Simple Harmonic Motion

Unit: Simple Harmonic Motion

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 7.1.A, 7.1.A.1, 7.1.A.2, 7.1.A.2.i, 7.1.A.2.ii, 7.1.A.2.iii, 7.2.A, 7.2.A.1, 7.2.A.1.i, 7.2.A.1.ii, 7.3.A, 7.3.A.1, 7.3.A.1.i, 7.3.A.1.ii, 7.3.A.2, 7.3.A.3

Mastery Objective(s): (Students will be able to…)

- Describe simple harmonic motion and explain the behaviors of oscillating systems such as springs & pendulums.

Success Criteria:

- Explanations are sufficient to predict the observed behavior.

Language Objectives:

- Explain why oscillating systems move back and forth by themselves.

Tier 2 Vocabulary: simple, harmonic

Labs, Activities & Demonstrations:

- Show & tell with springs & pendulums.

Notes:

simple harmonic motion: motion consisting of regular, periodic back-and-forth oscillation.

restoring force: a force that pushes or pulls an object in SHM toward its equilibrium position.

equilibrium position: a point in the center of an object's oscillation where the net force on the object is zero. If an object is placed at the equilibrium position with a velocity of zero, the object will remain there.

Because the restoring force is in the opposite direction from the displacement, acceleration is also in the opposite direction from displacement. This means the acceleration always slows down the motion and reverses the direction.

Applying Newton's Second Law gives $m\vec{a}_x = -k\Delta\vec{x}$. In this equation, *k* is an arbitrary constant that makes the units work. The units of this constant are $\frac{\text{N}}{\text{m}}$.

In an ideal system with no friction, simple harmonic motion would continue forever.

In a real system with nonzero friction, the oscillation will slow down and the system will eventually come to rest. This process is called *damped oscillation*.

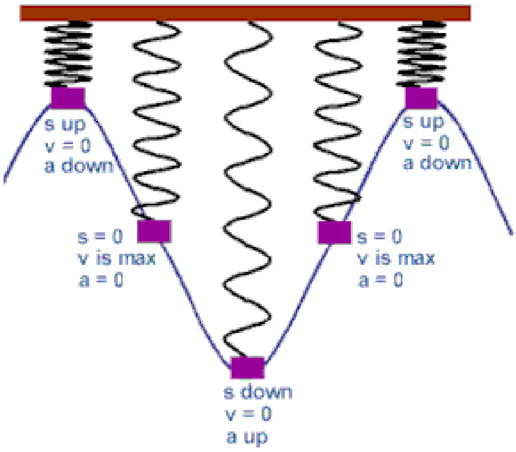
<u>(time) period</u> (T, unit = s): The amount of time that it takes for an object to complete one complete cycle of periodic (repetitive) motion.

<u>frequency</u> (f, unit = Hz = $\frac{1}{s}$): The number of cycles of repetitive motion per unit of time. Frequency and period are reciprocals of each other, *i.e.*, $f = \frac{1}{T}$ and $T = \frac{1}{f}$

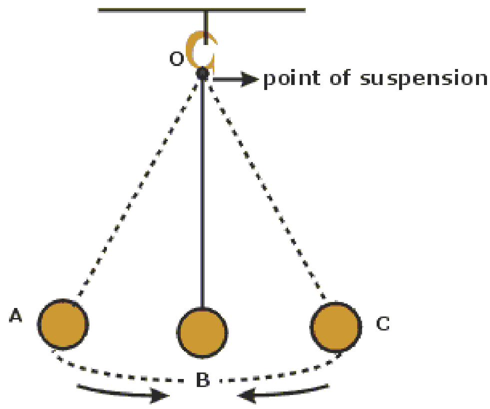
<u>amplitude</u>: the maximum displacement of the object from its equilibrium position.

Examples of Simple Harmonic Motion

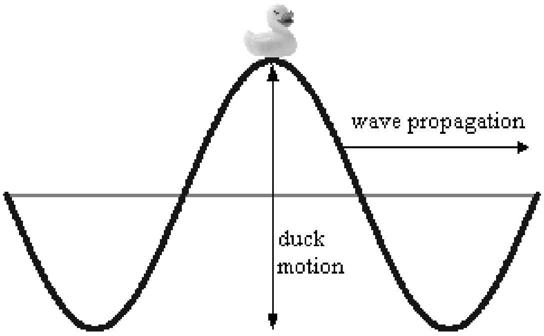
- **Springs**; as the spring compresses or stretches, the spring force accelerates it back toward its equilibrium position.



