Work

Unit: Energy, Work & Power

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 3.2.A, 3.2.A.1, 3.2.A.1.i, 3.2.A.1.ii, 3.2.A.1.iii, 3.2.A.1.iv, 3.2.A.1.v, 3.2.A.2, 3.2.A.3, 3.2.A.3.i, 3.2.A.3.ii, 3.2.A.4, 3.2.A.4.i, 3.2.A.4.i, 3.2.A.4.ii, 3.2.A.4.iii, 3.2.A.5

Mastery Objective(s): (Students will be able to…)

- Calculate the work done when a force displaces an object .

Success Criteria:

- Variables are correctly identified and substituted correctly into equation(s).
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why a longer lever arm is more effective.

Tier 2 Vocabulary: work, energy

Notes:

In high school physics, there are two ways that we will study of transferring energy into or out of a system:

work (*W*): mechanical *energy transferred* into or out of a system *by a net force acting over a distance*.

heat (*Q*): thermal energy transferred into or out of a system. Heat is covered in Physics 2.

If you lift a heavy object off the ground, you are giving the object gravitational potential energy (in the object-Earth system). The Earth's gravitational field can now cause the object to fall, turning the potential energy into kinetic energy. Therefore, we would say that you are doing work against the force of gravity.

Work is the amount of energy that was added to the object ($W = \Delta E$)*. (In this case, because the work was turned into *potential* energy, we would say that $W = \Delta U$.)

* Many texts start with work as the application of force over a distance, and then discuss energy. Those texts then derive the work-energy theorem, which states that the two quantities are equivalent. In these notes, we instead started with energy, and then defined work as the change in energy. This presentation makes the concept of work more intuitive, especially when studying other energy-related topics such as thermodynamics.

Mathematically, work is also the effect of a force applied over a distance (the dot product of the force vector and the displacement vector). Therefore:

$$\Delta E = W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}}$$

Recall that the dot product is one of three ways of multiplying vectors. The dot product is a scalar quantity (a number, including its units, but without a direction), and is equal to the product of the magnitudes of the force and distance, and the cosine of the angle between them. This means:

$$W = Fd\cos\theta = F_{||}d$$

Where F is the magnitude of the force vector $\vec{\boldsymbol{F}}$, d is the magnitude of the displacement vector $\vec{\boldsymbol{d}}$, and θ is the angle between the two vectors. Sometimes $F\cos\theta$ is written as $F_{||}$, which means "the component of the force that is parallel to the direction of motion."

Note that when the force and the displacement are in the same direction, the angle $\theta = 0°$ which means $\cos\theta = \cos(0°) = 1$. In this case, $F_{||} = F\cos\theta = (F)(1) = F$ and the equation reduces to $W = Fd$.

Work is measured in joules (J) or newton-meters (N·m), which are equivalent.

$$1\,\text{N·m} \equiv 1\,\text{J} \equiv 1\,\tfrac{\text{kg·m}^2}{\text{s}^2}$$

Positive *vs.* Negative Work

Recall that in physics, we use positive and negative numbers to indicate direction. So far, we have used positive and negative numbers for one-dimensional vector quantities (*e.g.,* velocity, acceleration, force) to indicate the direction of the vector. We can also use positive and negative numbers to indicate the direction for energy (and other scalar quantities), to indicate whether the energy is being transferred into or out of a system.

- If the *energy* of an object or system *increases* because of work (energy is transferred ***into*** the object or system), then the *work* is *positive* with respect to that object or system.
- If the *energy* of an object or system *decreases* because of work (energy is transferred ***out of*** the object or system), then the *work* is *negative* with respect to that object or system.

However, we often discuss work using the prepositions ***on*** (into) and ***by*** (out of).

- If energy is transferred ***into*** an object or system, then we can say that work was done ***on*** (into) the object or system, or that work was done ***by*** (out of) the surroundings.
- If energy was transferred ***out of*** an object or system, we can say that work was done ***by*** (out of) the object, or we can say that work was done ***on*** (into) the surroundings.

