Impulse

Unit: Momentum

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-2

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 4.2.A, 4.2.A.1, 4.2.A.2, 4.2.A.3, 4.2.A.4, 4.2.A.5, 4.2.B, 4.2.B.1, 4.2.B.2, 4.2.B.3

Mastery Objective(s): (Students will be able to…)

- Calculate the change in momentum of (impulse applied to) an object.
- Calculate impulse as a force applied over a period of time.
- Calculate impulse as the area under a force-time graph.

Success Criteria:

- Masses and velocities are correctly identified as before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the similarities and differences between impulse and work.

Tier 2 Vocabulary: momentum, impulse

Notes:

impulse ($\vec{\boldsymbol{J}}$): the effect of a force applied over a period of time; the accumulation of momentum.

Mathematically, impulse is a change in momentum, and is also equal to force times time:

$$\Delta\vec{\boldsymbol{p}} = \vec{\boldsymbol{J}} = \vec{\boldsymbol{F}}t \quad \text{and} \quad \vec{\boldsymbol{F}} = \frac{\vec{\boldsymbol{J}}}{t} = \frac{\Delta\vec{\boldsymbol{p}}}{t} = \frac{d\vec{\boldsymbol{p}}}{dt}$$

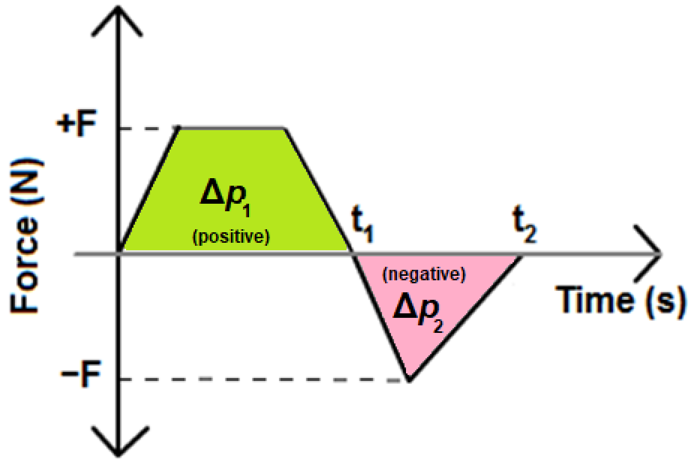
Where $\vec{\boldsymbol{F}}$ is the force vector and, t is time.

Impulse is measured in newton-seconds (N·s), just like momentum.

Impulse is analogous to work:

- Work is a change in energy;
  Impulse is a change in momentum.
- Work is the accumulation of force over a distance ($W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}}$);
  Impulse is the accumulation of force over a time ($\vec{\boldsymbol{J}} = \vec{\boldsymbol{F}}t$)

Just as work is the area under a graph of force *vs.* distance, impulse is the area under a graph of force *vs.* time:



In the above graph, the impulse from time zero to t_1 would be Δp_1. The impulse from t_1 to t_2 would be Δp_2, and the total impulse would be $\Delta p_1 + \Delta p_2$ (keeping in mind that Δp_2 is negative).

Sample Problem:

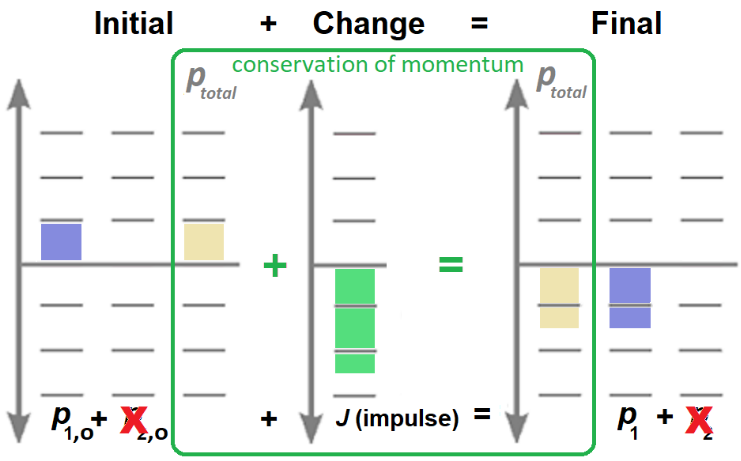
Q: A baseball has a mass of 0.145 kg and is pitched with a velocity of $38\,\frac{m}{s}$ toward home plate. After the ball is hit, its velocity is $52\,\frac{m}{s}$ in the opposite direction, toward the center field fence. If the impact between the ball and bat takes place over an interval of 3.0 ms (0.0030 s), find the impulse given to the ball by the bat, and the force applied to the ball by the bat.

