Conservation of Linear Momentum

Unit: Momentum

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-2

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 4.3.A, 4.3.A.1, 4.3.A.1.i, 4.3.A.1.ii, 4.3.A.2, 4.3.A.3, 4.3.A.3.i, 4.3.A.3.ii, 4.3.A.3.iii, 4.3.A.4, 4.3.B, 4.3.B.1, 4.3.B.2, 4.3.B.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving collisions in which momentum is conserved, with or without an external impulse.

Success Criteria:

- Masses and velocities are correctly identified for each object, both before and after the collision.
- Variables are correctly identified and substituted correctly into the correct part of the equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what happens before, during, and after a collision from the point of view of one of the objects participating in the collision.

Tier 2 Vocabulary: momentum, collision

Labs, Activities & Demonstrations:

- Collisions on air track.
- "Happy" and "sad" balls knocking over a board.
- Students riding momentum cart.

Notes:

collision: when two or more objects come together and hit each other.

elastic collision: a collision in which the objects bounce off each other (remain separate) after they collide, without any loss of kinetic energy.

inelastic collision: a collision in which the objects remain together after colliding. In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before *vs.* after the collision.

Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large-scale impacts are ever perfectly elastic.

Conservation of Momentum

Recall that in physics, if a quantity is "conserved", that means when some change happens to a system, there is the same amount of that quantity after the change as there was before.

In a closed system in which objects are free to move before and after a collision, momentum is ***conserved***. This means that unless there is an outside force, *the combined momentum of all of the objects after they collide is equal to the combined momentum of all of the objects before the collision*.

Solving Conservation of Momentum Problems

In plain English, the conservation of momentum law means that the total momentum before a collision, plus any momentum that we add (positive or negative impulse), must add up to the total momentum after.

In equation form, the conservation of momentum looks like this:

$$\textbf{Before} + \textbf{Impulse} = \textbf{After}$$
$$\sum \vec{\boldsymbol{p}}_i + \vec{\boldsymbol{J}} = \sum \vec{\boldsymbol{p}}_f$$
$$\sum \vec{\boldsymbol{p}}_i + \Delta\vec{\boldsymbol{p}} = \sum \vec{\boldsymbol{p}}_f$$
$$\sum m\vec{\boldsymbol{v}}_i + \Delta\vec{\boldsymbol{p}} = \sum m\vec{\boldsymbol{v}}_f$$

The symbol $\sum$ is the Greek capital letter "sigma". In mathematics, the symbol $\sum$ means "summation". $\sum \vec{\boldsymbol{p}}$ means the sum of the momentums. The subscript "*i*" means initial (before the collision), and the subscript "*f*" means final (after the collision). In plain English, $\sum \vec{\boldsymbol{p}}$ means find each individual value of $\vec{\boldsymbol{p}}$ (positive or negative, depending on the direction) and then add them all up to find the total.

In the last step, we replaced each $\vec{\boldsymbol{p}}$ with $m\vec{\boldsymbol{v}}$, because we are usually given the masses and velocities in collision problems.

(Note that most momentum problems do not mention the word "momentum." The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that any problem involving collisions is almost always a conservation of momentum problem.)

The problems that we will see in this course involve two objects. These objects will either bounce off each other and remain separate (elastic collision), or they will either start out or end up together (inelastic collision).

Elastic Collisions

An elastic collision occurs when two or more object come together in a collision and then separate. There are the same number of separate objects before and after the collision.

As stated above, the equation for the conservation of momentum in an elastic collision is:

$$\textbf{Before} = \textbf{After}$$

$$\sum \vec{p}_i + \vec{J} = \sum \vec{p}_f$$

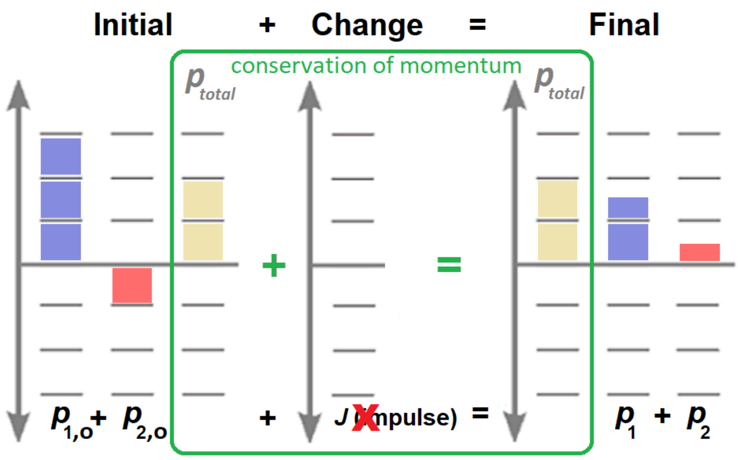
$$\vec{p}_{1,i} + \vec{p}_{2,i} + \vec{J} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

$$m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} + \vec{J} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}$$

Notice that we have two subscripts after each "$\vec{p}$" and each "$\vec{v}$", because we have two separate things to keep track of. The "1" and "2" mean object #1 and object #2, and the "*i*" and "*f*" mean "initial" and "final".

Notice also that there are six variables: the two masses $(m_1$ and $m_2)$, and the four velocities $(\vec{v}_{1,i}, \vec{v}_{2,i}, \vec{v}_{1,f}$ and $\vec{v}_{2,f})$. In a typical problem, you will be given five of these six values and use algebra to solve for the remaining one.

The following momentum bar chart is for an elastic collision. Imagine that two objects are moving in opposite directions and then collide. There is no external force on the objects, so there is no impulse.



Before the collision, the first object has a momentum of +3 N·s, and the second has a momentum of −1 N·s. The total momentum is therefore +3 + (−1) = +2 N·s.

Because there are no forces changing the momentum of the system, the final momentum must also be +2 N·s. If we are told that the first object has a momentum of +1.5 N·s after the collision, we can subtract the +1.5 N·s from the total, which means the second object must have a momentum of +0.5 N·s.

Inelastic Collisions

An inelastic collision occurs either when two or more objects come together in a collision and remain together, or when one object separates into two or more objects with different velocities (*i.e.,* moving with different speeds and/or directions).

The law of conservation of momentum for an inelastic collision (with no impulse) is either:

$$\begin{gathered}\textbf{Before} = \textbf{After}\\ \sum \vec{\boldsymbol{p}}_i + \vec{\boldsymbol{J}} = \sum \vec{\boldsymbol{p}}_f\\ \sum m\vec{\boldsymbol{v}}_i + \vec{\boldsymbol{J}} = \sum m\vec{\boldsymbol{v}}_f\\ m_1\vec{\boldsymbol{v}}_{1,i} + m_2\vec{\boldsymbol{v}}_{2,i} + \vec{\boldsymbol{J}} = m_T\vec{\boldsymbol{v}}_f\end{gathered}$$

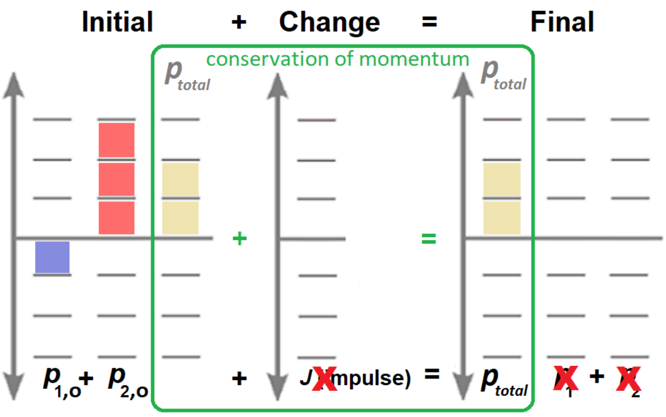
or

$$\begin{gathered}\textbf{Before} = \textbf{After}\\ \sum \vec{\boldsymbol{p}}_i + \vec{\boldsymbol{J}} = \sum \vec{\boldsymbol{p}}_f\\ \sum m\vec{\boldsymbol{v}}_i + \vec{\boldsymbol{J}} = \sum m\vec{\boldsymbol{v}}_f\\ m_T\vec{\boldsymbol{v}}_i + \vec{\boldsymbol{J}} = m_1\vec{\boldsymbol{v}}_{1,f} + m_2\vec{\boldsymbol{v}}_{2,f}\end{gathered}$$