Example:

A truck pushes a 1000 kg car up a 50 m hill. The car gained $U_g = mgh = (1000)(10)(50) = 500\,000$ J of potential energy. We could say that:

- 500 000 J of work was done ***on*** the car (by the truck).
- 500 000 J of work was done ***by*** the truck (on the car).
- −500 000 J of work was done ***on*** the truck (by the car).

A simple way to tell if a force does positive or negative work on an object is to use the equation $W = \vec{F} \bullet \vec{d}$. If the force and the displacement are in the ***same*** direction, then the work done by the force is ***positive***. If the force and displacement are in ***opposite*** directions, then the work done by the force is ***negative***.

Example:

Suppose a force of 750 N is used to push a cart against 250 N of friction for a distance of 20 m. The work done ***by*** the force is $W = F_{||}d = (750)(20) = 15\,000$ J. The work done ***by*** friction is $W = F_{||}d = (-250)(20) = -5\,000$ J (negative because friction is in the negative direction). The total (net) work done on the cart is $15\,000 + (-5\,000) = 10\,000$ J.

We could also figure out the net work done on the cart directly by using the net force: $W_{net} = F_{net,||}d = (750 - 250)(20) = (500)(20) = 10\,000$ J

Notes:

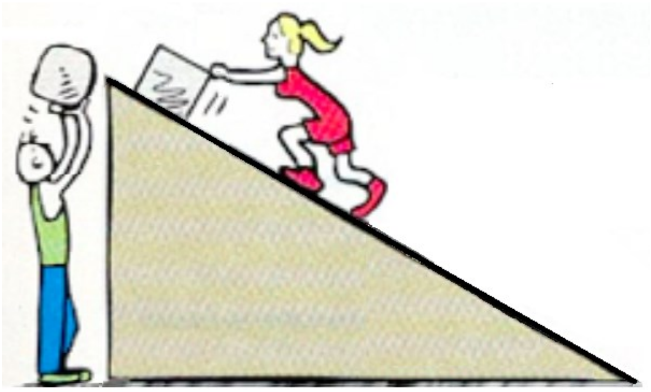
- If the displacement is zero, no work is done by the force. *E.g.,* if you hold a heavy box without moving it, you are exerting a force (counteracting the force of gravity) but you are not doing work.
- If the net force is zero, no work is done by the displacement (change in location) of the object. *E.g.,* if a cart is sliding across a frictionless air track at a constant velocity, the net force on the cart is zero, which means no work is being done.
- If the displacement is perpendicular to the direction of the applied force (θ = 90°, which means cos θ = 0), no work is done by the force. *E.g.,* you can slide a very heavy object along a roller conveyor, because the force of gravity is acting vertically and the object's displacement is horizontal, which means gravity and the normal force cancel, and you therefore do not have to do any work against gravity.

Work Done by Conservative *vs.* Nonconservative Forces

conservative force: a force for which the amount of work done does not depend on the path.

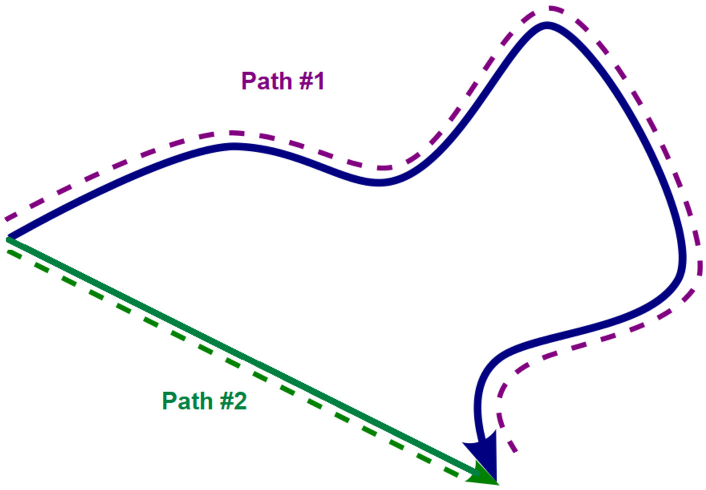
nonconservative force: a force for which the amount of work done depends on the path.

