Angular Momentum

Unit: Momentum

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP Physics 1 Learning Objectives/Essential Knowledge (2024): 6.4.A, 6.4.A.1, 6.4.A.2, 6.4.A.2.i, 6.4.A.2.ii, 6.4.A.2.iii, 6.4.A.2.iv, 6.4.B, 6.4.B.1, 6.4.B.2, 6.4.B.3

Mastery Objective(s): (Students will be able to…)

- Explain and apply the principle of conservation of angular momentum.

Success Criteria:

- Explanation takes into account the factors affecting the angular momentum of an object before and after some change.

Language Objectives:

- Explain what happens when linear momentum is converted to angular momentum or *vice versa*.

Tier 2 Vocabulary: momentum

Labs, Activities & Demonstrations:

- Try to change the direction of rotation of a bicycle wheel.
- Spin on a turntable with weights at arm's length.
- Sit on a turntable with a spinning bicycle wheel and invert the wheel.

Notes:

angular momentum ($\vec{L}$) : the momentum of a rotating object in the direction of rotation. Angular momentum is the property of an object that resists changes in the speed or direction of rotation. Angular momentum is measured in units of $\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$.

Just as linear momentum is the product of mass (linear inertia) and (linear) velocity, angular momentum is also the product of the moment of inertia (rotational inertia) and angular (rotational) velocity:

$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$ *
linear	**rotational**

* CP1 and honors physics students are responsible only for a qualitative understanding of angular momentum. AP® Physics students need to solve quantitative problems.

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Angular momentum can also be converted to linear momentum, and *vice versa*. Angular momentum is the cross-product of radius and linear momentum:

$$\vec{L} = \vec{r} \times \vec{p} = rp\sin\theta = rmv\sin\theta$$

E.g., if you shoot a bullet into a door:

1. As soon as the bullet embeds itself in the door, it is constrained to move in an arc, so the linear momentum of the bullet becomes angular momentum.
2. The total angular momentum of the bullet just before impact equals the total angular momentum of the bullet and door after impact.

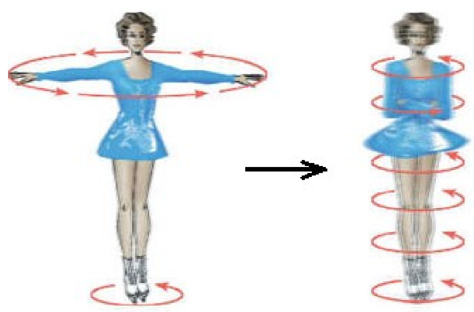
Just as a force produces a change in linear momentum, a torque produces a change in angular momentum. The net external torque on an object is its change in angular momentum with respect to time:

$$\vec{\tau}_{net} = \frac{\Delta\vec{L}}{t} = \frac{d\vec{L}}{dt} \quad \text{and} \quad \Delta\vec{L} = \vec{\tau}_{net}t$$

Conservation of Angular Momentum

Just as linear momentum is conserved unless an external force is applied, angular momentum is conserved unless an external torque is applied. This means that the total angular momentum before some change (that occurs entirely within the system) must equal the total angular momentum after the change.

An example of this occurs when a person spinning (*e.g.,* an ice skater) begins the spin with arms extended, then pulls the arms closer to the body. This causes the person to spin faster. (In physics terms, it increases the angular velocity, which means it causes angular acceleration.)

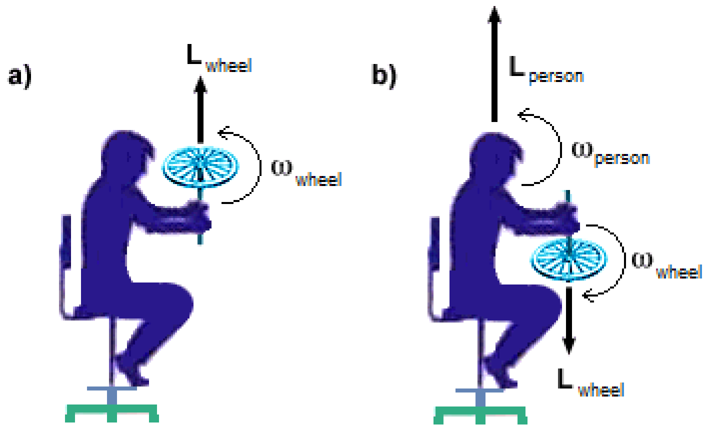


When the skater's arms are extended, the moment of inertia of the skater is greater (because there is more mass farther out) than when the arms are close to the body. Conservation of angular momentum tells us that:

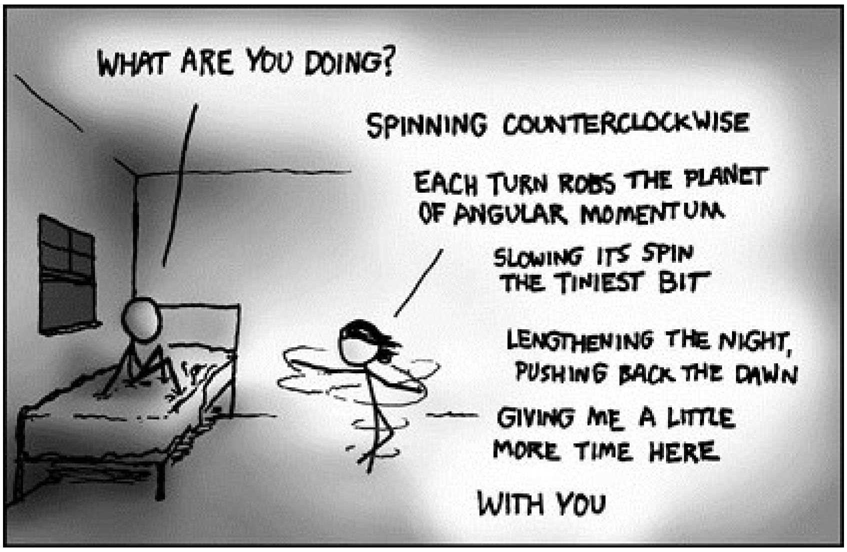
$$L_i = L_f$$
$$I_i\omega_i = I_f\omega_f$$

I.e., if I decreases, then ω must increase.

