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Buoyancy

Unit: Fluids & Pressure

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-10(MA), HS-PS2-1

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 8.3.B, 8.3.B.1, 8.3.B.2, 8.3.B.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving the buoyant force on an object.
- Use a free-body diagram to represent the forces on an object surrounded by a fluid.

Success Criteria:

- Problems are set up & solved correctly with the correct units.

Language Objectives:

- Explain why a fluid exerts an upward force on an object surrounded by it.

Tier 2 Vocabulary: float, displace

Labs, Activities & Demonstrations:

- Upside-down beaker with tissue
- Ping-pong ball or balloon under water
- beaker floating in water
  - right-side-up with weights
  - upside-down with trapped air
- Spring scale with mass in & out of water on a balance
- Cartesian diver
- Aluminum foil & weights
- Cardboard & duct tape canoes

Notes:

<u>displace</u>: to push out of the way

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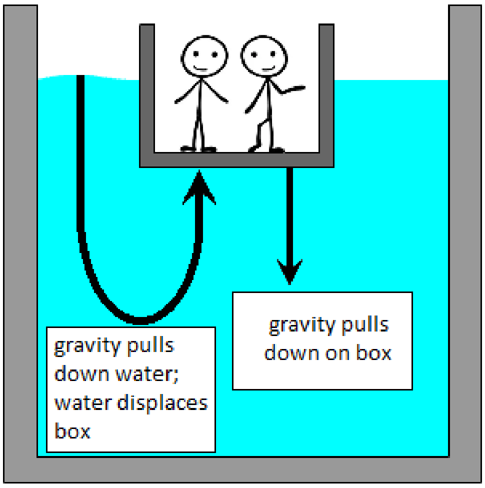
<u>buoyancy</u>: a net upward force caused by the differences in hydrostatic pressure at different levels within a fluid.

Buoyancy is ultimately caused by gravity:

1. Gravity pulls down on an object.
2. The object displaces water (or whatever fluid it's in).
3. Gravity pulls down on the water.
4. The water attempts to displace the object.

The force of the water attempting to displace the object is the buoyant force $(\vec{F}_B)$.



If the object floats, it reaches its equilibrium when the weight of the object and the weight of the water that was displaced (and is trying to displace the object) are equal.

If the object sinks, it is because the object can only displace its own volume. If an equal volume of water would weigh less than the object, the weight of the water is unable to apply enough force to lift the object.

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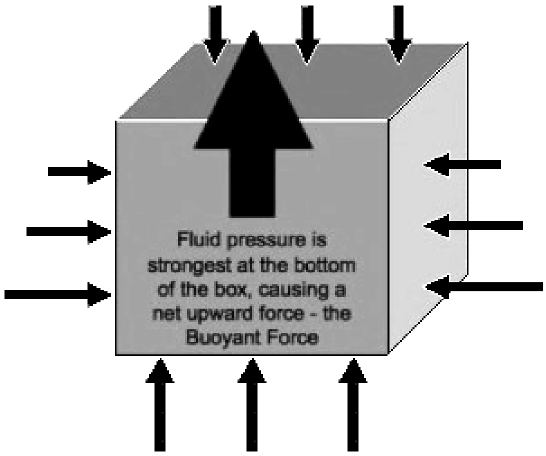
The reason the object moves upwards is because the hydrostatic pressure is stronger at the bottom of the object than at the top. This slight difference causes a net upward force on the object.



When an object displaces a fluid:

1. The volume of the fluid displaced equals the volume of the submerged part of the object: $V_{fluid\ displaced} = V_{submerged\ part\ of\ object}$
2. The weight of the fluid displaced equals the buoyant force (F_B).
3. The net force on the object, if any, is the difference between its weight and the buoyant force: $F_{net} = F_g - F_B$

The equation for the buoyant force is:

$$F_B = \rho V_d g$$

Where:

F_B = buoyant force (N)
ρ = density of fluid $\left(\frac{\text{kg}}{\text{m}^3}\right)$; fresh water = $1000\,\frac{\text{kg}}{\text{m}^3}$
V_d = volume of fluid displaced (m^3)
g = strength of gravitational field $\left(g = 10\,\frac{\text{N}}{\text{kg}}\right)$

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Maximum Buoyant Force

The maximum buoyant force on an object is conceptually similar to the maximum force of static friction.

Friction	Buoyancy
Static friction is a reaction force that is equal to the force that caused it.	Buoyancy is a reaction force that is equal to the force that caused it (the weight of the object).
When static friction reaches its maximum value, the object starts moving.	When the buoyant force reaches its maximum value (*i.e.,* when the volume of water displaced equals the volume of the object), the object sinks.
When the object is moving, there is still friction, but the force is not strong enough to stop the object from moving.	When an object sinks, there is still buoyancy, but the force is not strong enough to cause the object to float.

