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# Fluid Motion & Bernoulli's Law

**Unit:** Fluids & Pressure

**NGSS Standards/MA Curriculum Frameworks (2016):** HS-PS2-10(MA), HS-PS2-1

**AP® Physics 1 Learning Objectives/Essential Knowledge (2024):** 8.4.B, 8.4.B.1, 8.4.B.2, 8.4.B.3

**Mastery Objective(s):** (Students will be able to…)

- Solve problems involving fluid flow using Bernoulli's Equation.

**Success Criteria:**

- Problems are set up & solved correctly with the correct units.

**Language Objectives:**

- Explain why a fluid has less pressure when the flow rate is faster.

**Tier 2 Vocabulary**: fluid, velocity

## Labs, Activities & Demonstrations:

- Blow across paper (unfolded & folded)
- Blow between two empty cans.
- Ping-pong ball and air blower (without & with funnel)
- Venturi tube
- Leaf blower & large ball

## Notes:

dynamic pressure ($P_D$): the pressure caused when particles of a moving fluid entrain adjacent fluid particles and push them along.

When a fluid is flowing, the fluid must have kinetic energy, which equals the work that it takes to move that fluid.

The following are the equations for work and kinetic energy:

$$K = \tfrac{1}{2}mv^2$$
$$W = \Delta K = F_{\parallel}d$$

(These equations were studied in detail in the *Introduction: Energy, Work* & Power unit, starting on page 407.)

Combining these equations (the work-energy theorem) gives $\tfrac{1}{2}mv^2 = F_{\parallel}d$.

Solving $P_D = \frac{F}{A}$ for force gives $F = P_D A$. Substituting this into the above equation gives:

$$\tfrac{1}{2}mv^2 = F_{\parallel}d = P_D A d$$

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Rearranging the above equation to solve for dynamic pressure gives the following. Because volume is area times distance ($V = Ad$), we can then substitute $V$ for $Ad$:

$$P_D = \frac{\frac{1}{2}mv^2}{Ad} = \frac{\frac{1}{2}mv^2}{V}$$

Finally, rearranging $\rho = \frac{m}{V}$ to solve for mass gives $m = \rho V$. This means our equation becomes:

$$P_D = \frac{\frac{1}{2}mv^2}{V} = \frac{\frac{1}{2}\rho \cancel{V} v^2}{\cancel{V}} = \frac{1}{2}\rho v^2$$

$$P_D = \frac{1}{2}\rho v^2$$

## Bernoulli's Principle

Bernoulli's Principle, named for Dutch-Swiss mathematician Daniel Bernoulli states that the pressures in a moving fluid are caused by a combination of:

- The hydrostatic pressure: $P_H = \rho gh$
- The dynamic pressure: $P_D = \frac{1}{2}\rho v^2$
- The "external" pressure, which is the pressure that the fluid exerts on its surroundings. (This is the pressure we would measure with a pressure gauge.)

A change in any of these pressures affects the others, which means:

$$P_{ext.} + P_H + P_D = \text{constant}$$

$$P_{ext.} + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

The above equation is Bernoulli's equation.

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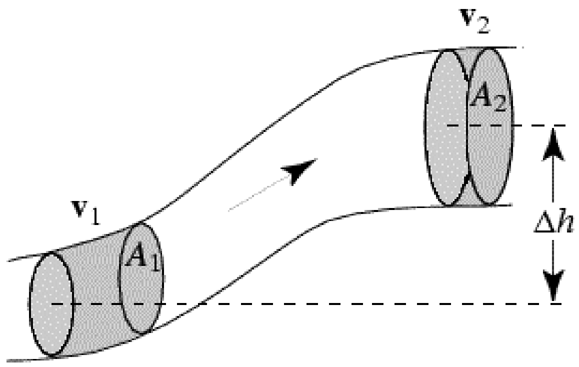
For example, consider the following situation:




- The velocity of the fluid is changing (because the cross-sectional area is changing—remember the continuity equation $A_1v_1 = A_2v_2$). This means the dynamic pressure, $P_D = \frac{1}{2}\rho v^2$ is changing.
- The height is changing, which means the hydrostatic pressure, $P_H = \rho gh$ is changing.
- The external pressures will also be different, in order to satisfy Bernoulli's Law.