Another popular example, which shows the vector nature of angular momentum, is the demonstration of a person holding a spinning bicycle wheel on a rotating chair. The person then turns over the bicycle wheel, causing it to rotate in the opposite direction:



Initially, the direction of the angular momentum vector of the wheel is upwards. When the person turns over the wheel, the angular momentum of the wheel reverses direction. Because the person-wheel-chair system is an isolated system, the total angular momentum must be conserved. This means the person must rotate in the opposite direction as the wheel, so that the total angular momentum (magnitude and direction) of the person-wheel-chair system remains the same as before.



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Sample Problem:

Q: A "Long-Playing" (LP) phonograph record has a radius of 15 cm and a mass of 150 g. A typical phonograph could accelerate an LP from rest to its final speed in 0.35 s.

a. Calculate the angular momentum of a phonograph record (LP) rotating at $33\frac{1}{3}$ RPM.

b. What average torque would be exerted on the LP?

A: The angular momentum of a rotating body is $L = I\omega$. This means we need to find I (the moment of inertia) and ω (the angular velocity).

An LP is a solid disk, which means the formula for its moment of inertia is:

$$I = \tfrac{1}{2}mr^2$$
$$I = (\tfrac{1}{2})(0.15\,\text{kg})(0.15\,\text{m})^2 = 1.69\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$\omega = \frac{33\frac{1}{3}\,\text{rev}}{1\,\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,\text{rad}}{1\,\text{rev}} = 3.49\ \tfrac{\text{rad}}{\text{s}}$$

$$L = I\omega$$
$$L = (1.69\times10^{-3}\,\text{kg}\cdot\text{m}^2)(3.49\ \tfrac{\text{rad}}{\text{s}})$$
$$L = 5.89\times10^{-3}\ \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L - L_o}{\Delta t} = \frac{5.89\times10^{-3} - 0}{0.35} = 1.68\times10^{-2}\,\text{N}\cdot\text{m}$$

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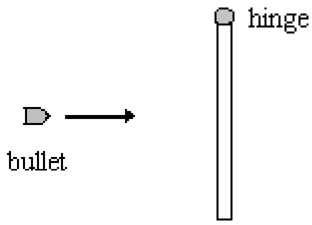
Homework Problems

1. **(M – AP®; A – honors & CP1)** A cylinder of mass 250 kg and radius 2.60 m is rotating at $4.00\,\frac{\text{rad}}{\text{s}}$ on a frictionless surface. A 500 kg cylinder of the same diameter is then placed on top of the cylinder. What is the new angular velocity?

   Answer: $1.33\,\frac{\text{rad}}{\text{s}}$

2. **(M – AP®; A – honors & CP1)** A solid oak door with a width of 0.75 m and mass of 50 kg is hinged on one side so that it can rotate freely. A bullet with a mass of 30 g is fired into the exact center of the door with a velocity of $400\,\frac{\text{m}}{\text{s}}$, as shown in the diagram at the right.



   What is the angular velocity of the door with respect to the hinge just after the bullet embeds itself in the door?
   (*Hint: Treat the bullet as a point mass. Consider the door to be a rod rotating about its end.*)

   Answer: $0.45\,\frac{\text{rad}}{\text{s}}$

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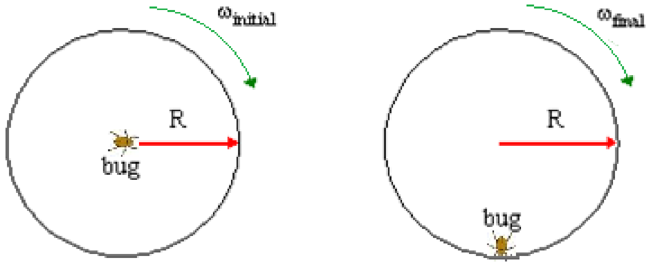
Use the following diagram for questions #3 & 3 below



3. **(M – AP®; A – honors & CP1)** A bug with mass *m* crawls from the center to the outside edge of a disc of mass *M* and radius *r*, rotating with angular velocity ω, as shown in the diagram above.

   Write an expression for the angular velocity of the disc when the bug reaches the edge. You may use your work from problem #3 to guide your algebra. (*Hint: Treat the bug as a point mass.*)

   (*If you are not sure how to do this problem, do #4 below and use the steps to guide your algebra.*)

   Answer: $\omega_f = \dfrac{M\omega_i}{M+2m}$

4. **(S – AP®; A – honors & CP1)** A 12.5 g bug crawls from the center to the outside edge of a 130. g disc of radius 15.0 cm that is rotating at $11.0\,\frac{\text{rad}}{\text{s}}$, as shown in the diagram above.

   What will be the angular velocity of the disc when the bug reaches the edge? (*Hint: Treat the bug as a point mass.*)

   (<u>*You must start with the equations in your* Physics Reference Tables *and show all of the steps of GUESS.*</u> *You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)

   Answer: $9.23\,\frac{\text{rad}}{\text{s}}$