AP®

Rotational Kinetic Energy

Unit: Energy, Work & Power

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 6.1.A, 6.1.A.1, 6.1.A.1.i, 6.1.A.1.ii, 6.1.A.2, 6.1.A.3

Mastery Objective(s): (Students will be able to…)

- Solve problems that involve kinetic energy of a rotating object.

Success Criteria:

- Correct equations for *both* translational *and* rotational kinetic energy are used in the problem.
- Variables are correctly identified and substituted correctly into the appropriate equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe how an object can have both rotational and translational kinetic energy.
- Explain the relationship between rotational and translational kinetic energy for a rolling object.

Tier 2 Vocabulary: energy, translational, rotational

Labs, Activities & Demonstrations:

- Calculate the exact landing spot of golf ball rolling down a ramp.

Notes:

Just as an object that is moving in a straight line has kinetic energy, a rotating object also has kinetic energy.

The angular velocity (rate of rotation) and the translational velocity are related, because distance that the object must travel (the arclength) is the object's circumference $(s = 2\pi r)$, and the object must make one complete revolution $(\Delta\theta = 2\pi \text{ radians})$ in order to travel this distance. This means that for a rolling object:

$$\Delta\theta = 2\pi r$$

Just as energy can be converted from one form to another and transferred from one object to another, rotational kinetic energy can be converted into any other form of energy, including translational kinetic energy.

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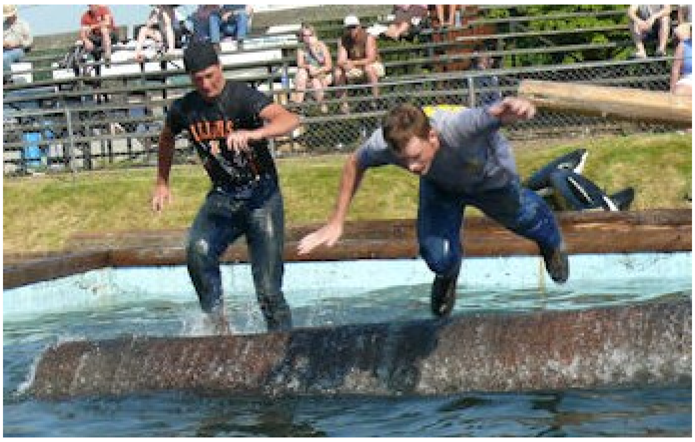
This is the principle behind log rolling. The two contestants get the log rolling quite fast. When one contestant fails to keep up with the log, some of the log's rotational kinetic energy is converted to that contestant's translational kinetic energy, which catapults them into the water:



In a rotating system, the formula for kinetic energy looks similar to the equation for kinetic energy in linear systems, with mass (translational inertia) replaced by moment of inertia (rotational inertia), and linear (translational) velocity replaced by angular velocity:

$K_t = \frac{1}{2}mv^2$	$K_r = \frac{1}{2}I\omega^2$
translational	**rotational**

In the rotational equation, *I* is the object's moment of inertia (see Rotational Inertia starting on page 356), and *ω* is the object's angular velocity.

Note: these problems make use of three relationships that you need to *memorize*:

$$s = r\Delta\theta \qquad v_t = r\omega \qquad a_t = r\alpha$$

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Sample Problem:

Q: What is the rotational kinetic energy of a tenpin bowling ball that has a mass of 7.25 kg and a radius of 10.9 cm as it rolls down a bowling lane at $8.0\,\frac{m}{s}$?

A: The equation for rotational kinetic energy is:

$$K_r = \frac{1}{2} I\omega^2$$

We can find the angular velocity from the translational velocity:

$$v = r\omega$$
$$8.0 = (0.109)\omega$$
$$\omega = \frac{8.0}{0.109} = 73.3\,\frac{rad}{s}$$

The bowling ball is a solid sphere. The moment of inertia of a solid sphere is:

$$I = \frac{2}{5} mr^2$$
$$I = \left(\frac{2}{5}\right)(7.25)(0.109)^2$$
$$I = 0.0345\,\text{kg}\cdot\text{m}^2$$

To find the rotational kinetic energy, we plug these numbers into the equation:

$$K_r = \frac{1}{2} I\omega^2$$
$$K_r = (\tfrac{1}{2})(0.0345)(73.3)^2$$
$$K_r = 185.6\ \text{J}$$

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Total Kinetic Energy

If an object (such as a ball) is rolling, then it is rotating and also moving (translationally). Its total kinetic energy must therefore be the sum of its translational kinetic energy and its rotational kinetic energy:

$$K_{total} = K_t + K_r$$
$$K_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Rolling Without Slipping

<u>rolling</u>: when a round object is in contact with a surface as it rotates and moves along the surface.

<u>rolling without slipping</u>: when the outside edge of a rolling object does not move (slide) relative to the surface at the point of contact.

