Pendulums

Unit: Simple Harmonic Motion

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 7.2.A, 7.2.A.1, 7.2.A.1.ii

Mastery Objective(s): (Students will be able to…)

- Calculate the period of oscillation of a pendulum.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the mass of the pendulum does not affect its period.

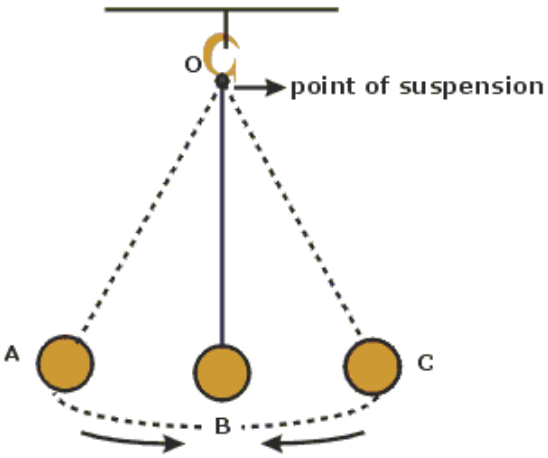
Tier 2 Vocabulary: pendulum

Labs, Activities & Demonstrations:

- Pendulum made from a mass hanging from a lab stand.

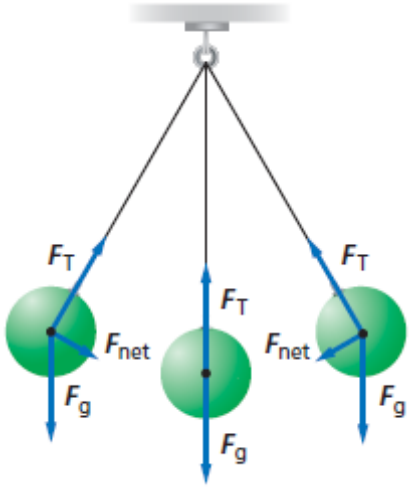
Notes:

pendulum: a lever that is suspended from a point such that it can swing back and forth.



The Forces on a Pendulum

As the pendulum swings, its mass remains constant, which means the force of gravity pulling it down remains constant. The tension on the pendulum (which we can think of as a rope or string, though the pendulum can also be solid) also remains constant as it swings.



However, as the pendulum swings, the angle of the tension force changes. When the pendulum is not in the center (bottom), the vertical component of the tension is $F_T \cos\theta$, and the horizontal component is $F_T \sin\theta$. Because the angle is between 0° and 90°, $\cos\theta < 1$, which means F_g is greater than the upward component of F_T. This causes the pendulum to eventually stop. Also because the angle is between 0° and 90°, $\sin\theta > 0$, This causes the pendulum to start swinging in the opposite direction.

The Period of a Pendulum

period or period of oscillation: the time it takes a pendulum to travel from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is *T*, and the unit is usually seconds.

Note that the time between pendulum "beats" (such as the tick-tock of a pendulum clock) are ½ of the period of the pendulum. Thus a "grandfather" clock with a pendulum that beats seconds has a period T = 2 s.

The period of a pendulum depends on the force of gravity, the length of the pendulum, and the maximum angle of displacement. For small angles ($\theta \leq 15°$), the period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where T is the period of oscillation, ℓ is the length of the pendulum in meters, and g is the acceleration due to gravity (approximately $10\,\frac{m}{s^2}$ on Earth).

Note that the potential energy of a pendulum is simply the gravitational potential energy of the pendulum's center of mass.

The velocity of the pendulum at its lowest point (where the potential energy is zero and all of the energy is kinetic) can be calculated using conservation of energy.

Sample Problem:

Q: An antique clock has a pendulum that is 0.20 m long. What is its period?

A: The period is given by the equation:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$
$$T = 2(3.14)\sqrt{\frac{0.20}{10}}$$
$$T = 6.28\sqrt{0.02}$$
$$T = (6.28)(0.141)$$
$$T = 0.889\text{ s}$$

Homework Problems

1. **(S)** A 20.0 kg chandelier is suspended from a high ceiling with a cable 6.0 m long. What is its period of oscillation as it swings?

   Answer: 4.87 s

2. **(M)** What is the length of a pendulum that oscillates 24.0 times per minute?

   Answer: 1.58 m

3. **(M)** The ceiling in a physics classroom is approximately 3.6 m high. If a bowling ball pendulum reaches from the ceiling to the floor, how long does it take the bowling ball pendulum to swing across the room and back?

   Answer: 3.77 s