Again we have two subscripts after each "$\vec{\boldsymbol{p}}$" and each "$\vec{\boldsymbol{v}}$", because we have two separate things to keep track of. The "1" and "2" mean object #1 and object #2, and "T" means total (when they are combined). The "*i*" and "*f*" mean "initial" and "final" as before

This time there are five variables: the two masses $(m_1$ and $m_2)$, and the three velocities (either $\vec{\boldsymbol{v}}_{1,i}$ & $\vec{\boldsymbol{v}}_{2,i}$ and $\vec{\boldsymbol{v}}_f$ or $\vec{\boldsymbol{v}}_i$ and $\vec{\boldsymbol{v}}_{1,f}$ & $\vec{\boldsymbol{v}}_{2,f}$). In a typical problem, you will be given four of these five values and use algebra to solve for the remaining one. (Remember that $m_1 + m_2 = m_T$).

The following momentum bar chart is for an inelastic collision. Two objects are moving in the same direction, and then collide.



Before the collision, the first object has a momentum of −1 N·s, and the second has a momentum of +3 N·s. The total momentum before the collision is therefore −1 + (+3) = +2 N·s.

There is no external force (*i.e.,* no impulse), so the total final momentum must still be +2 N·s. Because the objects remain together after the collision, the total momentum is the momentum of the combined objects.

Sample Problems:

Q: An object with a mass of 8.0 kg moving with a velocity of $+5.0\,\frac{m}{s}$ collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?

A: The momentum of the moving object before the collision is:

$$\vec{p} = m\vec{v} = (8.0)(+5.0) = +40\,\text{N}\cdot\text{s}$$

The stationary object has a momentum of zero, so the total momentum of the two objects combined is +40 N·s.

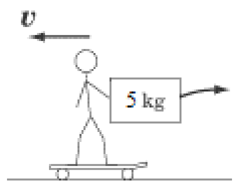
After the collision, the total mass is 8.0 kg + 12 kg = 20 kg. The momentum after the collision must still be +40 N·s, which means the velocity is:

$$\vec{p} = m\vec{v} \qquad 40 = 20\vec{v} \qquad \vec{v} = +2\,\tfrac{m}{s}$$

Using the equation, we would solve this as follows:

$$\begin{aligned}
\textbf{Before} \quad &= \textbf{After} \\
\vec{p}_{1,i} + \vec{p}_{2,i} &= \vec{p}_f \\
m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} &= m_T\vec{v}_f \\
(8)(5) + (12)(0) &= (8+12)\,\vec{v}_f \\
40 &= 20\,\vec{v}_f \\
\vec{v}_f &= \frac{40}{20} = +2\,\tfrac{m}{s}
\end{aligned}$$

Q: Stretch* has a mass of 60. kg and is holding a 5.0 kg box as they ride on a skateboard toward the west at a speed of $3.0\,\frac{\text{m}}{\text{s}}$. (Assume the 60. kg is the mass of Stretch and the skateboard combined.) Stretch throws the box behind them, giving the box a velocity of $2.0\,\frac{\text{m}}{\text{s}}$ to the east. What is Stretch's velocity after throwing the box?



A: This problem is like an inelastic collision in reverse; Stretch and the box are together before the "collision" and apart afterwards. The equation would therefore look like this:

$$m_T\vec{\boldsymbol{v}}_i = m_s\vec{\boldsymbol{v}}_{s,f} + m_b\vec{\boldsymbol{v}}_{b,f}$$

Where the subscript "*s*" is for Stretch, and the subscript "*b*" is for the box. Note that after Stretch throws the box, they are moving one direction and the box is moving the other, which means we need to be careful about our signs. Let's choose the direction Stretch is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be:

$$\vec{\boldsymbol{v}}_{b,f} = -2.0\,\tfrac{\text{m}}{\text{s}}$$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned}
\textbf{BEFORE} &= \textbf{AFTER} \\
\vec{\boldsymbol{p}}_i &= \vec{\boldsymbol{p}}_{s,f} + \vec{\boldsymbol{p}}_{b,f} \\
m_T\vec{\boldsymbol{v}}_i &= m_s\vec{\boldsymbol{v}}_{s,f} + m_b\vec{\boldsymbol{v}}_{b,f} \\
(60+5)(+3) &= 60\,\vec{\boldsymbol{v}}_{s,f} + (5)(-2) \\
+195 &= 60\,\vec{\boldsymbol{v}}_{s,f} + (-10) \\
+205 &= 60\,\vec{\boldsymbol{v}}_{s,f} \\
\vec{\boldsymbol{v}}_{s,f} &= \frac{+205}{60} = +3.4\,\tfrac{\text{m}}{\text{s}}
\end{aligned}$$