When an object is rolling without slipping, its translational and rotational kinetic energy are related, because its translational velocity are related:

- The displacement of the wheel $(\Delta\vec{x}_{cm})$ must be equal to the arclength of the section of the wheel that is in contact with the ground as it rolls. $\Delta x_{cm} = r\Delta\theta$
- The translational velocity of the wheel $(\vec{v}_{cm})$ must be the same as its tangential velocity. $v_{cm} = r\omega$
- The translational acceleration of the wheel $(\vec{a}_{cm})$ must be the same as its tangential acceleration. $a_{cm} = r\alpha$

When a wheel rolls without slipping, friction does not dissipate any energy from the rolling system.

Rolling With Slipping

<u>rolling with slipping</u>: when the outside edge of a rolling object moves (slides) relative to the surface at the point of contact.

When a wheel rolls with slipping, the motion of the system’s center of mass cannot be directly related to its rotational motion.

When a wheel rolls with slipping, there is kinetic friction between the edge of the wheel and the surface, which means energy will be dissipated in the form of heat.

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Sample problem:

Q: A standard Type 2 (medium) tennis ball is hollow and has a mass of 58 g and a diameter of 6.75 cm. If the tennis ball rolls 5.0 m across a floor without slipping in 1.25 s, how much total energy does the ball have?

A: The translational velocity of the tennis ball is:

$$v = \frac{d}{t} = \frac{5.0}{1.25} = 4.0\,\tfrac{\text{m}}{\text{s}}$$

The translational kinetic energy of the ball is therefore:

$$K_t = \tfrac{1}{2}mv^2 = (\tfrac{1}{2})(0.058)(4)^2 = 0.464\text{ J}$$

The angular velocity of the tennis ball can be calculated from:

$$v = r\omega$$

$$4 = (0.03375)\,\omega$$

$$\omega = \frac{4}{0.03375} = 118.5\,\tfrac{\text{rad}}{\text{s}}$$

The moment of inertia of a hollow sphere is:

$$I = \tfrac{2}{3}mr^2 = \left(\frac{2}{3}\right)(0.058)(0.03375)^2 = 4.40\times 10^{-5}\text{ kg}\cdot\text{m}^2$$

The rotational kinetic energy is therefore:

$$K_r = \tfrac{1}{2}I\omega^2 = (\tfrac{1}{2})(4.40\times 10^{-5})(118.5)^2 = 0.309\text{ J}$$

Finally, the total kinetic energy is the sum of the translational and rotational kinetic energies:

$$K = K_t + K_r = 0.464 + 0.309 = 0.773\text{ J}$$

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Homework Problems

1. **(M – AP®; A – honors & CP1)** A solid ball with a mass of 100 g and a radius of 2.54 cm rolls with a rotational velocity of $1.0\,\frac{\text{rad}}{\text{s}}$.

   a. **(M – AP®; A – honors & CP1)** What is its rotational kinetic energy?

   Answer: 1.29×10^{-5} J

   b. **(M – AP®; A – honors & CP1)** What is its translational kinetic energy?

   Answer: 3.23×10^{-5} J

   c. **(M – AP®; A – honors & CP1)** What is its total kinetic energy?

   Answer: 4.52×10^{-5} J

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2. **(M – AP®; A – honors & CP1)** How much work is needed to stop a 25 cm diameter solid cylindrical flywheel rotating at 3 600 RPM? The flywheel has a mass of 2 000 kg.

   (*Hint: Note that the problem gives the diameter, not the radius, and that the diameter is in centimeters, not meters.*)

   Answer: $1.11\times10^{6}\ \text{N}\cdot\text{m}$

3. **(M – AP®; A – honors & CP1)** An object is initially at rest. When $250\,\text{N}\cdot\text{m}$ of work is done on the object, it rotates through 20 revolutions in 4.0 s. What is its moment of inertia?

   Answer: $5.066\,\text{kg}\cdot\text{m}^2$

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4. **(M – AP®; A – honors & CP1)** How much work is required to slow a 20 cm diameter solid ball that has a mass of 2.0 kg from $5.0\,\frac{\text{m}}{\text{s}}$ to $1.0\,\frac{\text{m}}{\text{s}}$?

   (*Hint: Again, note that the problem gives the diameter, not the radius, and that the diameter is in centimeters, not meters.*)

   Answer: 33.6 J

5. **(M – AP®; A – honors & CP1)** A flat disc that has a mass of 1.5 kg and a diameter of 10 cm rolls down a 1 m long incline with an angle of 15°. What is its linear speed at the bottom?

   (*Hint: Again, note that the problem gives the diameter, not the radius, and that the diameter is in centimeters, not meters.*)

   Answer: $1.86\,\frac{\text{m}}{\text{s}}$