Linear Momentum

Unit: Momentum

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-2

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 4.1.A, 4.1.A.1, 4.1.A.2, 4.1.A.3, 4.1.A.3.i, 4.1.A.3.ii, 4.1.A.3.iii, 4.4.A, 4.4.A.1, 4.4.A.2, 4.4.A.3, 4.4.A.4, 4.4.A.5

Mastery Objective(s): (Students will be able to…)

- Calculate the momentum of an object.
- Solve problems involving collisions in which momentum is conserved.

Success Criteria:

- Masses and velocities are correctly identified as before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the difference between momentum and kinetic energy.

Tier 2 Vocabulary: momentum

Labs, Activities & Demonstrations:

- Collisions on air track.
- Newton's Cradle.
- Ballistic pendulum.

Notes:

In the 17th century, the German mathematician Gottfried Leibnitz recognized the fact that in some cases, the mass and velocity of objects before and after a collision were related by kinetic energy ($\frac{1}{2}mv^2$, which he called the "quantity of motion"); in other cases, however, the "quantity of motion" was not preserved but another quantity (*mv*, which he called the "motive force") was the same before and after. Debate about whether "quantity of motion" or "motive force" was the correct quantity to use for these types of problems continued through the 17th and 18th centuries.

We now realize that both quantities are relevant. "Quantity of motion" is what we now call kinetic energy, and "motive force" is what we now call momentum. The two quantities are different but related.

Momentum is the quantity that is transferred in a ***collision***.

<u>collision</u>: when two or more objects come together, at least one of which is moving, make contact with each other. Momentum is always transferred in a collision.

<u>elastic collision</u>: when two or more objects collide and then separate, with no loss of total kinetic energy. In the real world, elastic collisions are an idealization; only collisions between the particles (molecules) of a gas are perfectly elastic.

<u>inelastic collision</u>: when two or more objects collide, but the total kinetic energy of the objects after the collision is less than it was before it. All real-world collisions are inelastic to some extent. Note that in an inelastic collision, the objects may remain separate or may stick together.

A perfectly inelastic collision is one in which the objects stick together after the collision.

<u>coefficient of restitution</u> (COR) (*e*): a measure of how close a collision is to being perfectly elastic. Sometimes called the "coefficient of elasticity". The restitution equation was developed by Isaac Newton in 1687:

$$e = \frac{|\text{relative velocity of separation after collision}|}{|\text{relative velocity of separation before collision}|} = \frac{|\vec{\boldsymbol{v}}_{2,f} - \vec{\boldsymbol{v}}_{1,f}|}{|\vec{\boldsymbol{v}}_{2,i} - \vec{\boldsymbol{v}}_{1,i}|}$$

where:

- e = coefficient of restitution
- $\vec{\boldsymbol{v}}_{1,i}$ & $\vec{\boldsymbol{v}}_{2,i}$ = initial velocities of objects #1 & #2 (before the collision)
- $\vec{\boldsymbol{v}}_{1,f}$ & $\vec{\boldsymbol{v}}_{2,f}$ = final velocities of objects #1 & #2 (after the collision)

A COR of $e = 1$ represents a (perfectly) elastic collision.

A COR of $e = 0$ represents a perfectly inelastic collision, in which the objects stick together.

A COR between 0 and 1 represents a real-world (inelastic) collision, in which the objects separate after the collision, but with a decrease in total kinetic energy.

Note that in the case of a single object colliding with an immovable object, such as a rubber ball bouncing off the floor, $\vec{\boldsymbol{v}}_{2,i}$ & $\vec{\boldsymbol{v}}_{2,f}$ would both be zero (because object #2 does not exist), and the COR would be simply $e = \frac{|\vec{\boldsymbol{v}}_f|}{|\vec{\boldsymbol{v}}_i|}$

<u>momentum</u> ($\vec{\boldsymbol{p}}$): the amount of force that a moving object could transfer in a given amount of time in a collision. (Formerly called "motive force".)

