Springs

Unit: Simple Harmonic Motion

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 7.2.A, 7.2.A.1, 7.2.A.1.i

Mastery Objective(s): (Students will be able to…)

- Calculate the period of oscillation of a spring.
- Calculate the force from and potential energy stored in a spring.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what a spring constant measures.

Tier 2 Vocabulary: spring

Labs, Activities & Demonstrations:

- Spring mounted to lab stands with paper taped somewhere in the middle as an indicator.

Notes:

spring: a coiled object that resists motion parallel with the direction of propagation of the coil.

Spring Force

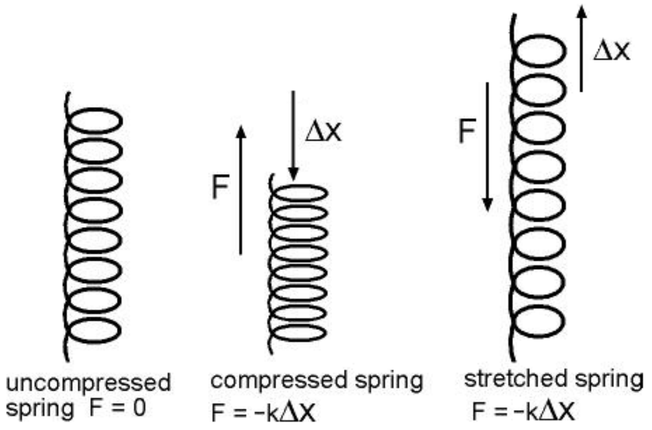
The equation for the force (vector) from a spring is given by Hooke's Law, named for the British physicist Robert Hooke:

$$\vec{F}_s = -k\vec{x}$$

Where $\vec{F}_s$ is the spring force (vector quantity representing the force exerted by the spring), $\vec{x}$ is the displacement of the end of the spring (also a vector quantity), and k is the spring constant, an intrinsic property of the spring based on its mass, thickness, and the elasticity of the material that it is made of.

The negative sign in the equation is because the force is always in the opposite (negative) direction from the displacement.

A Slinky has a spring constant of $0.5\,\frac{\text{N}}{\text{m}}$, while a heavy garage door spring might have a spring constant of $500\,\frac{\text{N}}{\text{m}}$.



Potential Energy

The potential energy stored in a spring is given by the equation:

$$U = \tfrac{1}{2} kx^2$$

Where U is the potential energy (measured in joules), k is the spring constant, and x is the displacement. Note that the potential energy is always positive (or zero); this is because energy is a scalar quantity. A stretched spring and a compressed spring both have potential energy.

The total mechanical energy in a spring-object system is given by the equation:

$$E_{total} = \tfrac{1}{2} kA^2$$

where A is the amplitude (maximum displacement). This makes sense, because when $x = A$, all of the energy is potential, and the equation becomes the same as above.

Period

<u>period</u> or <u>period of oscillation</u>: the time it takes a spring to move from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is T, and the unit is usually seconds.

The period of a spring-object system depends on the mass of the object and the spring constant of the spring, and is given by the equation:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

Frequency

frequency: the number of times something occurs in a given amount of time. Frequency is usually given by the variable f, and is measured in units of hertz (Hz). One hertz is the inverse of one second:

$$1\,\text{Hz} \equiv \frac{1}{1\,\text{s}} \equiv 1\,\text{s}^{-1}$$

Note that the period and frequency are reciprocals of each other:

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

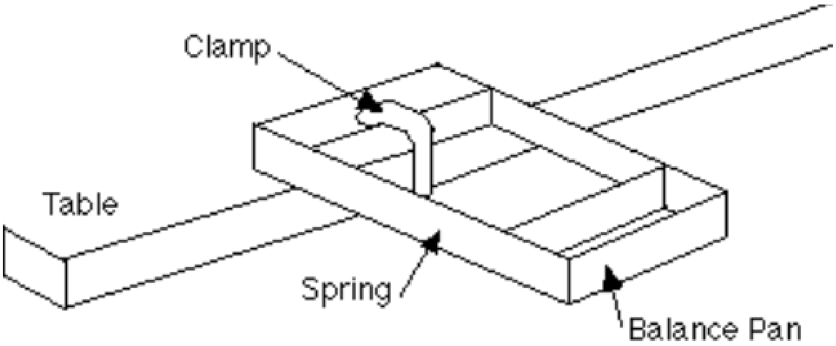
Measuring Inertial Mass

As described in *Newton's Laws of Motion*, starting on page 262, inertia is the property of an object that resists forces that attempt to change its motion. An object's translational inertia is the same as its mass:

gravitational mass: the property of an object that is attracted by a gravitational field. Measured in kg.