* The stick figure is called "Stretch" because the author is terrible at drawing, and most of his stick figures have a body part that is stretched out.

Q: A soccer ball that has a mass of 0.43 kg is rolling *east* with a velocity of $5.0\,\frac{m}{s}$. It collides with a volleyball that has a mass of 0.27 kg that is rolling *west* with a velocity of $6.5\,\frac{m}{s}$. After the collision, the soccer ball is rolling to the *west* with a velocity of $3.87\,\frac{m}{s}$. Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

$$m_s\vec{\boldsymbol{v}}_{s,i} + m_v\vec{\boldsymbol{v}}_{v,i} = m_s\vec{\boldsymbol{v}}_{s,f} + m_v\vec{\boldsymbol{v}}_{v,f}$$

Where the subscript "*s*" is for the soccer ball and the subscript "*v*" is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don't know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

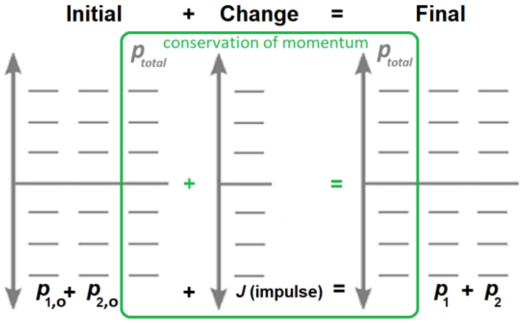
quantity	**direction**	**value**
initial velocity of soccer ball	east	$+5.0\,\frac{m}{s}$
initial velocity of volleyball	west	$-6.5\,\frac{m}{s}$
final velocity of soccer ball	west	$-3.87\,\frac{m}{s}$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{gathered}
\textbf{Before} = \textbf{After}\\
\vec{\boldsymbol{p}}_{s,i} + \vec{\boldsymbol{p}}_{v,i} = \vec{\boldsymbol{p}}_{s,f} + \vec{\boldsymbol{p}}_{v,f}\\
m_s\vec{\boldsymbol{v}}_{s,i} + m_v\vec{\boldsymbol{v}}_{v,i} = m_s\vec{\boldsymbol{v}}_{s,f} + m_v\vec{\boldsymbol{v}}_{v,f}\\
(0.43)(5.0) + (0.27)(-6.5) = (0.43)(-3.87) + (0.27)\,\vec{\boldsymbol{v}}_{v,f}\\
2.15 + (-1.755) = -1.664 + 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
0.395 = -1.664 + 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
2.059 = 0.27\,\vec{\boldsymbol{v}}_{v,f}\\
\vec{\boldsymbol{v}}_{s,f} = \frac{+2.059}{0.27} = +7.63\,\tfrac{m}{s} \text{ or } 7.63\,\tfrac{m}{s} \text{ to the east.}
\end{gathered}$$

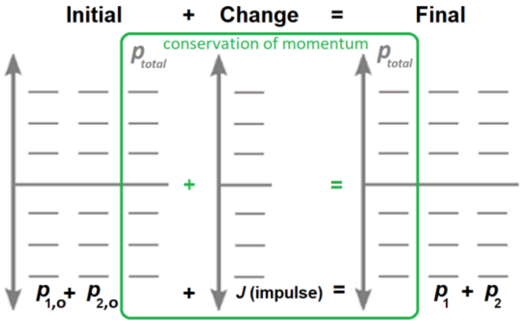
Homework Problems

1. **(M)** A turkey toss is a bizarre "sport" in which a person tries to catch a frozen turkey that is thrown through the air. A frozen turkey has a mass of 10. kg, and a 70. kg person jumps into the air to catch it. If the turkey was moving at $4.0\,\frac{\text{m}}{\text{s}}$ and the person's velocity was zero just before catching it, how fast will the person be moving after catching the frozen turkey?