Momentum is a vector quantity given by the formula:

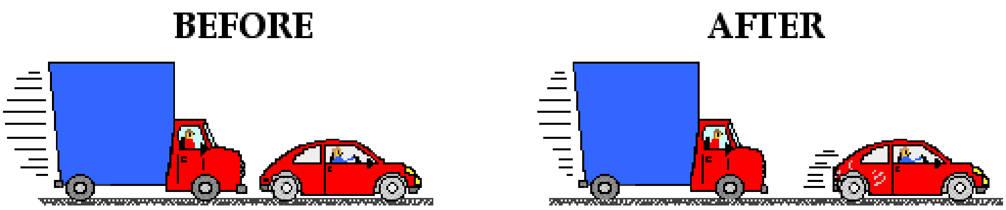
$$\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$$

and is measured in units of $\text{N}\cdot\text{s}$, or $\frac{\text{kg}\cdot\text{m}}{\text{s}}$.

Because momentum is a vector quantity, its sign (positive or negative)* indicates its direction. An object's momentum is in the same direction as (and therefore has the same sign as) its velocity.

An object at rest has a momentum of zero because $\vec{\boldsymbol{v}} = 0$.

As stated above, ***momentum*** is the quantity that is transferred between objects in a collision. For example, consider a collision between a moving truck and a stopped car:



Before the above collision, the truck was moving, so it had momentum; the car was not moving, so it did not have any momentum. After the collision, some of the truck's momentum was transferred to the car. After the collision, both vehicles were moving, which means both vehicles had momentum.

Of course, *total energy* is also conserved in a collision. However, the form of energy can change. Before the above collision, all of the energy in the system was the initial kinetic energy of the truck. Afterwards, some of the energy is the final kinetic energy of the truck, some of the energy is the kinetic energy of the car, and some of the energy is converted to heat, sound, *etc.* during the collision.

* Remember that the use of positive and negative numbers to indicate direction applies only to vectors in one dimension.

inertia: an object's ability to resist the action of a force.

Recall that a net force causes acceleration, which means the inertia of an object is its ability to resist a change in velocity. This means that in linear (translational) systems, inertia is simply mass. In rotating systems, inertia is the moment of inertia, which depends on the mass and the distance from the center of rotation. (See Rotational Inertia on page 365.)

Inertia and momentum are related but are not the same thing; an object has inertia even at rest, when its momentum is zero. An object's momentum changes if either its mass or its velocity changes, but the inertia of an object can change only if its mass changes, or, in the case of rotation, its distribution of mass changes.

Momentum and Kinetic Energy

We have the following equations, both of which relate mass and velocity:

momentum: $\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}$

kinetic energy: $K = \frac{1}{2}mv^2$

We can combine these equations to eliminate v, giving the equation:

$$K = \frac{p^2}{2m}$$

Momentum & Kinetic Energy in Elastic Collisions

Because kinetic energy and momentum must *both* be conserved in an elastic collision, the two final velocities are actually determined by the masses and the initial velocities. The masses and initial velocities are determined before the collision. The only variables are the two velocities after the collision. This means there are two equations (conservation of momentum and conservation of kinetic energy) and two unknowns ($\vec{\boldsymbol{v}}_{1,f}$ and $\vec{\boldsymbol{v}}_{2,f}$).

For a perfectly elastic collision, conservation of momentum states:

$$m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} = m_1\vec{\boldsymbol{v}}_{1,f} + m_2\vec{\boldsymbol{v}}_{2,f}$$

and conservation of kinetic energy states:

$$\tfrac{1}{2}m_1v_{1,i}^2 + \tfrac{1}{2}m_2v_{2,i}^2 = \tfrac{1}{2}m_1v_{1,f}^2 + \tfrac{1}{2}m_2v_{2,f}^2$$

If we use these two equations to solve for $\vec{\boldsymbol{v}}_{1,f}$ and $\vec{\boldsymbol{v}}_{2,f}$ in terms of the other variables, the result is the following:

$$\vec{\boldsymbol{v}}_{1,f} = \frac{\vec{\boldsymbol{v}}_{1,i}(m_1 - m_2) + 2m_2\vec{\boldsymbol{v}}_{2,i}}{m_1 + m_2}$$