This means Bernoulli's equation becomes:

$$P_1 + \rho gh_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \tfrac{1}{2}\rho v_2^2$$

where variables with the subscript "1" are the initial conditions, and variables with the subscript "2" are the final conditions.

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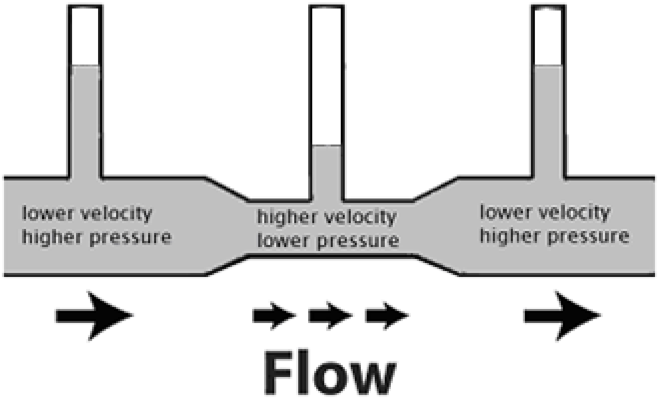
Note particularly that Bernoulli's equation tells us that increasing the fluid velocity ($v$) increases the dynamic pressure. (If $v$ increases, then $P_D = \frac{1}{2}\rho v^2$ increases.)

This means if more of the total pressure is in the form of dynamic pressure, that means the hydrostatic and/or external pressures will be less.

Consider the following example:




This pipe is horizontal, which means $h$ is constant; therefore $\rho gh$ is constant. This means that if $\frac{1}{2}\rho v^2$ increases, then pressure ($P$) must decrease so that $P_{ext.} + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$.

Although Bernoulli published his principle in 1738, the application to fluids in constricted channels was not published until 1797 by Italian physicist Giovanni Venturi. The above apparatus is named after Venturi and is called a Venturi tube.

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# Bernoulli's Equation and Conservation of Energy

Although we have not yet covered the Energy unit, Bernoulli's equation is essentially the conservation of energy per unit volume.

Briefly:

kinetic energy (*K* or *KE*): energy that an object or system has due to its motion

potential energy (*U* or *PE*): energy due to a force of attraction between two objects within a system. In the case of potential energy due to gravity, one of the objects is the Earth.

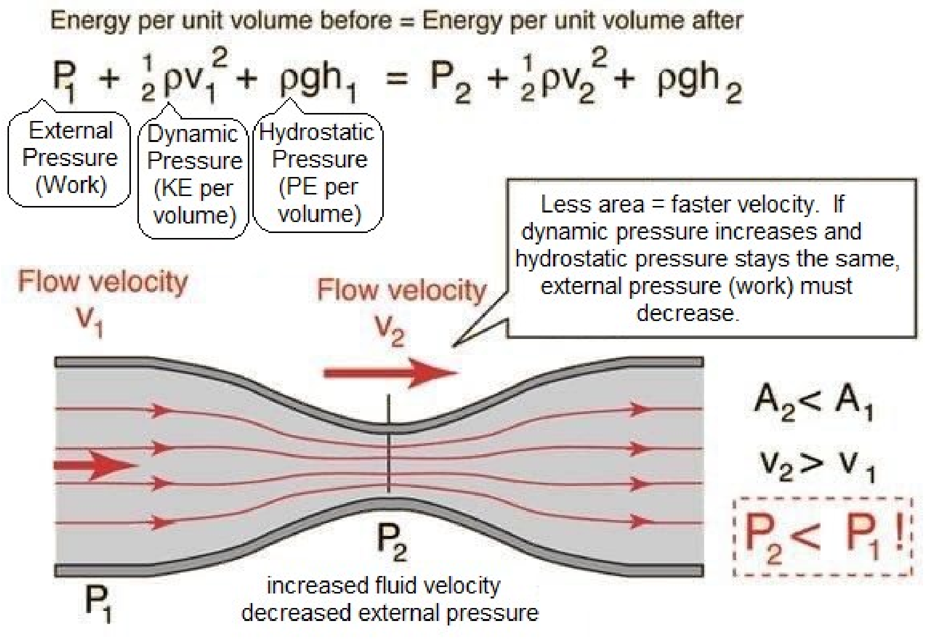
work (*W*): energy transferred into or out of a system