Detailed Explanation

If the object floats, there is no net force, which means the weight of the object is equal to the buoyant force. This means:

$$F_g = F_B$$
$$mg = \rho V_d g$$

Cancelling *g* from both sides gives $m = \rho V_d$, which can be rearranged to give the equation for density:

$$\rho = \frac{m}{V_d}$$

Therefore, if the object floats:

- The mass of the object equals the mass of the fluid displaced.
- The volume of the fluid displaced equals the volume of the object that is submerged.
- The density of the object (including any air inside of it that is below the fluid level) is less than the density of the fluid. (This is why a ship made of steel can float.)

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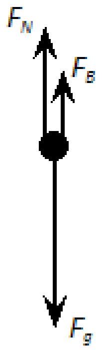
If the object sinks, the weight of the object is greater than the buoyant force. This means:

$$F_B = \rho V_d g$$
$$F_g = mg$$



Therefore:

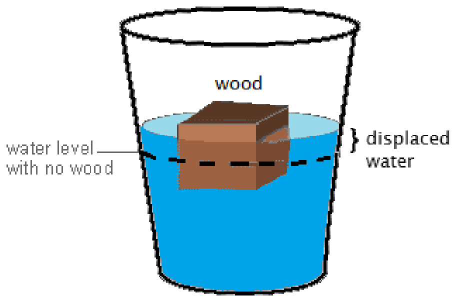
- The apparent weight of the submerged object is $F_{net} = F_g - F_B$

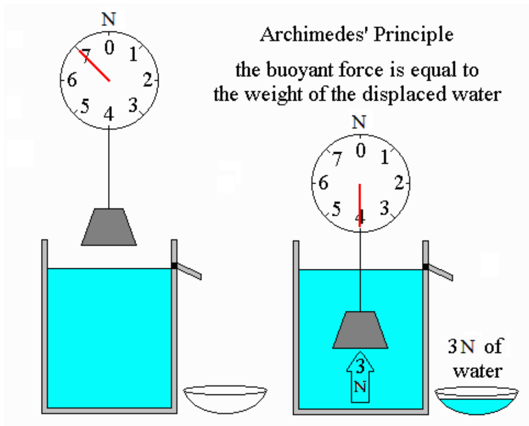
Note that if the object is resting on the bottom of the container, the net force must be zero, which means the normal force and the buoyant force combine to supply the total upward force. *I.e.,* for an object resting on the bottom:

$$F_{net} = 0 = F_g - (F_B + F_N)$$

which means:

$$F_g = F_B + F_N$$





This concept is known as Archimedes' Principle, named for the ancient Greek scientist who discovered it.

The buoyant force can be calculated from the following equation:

$$F_B = m_d g = \rho V_d g$$

where:

F_B = buoyant force
m_d = mass of fluid displaced by the object
g = strength of gravitational field ($10\,\frac{\text{N}}{\text{kg}}$ on Earth)
ρ = density of the fluid applying the buoyant force (*e.g.,* water, air)
V_d = volume of fluid displaced by the object

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Sample Problems:

Q: A cruise ship displaces 35 000 tonnes of water when it is floating. (1 tonne = 1 000 kg) If sea water has a density of $1025\,\frac{\text{kg}}{\text{m}^3}$, what volume of water does the ship displace? What is the buoyant force on the ship?

A:
$$\rho = \frac{m}{V_d}$$
$$1025 = \frac{(35\,000)(1000)}{V_d}$$
$$V_d = 34\,146\ \text{m}^3$$
$$F_B = \rho V_d = (1025)(34\,146)(10) = 3.5 \times 10^8\ \text{N}$$

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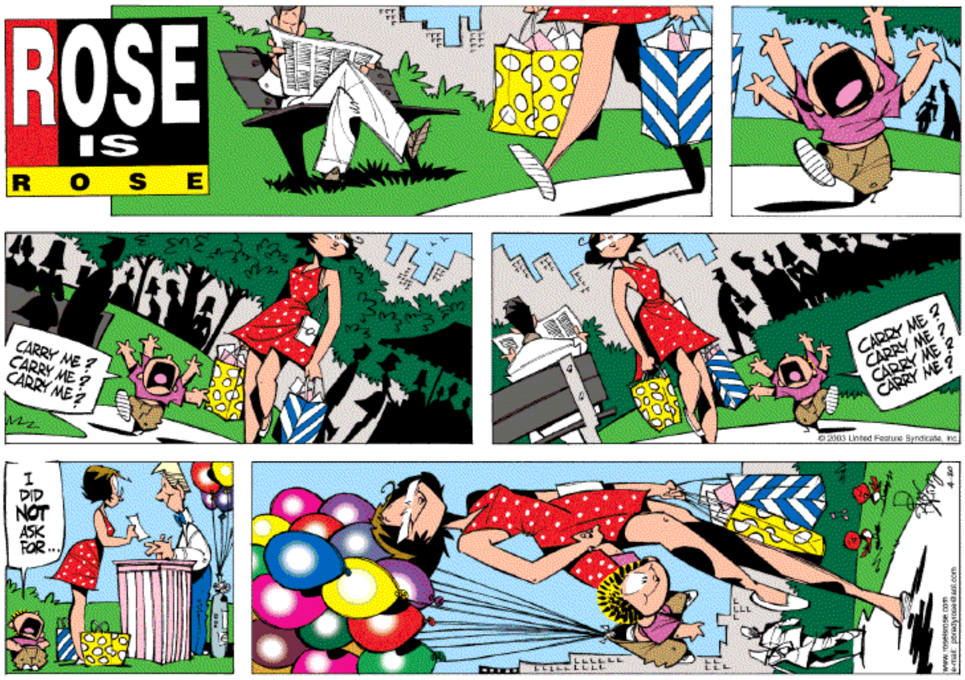
Q: Consider the following cartoon:



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Given the following assumptions:

- The balloons are standard 11" balloons, meaning that they have a diameter of 11 inches (28 cm), which equals a radius of 14 cm = 0.14 m.
- The temperature is 20°C. At this temperature, air has a density of $1.200\,\frac{\text{kg}}{\text{m}^3}$, and helium has a density of $0.166\,\frac{\text{kg}}{\text{m}^3}$.
- Pasquale (the child) is probably about four years old. The average mass of a four-year-old boy is about 16 kg.
- The mass of an empty balloon plus string is 2.37 g = 0.00237 kg

How many balloons would it actually take to lift Pasquale?

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A: In order to lift Pasquale, $F_B = F_g$.

$F_g = mg = (16)(10) = 160\,\text{N}$

$F_B = \rho_{air} V_d g = (1.2)\, V_d\, (10)$

Because $F_B = F_g$, this means:

$$160 = 12\,V_d$$
$$V_d = 13.\overline{3}\,\text{m}^3$$

Assuming spherical balloons, the volume of one balloon is:

$$V = \frac{4}{3}\pi r^3 = (\frac{4}{3})(3.14)(0.14)^3 = 0.0115\,\text{m}^3$$

Therefore, we need $\frac{13.\overline{3}}{0.0115}$ = 1 160 balloons to lift Pasquale.

However, the problem with this answer is that it doesn't account for the mass of the helium, the balloons and the strings.

Each balloon contains $0.0115\,\text{m}^3 \times 0.166\,\frac{\text{kg}}{\text{m}^3} = 0.00191\,\text{kg}$ of helium.

Each empty balloon (including the string) has a mass of 2.37 g = 0.00237 kg.

The total mass of each balloon full of helium is
1.91 g + 2.37 g = 4.28 g = 0.00428 kg.

This means if we have *n* balloons, the total mass of Pasquale plus the balloons is 16 + 0.00428*n* kilograms. The total weight (in newtons) of Pasquale plus the balloons is therefore this number times 10, which equals 160 + 0.0428*n.*

The buoyant force of one balloon is:

$$F_B = \rho_{air} V_d g = (1.2)(0.0115)(10) = 0.138\,\text{N}$$

Therefore, the buoyant force of *n* balloons is 0.138*n* newtons.

For Pasquale to be able to float, $F_B = F_g$, which means

$$0.138n = 0.0428n + 160$$
$$0.0952n = 160$$
$$n = \boxed{1\,680 \text{ balloons}}$$

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Homework Problems

1. **(M)** A block is 0.12 m wide, 0.07 m long and 0.09 m tall and has a mass of 0.50 kg. The block is floating in water with a density of $1000\,\frac{\text{kg}}{\text{m}^3}$.

   a. What volume of the block is below the surface of the water?

   Answer: $5\times10^{-4}\ \text{m}^3$

   b. If the entire block were pushed under water, what volume of water would it displace?

   Answer: $7.56\times10^{-4}\ \text{m}^3$

   c. How much *additional* mass could be piled on top of the block before it sinks?

   Answer: 0.256 kg

2. **(S)** The SS United Victory was a cargo ship launched in 1944. The ship had a mass of 15 200 tonnes fully loaded. (1 tonne = 1 000 kg). The density of sea water is $1025\,\frac{\text{kg}}{\text{m}^3}$. What volume of sea water did the SS United Victory displace when fully loaded?

   Answer: 14 829 m^3

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3. **(S)** An empty box is 0.11 m per side. It will slowly be filled with sand that has a density of $3500\,\frac{\text{kg}}{\text{m}^3}$. What volume of sand will cause the box to sink in water? Assume water has a density of $1000\,\frac{\text{kg}}{\text{m}^3}$. Assume the box is neutrally buoyant, which means you may neglect the weight of the box.

   Strategy:

   a. *Find the volume of the box.*
   b. *Find the mass of the water displaced.*
   c. *Find the volume of that same mass of sand.*

   Answer: $3.80\times10^{-4}\ \text{m}^3$