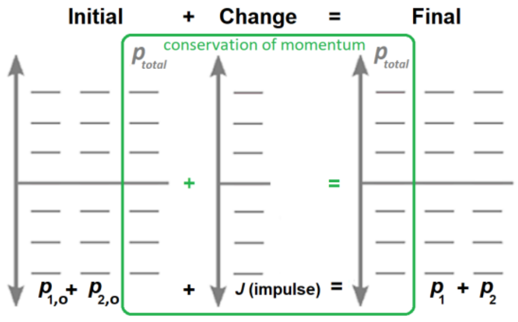
Answer: $0.5\,\frac{\text{m}}{\text{s}}$

2. **(M)** A 6.0 kg bowling ball moving at $3.5\,\frac{\text{m}}{\text{s}}$ toward the back of the alley makes a collision, head-on, with a stationary 0.70 kg bowling pin. If the ball is moving $2.77\,\frac{\text{m}}{\text{s}}$ toward the back of the alley after the collision, what will be the velocity (magnitude and direction) of the pin?



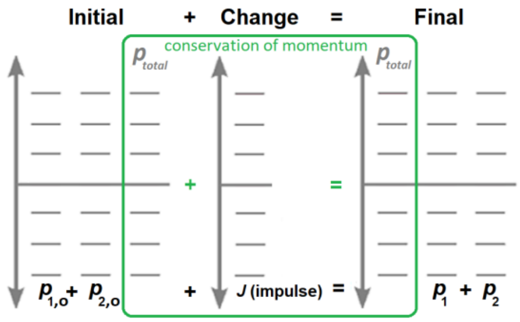
Answer: $6.25\,\frac{\text{m}}{\text{s}}$ toward the back of the alley

3. **(S)** An 80 kg student is standing on a stationary cart on wheels that has a mass of 40 kg. If the student jumps off with a velocity of $+3\,\frac{\text{m}}{\text{s}}$, what will the velocity of the cart be?



Answer: $-6\,\frac{\text{m}}{\text{s}}$

4. **(S)** A 730 kg Mini (small car) runs into a stationary 2 500 kg sport utility vehicle (large car). If the Mini was moving at $10.\,\frac{\text{m}}{\text{s}}$ initially, how fast will it be moving after making a completely inelastic collision with the SUV?



Answer: $2.3\,\frac{\text{m}}{\text{s}}$

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5. **(S – honors & AP®; A – CP1)** A billiard ball with mass m_b makes an elastic collision with a cue ball with mass m_c. Before the collision, the billiard ball was moving with a velocity of $\vec{\boldsymbol{v}}_{b,i}$, and the cue ball was moving with a velocity of $\vec{\boldsymbol{v}}_{c,i}$. After the collision, the cue ball is now moving with a velocity of $\vec{\boldsymbol{v}}_{c,f}$. What is the velocity of the billiard ball after the collision?

   (*If you are not sure how to do this problem, do #6 below and use the steps to guide your algebra.*)

   Answer: $\vec{\boldsymbol{v}}_{b,f} = \dfrac{m_b\vec{\boldsymbol{v}}_{b,i} + m_c\vec{\boldsymbol{v}}_{c,i} - m_c\vec{\boldsymbol{v}}_{c,f}}{m_b}$

6. **(M)** A billiard ball with a mass of 0.16 kg makes an elastic collision with a cue ball with a mass of 0.17 kg. Before the collision, the billiard ball was moving with a velocity of $0.50\,\frac{\text{m}}{\text{s}}$ to the east, and the cue ball was moving with a velocity of $1.0\,\frac{\text{m}}{\text{s}}$ to the west. After the collision, the cue ball is now moving with a velocity of $0.515\,\frac{\text{m}}{\text{s}}$ to the east. What is the velocity (magnitude and direction) of the billiard ball after the collision?

   Hint: Remember that east and west are opposite directions; one of them will be negative.

   (*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS. You may only use the answer to question #5 above as a starting point if you have already solved that problem.*)

   Answer: $1.11\,\frac{\text{m}}{\text{s}}$ to the west