The three terms in Bernoulli's equation are:

$P$ = external pressure = work that the fluid can do per unit volume

$P_D = \frac{1}{2}\rho v^2$ = kinetic energy of the fluid per unit volume

$P_H = \rho g h$ = gravitational potential energy of the fluid per unit volume




Energy is discussed in detail in the *Introduction: Energy, Work* & Power unit, starting on page 407.

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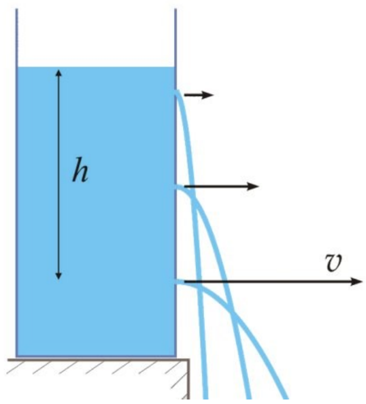
## Torricelli's Theorem

A special case of Bernoulli's Principle was discovered almost 100 years earlier, in 1643 by Italian physicist and mathematician Evangelista Torricelli. Torricelli observed that in a container with fluid effusing (flowing out) through a hole, the more fluid there is above the opening, the faster the fluid comes out.

Torricelli found that the velocity of the fluid was the same as the velocity would have been if the fluid were falling straight down, which can be calculated from the change of gravitational potential energy to kinetic energy:




$$\tfrac{1}{2}\cancel{m}v^2 = \cancel{m}gh \;\rightarrow\; v^2 = 2gh \;\rightarrow\; v = \sqrt{2gh}$$

Torricelli's theorem can also be derived from Bernoulli's equation*:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

- The external pressures ($P_1$ and $P_2$) are both equal—atmospheric pressure—so they cancel.
- The fluid level is going down slowly enough that the velocity of the fluid inside the container ($v_1$) is essentially zero.
- Once the fluid exits the container, the hydrostatic pressure is zero $(\rho g h_2 = 0)$.

This leaves us with:

$$\cancel{\rho} g h_1 = \tfrac{1}{2}\cancel{\rho} {v_2}^2 \;\rightarrow\; 2gh_1 = {v_2}^2 \;\rightarrow\; \sqrt{2gh_1} = v_2$$

We could do a similar proof from the kinematic equation: $v^2 - v_o^2 = 2ad$

Substituting $a = g, d = h, \text{and}\, v_o = 0$ gives $v^2 = 2gh$ and therefore $v = \sqrt{2gh}$

Note: as described in Hydrostatic Pressure, starting on page 526, hydrostatic pressure is caused by the fluid ***above*** the point of interest, meaning that height is measured *upward*, not downward. In the above situation, the two points of interest for the application of Bernoulli's law are actually:

- inside the container next to the opening, where there is fluid above, but essentially no movement of fluid (v = 0, but h ≠ 0)
- outside the opening where there is no fluid above, but the jet of fluid is flowing out of the container (h = 0, but v ≠ 0)

* On the AP® Physics exam, you must start problems from equations that are on the formula sheet. This means you may not use Torricelli's Theorem on the exam unless you first derive it from Bernoulli's Equation.

