac{\text{N}}{\text{kg}}\,^{*}$$

* In most places in this book, we round g to $10\,\frac{\text{N}}{\text{kg}}$ to simplify the math. However, if we are using 3 significant figures for the terms in this equation, we should express g to 3 significant figures as well. Note, however, that the value of g varies because the distance to the center (of mass) of the Earth varies. The Earth is a heterogeneous mixture, not a single solid object; the inertia of the particles as the Earth spins causes its equator to bulge ("equatorial bulge"), which takes mass from the poles ("polar flattening"). For example, the value of g in Boston, Massachusetts is approximately $9.80\,\frac{\text{N}}{\text{kg}}$.

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Sample Problems:

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is 5.97×10^{24} kg, and its radius is 6.37×10^{6} m.

A: $$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_g = \frac{(6.67\times10^{-11})(5.97\times10^{24})(75)}{(6.38\times10^{6})^2}$$

$$F_g = 736\,\text{N}$$

This is the same number that we would get using $F_g = mg$, with $g = 9.81\,\frac{\text{N}}{\text{kg}}$.

If we use the approximation $g = 10\,\frac{\text{N}}{\text{kg}}$ (which is about 2 % higher), we get $F_g = 750\,\text{N}$.

Q: Find the acceleration due to gravity on the moon.

A: $$g_{moon} = \frac{Gm_{moon}}{r^2_{moon}}$$

$$g_{moon} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2} = 1.62\,\frac{\text{N}}{\text{kg}} \equiv 1.62\,\frac{\text{m}}{\text{s}^2}$$

Q: If the distance between an object and the center of mass of a planet is tripled, what happens to the force of gravity between the planet and the object?

A: There are two ways to solve this problem.

Starting with $F_g = \frac{Gm_1m_2}{r^2}$, if we replace r with $3r$, we would get:

$$F'_g = \frac{Gm_1m_2}{(3r)^2} = \frac{Gm_1m_2}{9r^2} = \frac{1}{9}\cdot\frac{Gm_1m_2}{r^2}$$

A useful shortcut for these kinds of problems is to set them up as “before and after” problems, using the number 1 for every quantity on the “before” side, and replacing the ones that change with their new values on the “after” side. This shortcut is often called “the rule of 1s”:

Before	**After**
$F_g = \frac{1\cdot1\cdot1}{1^2} = 1$	$F'_g = \frac{1\cdot1\cdot1}{3^2} = \frac{1}{9}$

Thus F'_g is $\frac{1}{9}$ of the original F_g.

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Homework Problems

You will need to use data from *Table T. Planetary Data* and *Table U. Sun & Moon Data* on page 567 of your Physics Reference Tables.

1. **(M)** Find the force of gravity between the earth and the sun.

   Answer: 3.52×10^{22} N

2. **(M)** Find the acceleration due to gravity (the value of *g*) on the planet Mars.

   Answer: $3.70\,\frac{m}{s^2}$

3. **(S)** A mystery planet in another part of the galaxy has an acceleration due to gravity of $5.0\,\frac{m}{s^2}$. If the radius of this planet is 2.0×10^{6} m, what is its mass?

   Answer: 3.0×10^{23} kg

Use this space for summary and/or additional notes:

4. A person has a mass of 80. kg.

   a. **(S)** What is the weight of this person on the surface of the Earth?

   You may use $\vec{F}_g = m\vec{g}$ *for this problem, but use* $\vec{g} = 9.81\,\frac{\text{N}}{\text{kg}}$ *instead of* $\vec{g} = 10\,\frac{\text{N}}{\text{kg}}$ *so you will get the same answer as you would get with the universal gravitation equation.)*

   Answer: 785 N

   b. **(M – honors & AP®; S – CP1)** What is the weight of the same person when orbiting the Earth at a height of $4.0\times10^{6}\ \text{m}$ above its surface?

   (*Hint: Remember that Earth's gravity is calculated from the center of mass of the Earth. Therefore, the "radius" in this problem is the distance from the center of the Earth to the spaceship, which includes both the radius of the Earth and the distance from the Earth's surface to the spaceship. It may be helpful to draw a sketch.*)

   Answer: 296N

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