Escape Velocity & Orbits

Unit: Energy, Work & Power

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the velocity that a rocket or spaceship needs in order to escape the pull of gravity of a planet.

Success Criteria:

- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why we can't simply use $\vec{g} = 10\,\frac{m}{s^2}$ to calculate escape velocity.

Tier 2 Vocabulary: escape

Notes:

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth's gravity.

To explain the calculation, we measure height from Earth's surface and use $\vec{g} = 10\,\frac{m}{s^2}$ for the strength of the gravitational field. However, when we calculate the escape velocity of a rocket, the rocket has to go from the surface of the Earth to a point where $\vec{g}$ is small enough to be negligible.

We can still use the conservation of energy, but we need to calculate the potential energy that the rocket has based on its distance from the center of the Earth instead of the surface of the Earth. (When the distance from the Earth is great enough, the gravitational potential energy becomes zero, and the rocket has escaped.) Therefore, the spaceship needs to turn its kinetic energy into potential energy.

To solve this, we need to turn to Newton's Law of Universal Gravitation. Recall from Universal Gravitation starting on page 400 that:

$$F_g = \frac{Gm_1m_2}{r^2}$$

The potential energy equals the work that gravity could theoretically do on the rocket, based on the force of gravity and the distance to the center of the Earth:

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = F_g h = \left(\frac{Gm_1m_2}{r^2}\right)h$$

Because *h* is the distance to the center of the Earth, *h* = *r* and we can cancel, giving the equation:

$$U_g = -\frac{Gm_1m_2}{r}$$

Now, we can use the law of conservation of energy. The kinetic energy that the rocket needs to have at launch equals the potential energy that the rocket has due to gravity. Using m_1 for the mass of the Earth and m_2 for the mass of the spaceship:

$$\begin{aligned}
\textbf{Before} &= \textbf{After} \\
TME_i &= TME_f \\
K_i &= U_f \\
\tfrac{1}{2} \cancel{m_2} v_e^2 &= \frac{Gm_1 \cancel{m_2}}{r} \\
v_e^2 &= \frac{2Gm_E}{r} \\
v_e &= \sqrt{\frac{2Gm_E}{r}}
\end{aligned}$$

Therefore, at the surface of the Earth, where $m_E = 5.97 \times 10^{24}$ kg and $r = 6.37 \times 10^6$ m, this gives $v_e = 1.12 \times 10^4 \ \tfrac{\text{m}}{\text{s}} = 11200 \ \tfrac{\text{m}}{\text{s}}$. (If you're curious, this equals just over 25 000 miles per hour.)

Sample Problem:

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth's escape velocity of $11200\,\frac{m}{s}$ to escape Earth's gravity. What velocity did the spaceship need to achieve in order to escape the moon's gravity and return to Earth? (*I.e.,* what is the escape velocity on the surface of the moon?)

A: $$v_e = \sqrt{\frac{2Gm_{moon}}{d_{moon}}}$$

$$v_e = \sqrt{\frac{(2)(6.67\times10^{-11})(7.35\times10^{22})}{1.74\times10^{6}}}$$

$$v_e = 2370\,\tfrac{m}{s}$$

Orbits

When a satellite is orbiting a planet ("massive central object"), its motion can usually be approximated as a circle. For a satellite in a circular orbit, the following are all constant:

- total mechanical energy
- gravitational potential energy
- rotational kinetic energy
- angular momentum

(*Angular Momentum* is covered later, starting on page 490.)

For a satellite in an elliptical orbit, its total mechanical energy and its angular momentum are constant, but potential and kinetic energy change as distance between the satellite and planet change.