A: The ball is initially moving toward home plate. The bat applies an impulse in the ***opposite*** direction. As with any one-dimensional vector quantity, opposite directions means we will have opposite signs. If we choose the initial direction of the ball (toward home plate) as the positive direction, then the initial velocity is $+38\,\frac{m}{s}$, and the final velocity is $-52\,\frac{m}{s}$. Because mass is scalar and always positive, this means the initial momentum is positive and the final momentum is negative.

Furthermore, because the final velocity is about 1½ times as much as the initial velocity (in the opposite direction) and the mass doesn't change, this means the impulse needs to be enough to negate the ball's initial momentum plus enough in addition to give the ball about 1½ times as much momentum in the opposite direction.

Just like the energy bar charts (LOL charts) that we used for conservation of energy problems, we can create a momentum bar chart. However, because momentum is a vector, we use positive and negative numbers to indicate direction for collisions in one dimension, just like we used positive and negative numbers to indicate direction for velocity, acceleration and force. This means that our momentum bar chart needs to be able to accommodate positive and negative values.

In our problem, the pitcher initially threw the ball in the positive direction. When the batter hit the ball, the impulse on the ball caused it to change direction. The momentum bar chart would look like the following:



The chart shows us the equation so we can solve the problem mathematically:

$$\vec{p}_{1,o} + \vec{J} = \vec{p}_1$$
$$m\vec{v}_o + \vec{J} = m\vec{v}$$
$$(0.145)(38) + \vec{J} = (0.145)(-52)$$
$$5.51 + \vec{J} = -7.54$$
$$\vec{J} = -13.05\,\text{N}\cdot\text{s}$$

The negative value for impulse means that it was in the opposite direction from the baseball's original direction, which makes sense.

Now that we know the impulse, we can use $\vec{J} = \vec{F}t$ to find the force from the bat.

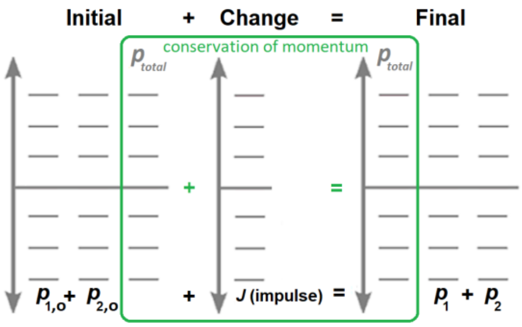
$$\vec{J} = \vec{F}t$$
$$-13.05 = \vec{F}(0.003)$$
$$\vec{F} = \frac{-13.05}{0.003} = -4350\,\text{N}$$

Therefore, the force was 4 350 N toward center field.

Homework Problems

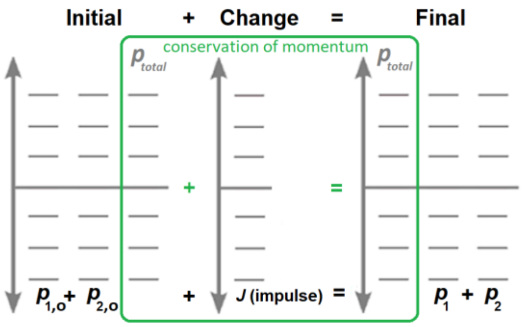
If a problem includes a momentum bar chart, you must fill it out *in addition to* calculating the numerical answer.

1. **(S)** A teacher who was standing still in a corridor was run into by a student rushing to class. The teacher has a mass of 100. kg and the collision lasted for 0.020 s. After the collision, the teacher's velocity was $0.67\frac{m}{s}$. What were the impulse and force applied to the teacher?



Answer: impulse: $67\,N\cdot s$; force: $3\,350\,N$

2. **(M)** An 800 kg car travelling at $10\frac{m}{s}$ comes to a stop in 0.50 s in an accident.



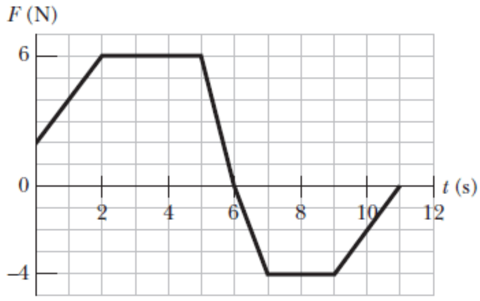
   a. **(M)** What was the impulse applied to the car?

   Answer: $-8000\,N\cdot s$

   b. **(M)** What was the average net force on the car as it came to a stop?

   Answer: $-16000\,N$

3. Force is applied to a 2.0 kg block on a frictionless surface, as shown on the graph below.



At time t = 0, the block has a velocity of $+3.0\frac{\text{m}}{\text{s}}$.

a. **(M)** What is the momentum of the block at time t = 0?