For example, consider a situation in which one person lifts a block, and another person slides a block of the same weight up a frictionless ramp.



Both blocks gained the same amount of gravitational potential energy, so the amount of work done on the blocks (against gravity) is the same. Work that is converted to potential energy is therefore done by a conservative force.

However, consider instead a block sliding on the ground, with friction:

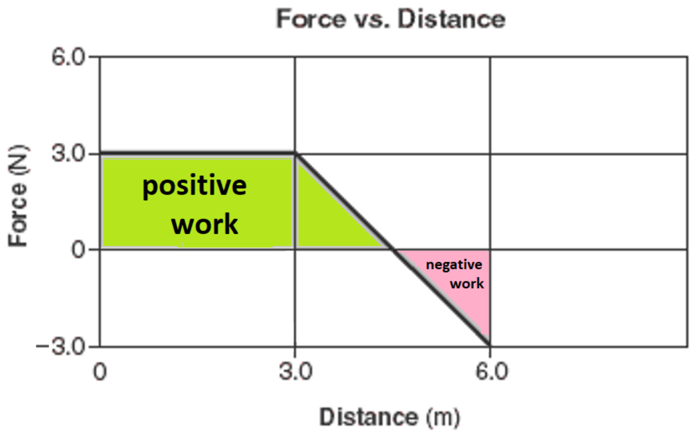


In this case, the amount of work done against friction depends on the distance traveled; Path #1 requires more work than Path #2. Friction is therefore a nonconservative force.

Force *vs.* Distance Graphs

Recall that on a graph, the area "under the graph" (between the graph and the *x*-axis)* represents what you get when you multiply the quantities on the *x* and *y*-axes by each other.

Because $W = F_{||}d$, if we plot force *vs.* distance, the area "under the graph" is therefore the work:



In the above example, $(3\,\text{N})(3\,\text{m}) = 9\,\text{N}\cdot\text{m} = 9\,\text{J}$ of work was done on the object in the interval from 0–3 s, 2.25 J of work was done on the object in the interval from 3–4.5 s, and −2.25 J of work was done on the object in the interval from 4.5–6 s. (Note that the work from 4.5–6 s is negative, because the force was applied in the negative direction during that interval.) The total work is therefore $9 + 2.25 + (-2.25) = +9\,\text{J}$.

* In most physics and calculus textbooks, the term "area under the graph" is used. This term ***always*** means the area ***between the graph and the x-axis***.

Sample Problems:

Q: How much work does it take to lift a 60. kg box 1.5 m off the ground at a constant velocity over a period of 3.0 s?

A: The box is being lifted, which means the work is done against the force of gravity.

$$W = F_{||} \cdot d = F_g d$$
$$W = F_g d = [mg]d = [(60)(10)](1.5) = [600](1.5) = 900 \text{ J}$$

Note that the amount of time it took to lift the box has nothing to do with the amount of work done.

It may be tempting to try to use the time to calculate velocity and acceleration in order to calculate the force. However, because the box is lifted at a constant velocity, the only force needed to lift the box is enough to overcome the weight of the box (F_g).

In general, if work is done to move an object vertically, the work is done against gravity, and you need to use $a = g = 10\frac{\text{m}}{\text{s}^2}$ for the acceleration when you calculate $F = ma$.

Similarly, if work is done to move an object horizontally, the work is *not* against gravity and either you need to know the force applied or you need to find it from the acceleration of the object using $F = ma$.