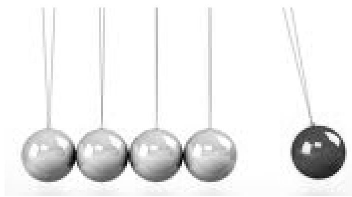
$$\vec{\boldsymbol{v}}_{2,f} = \frac{\vec{\boldsymbol{v}}_{2,i}(m_2 - m_1) + 2m_1\vec{\boldsymbol{v}}_{1,i}}{m_1 + m_2}$$

Momentum & Kinetic Energy in Inelastic Collisions

For an inelastic collision, there is no pair of final velocities that can satisfy both the conservation of momentum and the conservation of kinetic energy, because some of the kinetic energy is "lost" (converted to other forms) and the total kinetic energy after the collision is therefore less than the total kinetic energy before. This matches what we observe, which is that momentum is conserved, but some of the kinetic energy is converted to heat during the collision.

Newton's Cradle

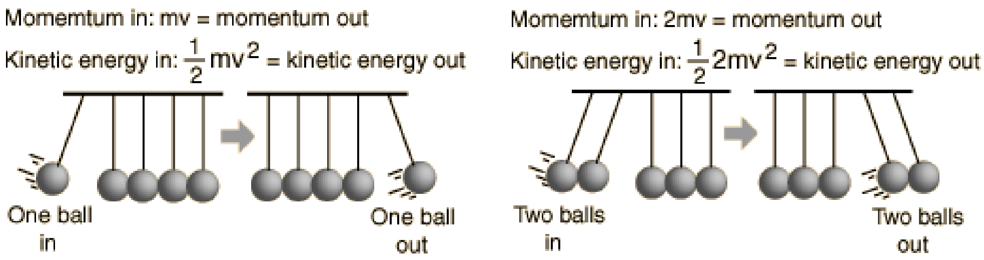
Newton's Cradle is the name given to a set of identical balls that are able to swing suspended from wires, as shown at the right.



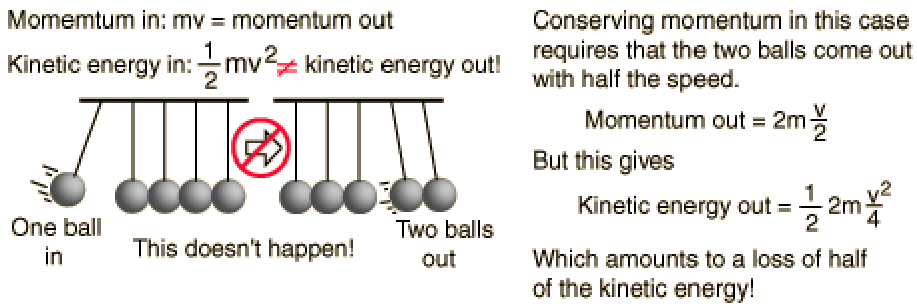
When one ball is swung and allowed to collide with the rest of the balls, the momentum transfers through the balls and one ball is knocked out from the opposite end. When two balls are swung, two balls are knocked out from the opposite end, and so on.

This apparatus demonstrates the relationship between the conservation of momentum and conservation of kinetic energy. When the balls collide, the collision is nearly elastic (has a high coëfficient of restitution), meaning that all of the momentum and most of the kinetic energy are conserved.

Before the collision, the moving ball(s) have momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$). There are no external forces, which means *momentum* must be conserved. The collision is nearly elastic, which means *kinetic energy* is nearly conserved. The only way for the same amount of momentum and almost the same amount of kinetic energy to be present after the collision is for the same number of balls to swing away from the opposite end with the same velocity.



If kinetic energy were not nearly conserved, it would be possible to pull back one ball but for two balls to come out the other side at ½ of the original velocity. However, this doesn't actually happen (unless you attach two of the balls together, *e.g.,* by taping them).



Note also that if there were no losses (friction, drag, *etc.*), the collisions would be perfectly elastic and the balls would continue to swing forever. However, because of friction (between the balls and air molecules, within the strings as they stretch, *etc.*) and conversion of some of the kinetic energy to other forms (such as heat), the balls in a real Newton's Cradle will, of course, slow down and eventually stop. As mentioned earlier in this unit, perfectly elastic collisions do not exist at a macroscopic scale.