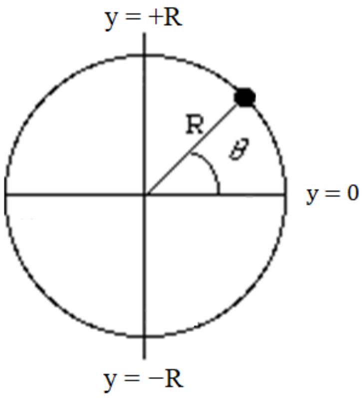
- **Pendulums**: as the pendulum swings, gravity accelerates it back toward its equilibrium position.



- **Waves**: waves passing through some medium (such as water or air) cause the medium to oscillate up and down, like a duck sitting on the water as waves pass by.

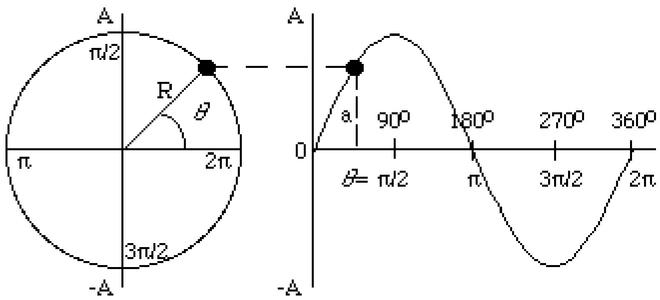


- **Uniform circular motion**: as an object moves around a circle, its vertical position (y-position) is continuously oscillating between +r and –r.

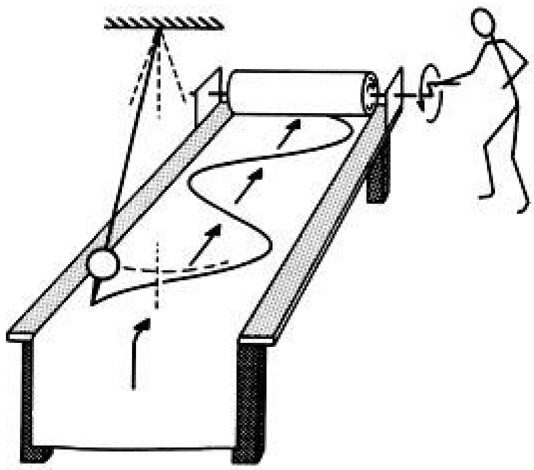


Graphs of Simple Harmonic Motion

If an object is in SHM, a graph representing the displacement of the object from its equilibrium position will be either $y = A\sin x$ or $y = A\cos x$, where A is the amplitude.



If you trace the displacement of a pendulum onto a moving paper, the resulting graph will be $y = A\sin x$ or $y = A\cos x$.



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Kinematics of Simple Harmonic Motion

As described above, the *y*-position of an object in simple harmonic motion as a function of time is the sine or cosine of an angle around the unit circle.

From the rotational kinematics equations (in the *Solving Linear & Rotational Motion Problems* topic starting on page 252), the object's change in position is given by the equation $\Delta\theta = \omega t$. Because the object's angular starting position is arbitrary, we can describe this starting position by an offset angle, ϕ. This angle is called the <u>*phase*</u>.

We can therefore describe the object's position using the equation:

$$\text{position:}\quad x = A\cos(\omega t + \phi)$$

From calculus, because velocity is the first derivative of position with respect to time and acceleration is the second derivative, the general equations for periodic motion are therefore:

$$\text{velocity:}\quad v = -A\omega\sin(\omega t + \phi) = \frac{dx}{dt}\,^{*}$$

$$\text{acceleration:}\quad a = -A\omega^2\cos(\omega t + \phi) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Because many simple harmonic motion problems (including AP® problems) are given in terms of the frequency of oscillation (number of oscillations per second), we can multiply the angular frequency by 2π (because one complete oscillation represents the distance around the unit circle) to use *f* instead of ω, *i.e.,* $\omega = 2\pi f$.

On the AP® formula sheet, ϕ is assumed to be zero, which results in the following versions of the position equation:

$$x = A\cos(2\pi f t) \quad \text{or} \quad x = A\sin(2\pi f t)$$

On the AP® exam, you are expected to understand and use the position equation above, but simple harmonic problems that involve the velocity and acceleration equations are beyond the scope of this course.

[*] The derivatives $\frac{dx}{dt}$, $\frac{dv}{dt}$, and $\frac{d^2x}{dt^2}$ are from calculus. The velocity and acceleration equations of SHM are beyond the scope of the AP® Physics course.

Energy in SHM

As in other situations, the total mechanical energy of a system is its potential plus kinetic energy:

$$E_{total} = U + K$$

In an oscillating system:

- As the restoring force moves the object toward the equilibrium position, potential energy decreases and kinetic energy increases.
- The system's potential energy is at a minimum when its kinetic energy is at a maximum.
- As the restoring force moves the object away from the equilibrium position, potential energy increases and kinetic energy decreases.
- The system's kinetic energy is at a minimum when its potential energy is at a maximum.