Q: In the picture to the right, the adult is pulling on the handle of the wagon with a force of 150. N at an angle of 60.0°.

If the adult pulls the wagon a distance of 500. m, how much work does he do?

A: $W = F_{||} d$

$$W = [F\cos\theta]d = [(150.)\cos 60.0°](500.) = [(150.)(0.500)](500.) = 37500 \text{ J}$$

Homework Problems

1. **(S)** How much work is done against gravity by a weightlifter lifting a 30. kg barbell 1.5 m upwards at a constant speed?

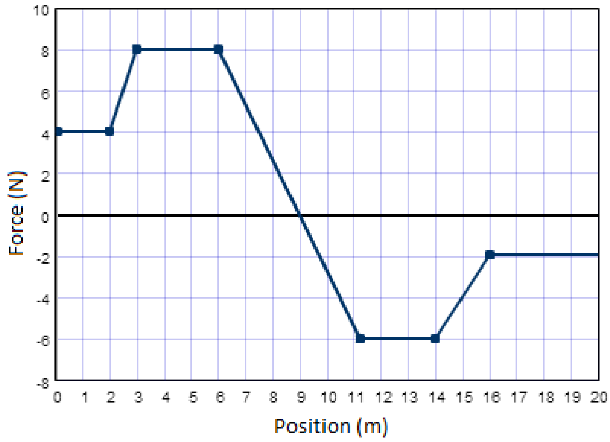
   Answer: 450 J

2. **(M)** A 3000. kg car is moving across level ground at $5.0\,\frac{m}{s}$ when it begins an acceleration that ends with the car moving at $15.0\,\frac{m}{s}$. Is work done in this situation? How do you know?

3. **(S)** A 60. kg man climbs a 3.0 m tall flight of stairs. How much work was done by the man against the force of gravity?

   Answer: 1 800 J

4. **(M)** The following graph shows the force on a 2.0 kg object *vs.* its position on a level surface.



The object has a velocity of $+4.0\,\frac{\text{m}}{\text{s}}$ at time $t = 0$.

a. **(M)** What is the kinetic energy of the object at time $t = 0$?

Answer: 16 J (*Note: this is the starting kinetic energy for parts (b) & (c).*)

b. **(M)** How much work was done on the object by the force during the interval from 0–2 m? What are the kinetic energy and velocity of the object at position $x = 2$ m?

Answer: $W = 8$ J; $K = 24$ J; $\vec{v} = +4.9\,\frac{\text{m}}{\text{s}}$

c. **(M)** How much work was done on the object by the force during the interval from 0–9 m? What are the kinetic energy and velocity of the object at position $x = 9$ m?

Answer: $W = 50$ J; $K = 66$ J; $\vec{v} = +8.1\,\frac{\text{m}}{\text{s}}$

(*Note: 66 J is the starting kinetic energy for part (d).*)

d. **(M)** How much work was done on the block during the interval from 9–20 m? What are the kinetic energy and velocity of the block at position x = 20 m?

Answer: W = −40 J; K = 26 J; $\vec{\boldsymbol{v}} = +5.1\,\frac{\text{m}}{\text{s}}$

5. **(M)** A dog pulls a sled using a 500. N force across a 10. m wide street. The force of friction on the 90. kg sled is 200. N. How much work is done by the dog? How much work is done by friction? How much work is done on the sled? How much work is done by gravity?

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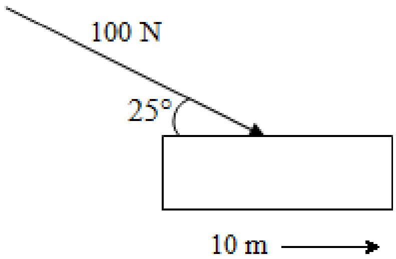
6. **(M – honors & AP®; A – CP1)** Find the work done when a 100. N force at an angle of 25° pushes a cart 10. m to the right, as shown in the diagram to the right.



Answer: 906 J