inertial mass: the ability of an object to resist changes to its motion. Also measured in kg, and equal to the object's gravitational mass.

Inertial mass is measured using an inertial balance, which is just an apparatus that consists of a pair of springs and a pan to hold the object whose mass is being measured:



The balance pan is pulled to one side, causing it to oscillate. The balance is calibrated with objects of known mass, and the period of oscillation is then used to determine the mass of the unknown object.

Inertial mass is useful because it does not depend on the gravitational force, and can be measured in space.

Sample Problem:

Q: A spring with a mass of 0.1 kg and a spring constant of $2.7\,\frac{\text{N}}{\text{m}}$ is compressed 0.3 m. Find the force needed to compress the spring, the potential energy stored in the spring when it is compressed, and the period of oscillation.

A: The force is given by Hooke's Law.

Substituting these values gives:

$$\vec{\boldsymbol{F}} = -k\vec{\boldsymbol{x}}$$
$$\vec{\boldsymbol{F}} = -(2.7\,\tfrac{\text{N}}{\text{m}})(+0.3\,\text{m}) = -0.81\,\text{N}$$

The potential energy is:

$$U_s = \tfrac{1}{2}kx^2$$
$$U_s = (0.5)(2.7\,\tfrac{\text{N}}{\text{m}})(0.3\,\text{m})^2 = 0.12\ \text{J}$$

The period is:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$
$$T_s = (2)(3.14)\sqrt{\frac{0.1}{2.7}}$$
$$T_s = 6.28\sqrt{0.037} = (6.28)(0.19) = 1.2\,\text{s}$$

Homework Problems

1. **(M)** A 100.0 g mass is suspended from a spring whose constant is $50.0\,\frac{\text{N}}{\text{m}}$. The mass is then pulled down 1.0 cm and then released.
   a. **(M)** How much force was applied in order to pull the spring down the 1.0 cm?

      Answer: 0.5 N

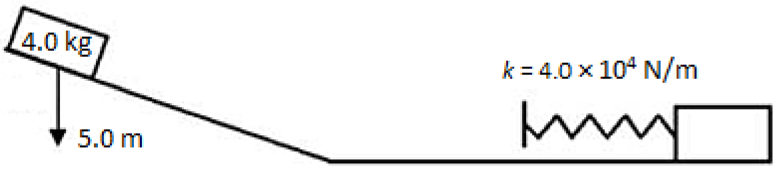
   b. **(M)** What is the frequency of the resulting oscillation?

      Answer: 3.56 Hz

2. **(M)** A 1000. kg car bounces up and down on its springs once every 2.0 s. What is the spring constant of its springs?

   Answer: $9870\,\frac{\text{N}}{\text{m}}$

3. **(M – honors & AP®; S – CP1)** A 4.0 kg block is released from a height of 5.0 m on a frictionless ramp. When the block reaches the bottom of the ramp, it slides along a frictionless surface and hits a spring with a spring constant of $4.0\times 10^4\ \frac{\text{N}}{\text{m}}$ as shown in the diagram below:



What is the maximum distance that the spring is compressed after the impact?

(*Hint: this is a conservation of energy problem.*)

Answer: 0.10 m

4. **(M – honors & AP®; S – CP1)** A 1.6 kg block is attached to a spring that has a spring constant of $1000\,\frac{\text{N}}{\text{kg}}$. The spring is compressed a distance of 2.0 cm (which equals 0.02 m), and the block is released from rest onto a frictionless surface. What is the speed of the block as it passes through the equilibrium position?

(*Hint: this is a conservation of energy problem.*)

Answer: $0.5\,\frac{\text{m}}{\text{s}}$