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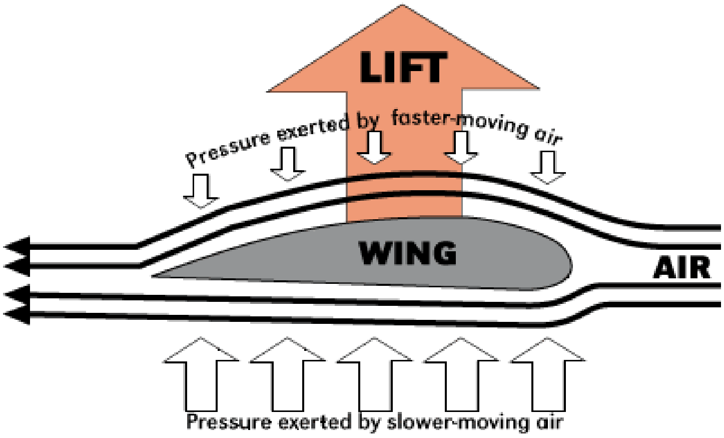
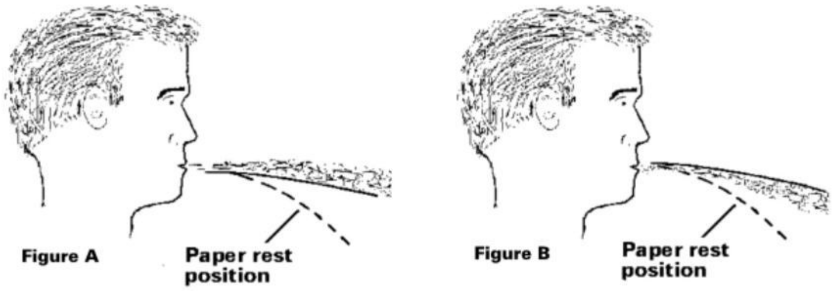
The decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:




(Of course, most of an airplane's lift comes from the fact that the wing is inclined with an angle of attack relative to its direction of motion, an application of Newton's third law.)

A common demonstration of Bernoulli's Law is to blow across a piece of paper:




The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

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## Sample Problems:

Q: A fluid in a pipe with a *diameter* of 0.40 m is moving with a velocity of $0.30\,\frac{m}{s}$. If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: Note that the diameter of the pipe was given, not the radius. The cross-sectional area of the first pipe is:

$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126\text{ m}^2$

The cross-sectional area of the second pipe is:

$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314\text{ m}^2$

Using the continuity equation:

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ (0.126)(0.30) &= (0.0314)v_2 \\ v_2 &= 1.2\,\tfrac{m}{s} \end{aligned}$$

Q: A fluid with a density of $1250\,\frac{kg}{m^3}$ has a pressure of 45 000 Pa as it flows at $1.5\,\frac{m}{s}$ through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of $3.4\,\frac{m}{s}$ through it?

A:

$$\begin{aligned} P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 &= P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 \\ 45\,000 + (1250)(10)(0) + \left(\tfrac{1}{2}\right)(1250)(1.5)^2 &= P_2 + (1250)(10)(2.5) + \left(\tfrac{1}{2}\right)(1250)(3.4)^2 \\ 45\,000 + 1406 &= P_2 + 31\,250 + 7225 \\ P_2 &= 7931\text{ Pa} \end{aligned}$$

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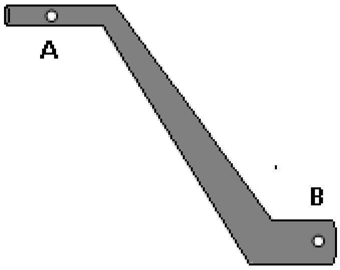
# Homework Problem




1. **(M)** At point A on the pipe to the right, the water's speed is $4.8\,\frac{m}{s}$ and the external pressure (the pressure on the walls of the pipe) is 52.0 kPa. The water drops 14.8 m to point B, where the pipe's cross-sectional area is twice that at point A.

   a. Calculate the velocity of the water at point B.

   Answer: $2.4\,\frac{m}{s}$

   b. Calculate the external pressure (the pressure on the walls of the pipe) at point B.

   Answer: 208 600 Pa or 208.6 kPa