Answer: $6\,\text{N}\cdot\text{s}$ (*Note: this is the starting momentum for parts (b) & (c).*)

b. **(S)** What is the impulse applied to the block during the interval from 0–2 s? What are the momentum and velocity of the block at time t = 2 s?

Answer: $\vec{J} = +8.0\,\text{N}\cdot\text{s}$; $\vec{p} = +14.0\,\text{N}\cdot\text{s}$; $\vec{v} = +7.0\frac{\text{m}}{\text{s}}$

c. **(M)** What is the impulse applied to the block during the interval from
0–6 s? What are the momentum and velocity of the block at time t = 6 s?

Answer: $\vec{J} = +29.0\,\text{N}\cdot\text{s}$; $\vec{p} = +35.0\,\text{N}\cdot\text{s}$; $\vec{v} = +17.5\frac{\text{m}}{\text{s}}$

(*Note:* $+35.0\,\text{N}\cdot\text{s}$ *will be the starting momentum for part (d).*)

d. **(S)** What is the impulse applied to the block during the interval from 6–11 s? What are the momentum and velocity of the block at time t = 11 s?

Answer: $\vec{J} = -14.0\,\text{N}\cdot\text{s}$; $\vec{p} = +21\,\text{N}\cdot\text{s}$; $\vec{v} = +10.5\frac{\text{m}}{\text{s}}$

4. **(M)** Two balls, each with a mass of 0.1 kg, are dropped from a height of 1.25 m and bounce off a table.

   a. **(M)** Calculate the velocity of each ball just before it collides the table. (*Hint: This is a conservation of energy problem.*)

   Answer: $-5\,\frac{m}{s}$ (*i.e.*, $5\,\frac{m}{s}$ downwards)

   b. **(M)** Calculate the momentum of each ball just before it collides with the table.

   Answer: $-0.5\,N\cdot s$ (*i.e.*, $0.5\,N\cdot s$ downwards)

   c. **(M)** Ball #1 (the "happy" ball) bounces back to a height of 0.8 m. Calculate the velocity of ball #1 immediately after the collision. (*Hint: This is a conservation of energy problem.*)

   Answer: $+4\,\frac{m}{s}$ (*i.e.*, $4\,\frac{m}{s}$ upwards)

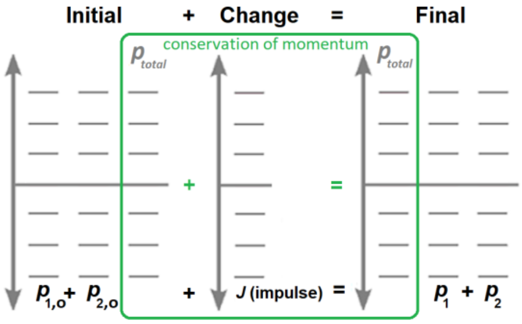
   d. **(M)** What is the coëfficient of restitution (COR) of ball #1? Was total kinetic energy conserved?

   Answer: 0.8; no

   e. **(M)** Calculate the momentum of ball #1 after the collision.

   Answer: $+0.4\,N\cdot s$ (*i.e.*, $0.4\,N\cdot s$ upwards)

f. **(M)** Calculate the impulse delivered to ball #1 by the table.

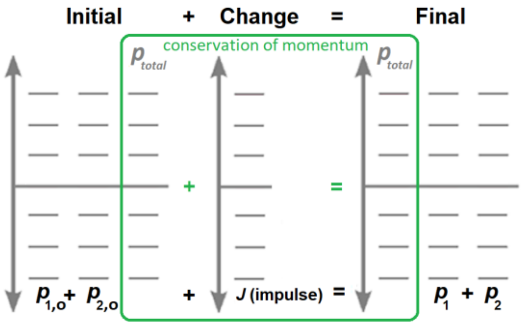


Answer: $+0.9\,\text{N·s}$

g. **(M)** Ball #2 (the "sad" ball) does not bounce when it hits the table. What is the momentum of ball #2 after the collision?

Answer: zero

h. **(M)** Calculate the impulse delivered to ball #2 by the table.



Answer: $+0.5\,\text{N·s}$
(*i.e.*, $0.5\,\text{N·s}$ upwards)

i. **(M)** Which ball experienced the greater impulse, the ball that bounced back (ball #1) or the ball that stopped upon impact (ball #2)?

j. **(M)** Older cars had bumpers that would recoil, which caused the car to bounce when it crashed into something. Modern cars are designed to crumple (while keeping the passenger compartment intact). This change in design reduces the force in two ways. Explain *both*, based on your answer to part (i) above and the equation relating force and impulse.