Class Notes for

Physics 2:

Non-Mechanics Topics

(including AP® Physics 2)

in Plain English

Jeff Bigler

September 2024

https://www.mrbigler.com/Physics-2/Notes-Physics-2.pdf

ISBN-13: 979-8545489268

ISBN-10: 8545489268

Note to Students

This is a set of class notes that can be used for an algebra-based, second-year high school Physics 2 course at the honors or AP® level. This hardcopy is provided so that you can fully participate in class discussions without having to worry about writing everything down.

While a significant amount of detail is included in these notes, they are intended as a supplement to textbooks, classroom discussions, experiments and activities. These class notes and any textbook discussion of the same topics are intended to be complementary. In some cases, the notes and the textbook differ in method or presentation, but the physics is the same. There may be errors and/or omissions in any textbook. There are almost certainly errors and omissions in these notes, despite my best efforts to make them clear, correct, and complete.

Topics

The AP® curriculum is, of course, set by the College Board. My decision was to have the same units in the honors course and the AP® course. However, the honors course has more flexibility with regard to pacing, difficulty, and topics.

Topics that are part of the curriculum for either the honors or AP® course but not both are marked in the left margin as follows:

honors only (not AP®) ¦

AP® (only) ‖

Topics that are not otherwise marked should be assumed to apply to both courses.

The first two units (*Laboratory & Measurement* and *Mathematics*) are repeated from the Physics 1 notes, so these notes can be used without having to refer to them.

About the Homework Problems

The homework problems include a mixture of easy and challenging problems. *The process of making yourself smarter involves challenging yourself, even if you are not sure how to proceed.* By spending at least 10 minutes attempting each problem, you build neural connections between what you have learned and what you are trying to do. Even if you are not able to get the answer, when we go over those problems in class, you will reinforce the neural connections that led in the correct direction.

Answers to most problems are provided so you can check your work and see if you are on the right track. Do not simply write those answers down in order to receive credit for work you did not do. This will give you a false sense of confidence, and will actively prevent you from using the problems to make yourself smarter. *You have been warned.*

Using These Notes

As we discuss topics in class, you will want to add your own notes to these. If you have purchased this copy, you are encouraged to write directly in it, just as you would write in your own notebook. If this copy was issued to you by the school and you intend to return it at the end of the year, you will need to write your supplemental notes on separate paper. If you do this, be sure to write down page numbers in your notes, to make cross-referencing easier.

You should bring these notes to class every day, because lectures and discussions will follow these notes, which will also be projected onto the SMART board.

Features

These notes, and the course they accompany, are designed to follow both the 2016 Massachusetts Curriculum Frameworks, which are based on the Next Generation Science Standards (NGSS), and the AP® Physics 1 curriculum. (Note that the AP® Learning Objectives/Essential Knowledge (2024) are the ones from 2014.) The notes also utilize strategies from the following popular teaching methods:

- Each topic includes Mastery Objectives and Success Criteria. These are based on the *Studying Skillful Teaching* course, from Research for Better Teaching (RBT), and are in "Students will be able to…" language.
- AP® topics include Learning Objectives and Essential Knowledge (2024) from the College Board.
- Each topic includes Next-Generation Science Standards (NGSS) and Massachusetts Curriculum Frameworks (2016) . The MA Curriculum Frameworks are the same as the NGSS standards with the exception of a few MA-specific frameworks, which include (MA) in the identifier.
- Each topic includes Language Objectives and Tier 2 vocabulary words for English Learners, based on the Massachusetts Rethinking Equity and Teaching for English Language Learners (RETELL) course.
- Notes are organized in Cornell notes format as recommended by Keys To Literacy.
- Problems in problem sets are designated "Must Do" (M), "Should Do" (S) and "Aspire to Do" (A), as recommended by the Modern Classrooms Project (MCP).

Conventions

Some of the conventions in these notes are different from conventions in some physics textbooks. Although some of these are controversial and may incur the ire of other physics teachers, here is an explanation of my reasoning:

- When working sample problems, the units are left out of the algebra until the end. While I agree that there are good reasons for keeping the units to show the dimensional analysis, many students confuse units for variables, *e.g.,* confusing the unit "m" (meters) with the variable "*m*" (mass).
- Problems are worked using $g = 10\,\frac{\text{m}}{\text{s}^2} = 10\,\frac{\text{N}}{\text{kg}}$. This is because many students are not adept with algebra, and have trouble seeing where a problem is going once they take out their calculators. With simpler numbers, students have an easier time following the physics.
- Vector quantities are denoted with arrows as well as boldface, *e.g.,* $\vec{\boldsymbol{v}}, \vec{\boldsymbol{d}}, \vec{\boldsymbol{F}}_g$. This is to help students keep track of which quantities are vectors and which are scalars. In some cases, this results in equations that are nonsensical as vector expressions, such as $\vec{\boldsymbol{v}}^2 - \vec{\boldsymbol{v}}_o^{\,2} = 2\vec{\boldsymbol{a}}\vec{\boldsymbol{d}}$. (A vector can't be "squared", and multiplying $\vec{\boldsymbol{a}}$ by $\vec{\boldsymbol{d}}$ would have to be either $\vec{\boldsymbol{a}} \bullet \vec{\boldsymbol{d}}$ or $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{d}}$.) It is good to point this out to students when they encounter these expressions, but in my opinion the benefits of keeping the vector notation even where it results in an incorrect vector expression outweigh the drawbacks.
- Forces are denoted as the variable $\vec{\boldsymbol{F}}$ with a subscript, e.g., $\vec{\boldsymbol{F}}_g, \vec{\boldsymbol{F}}_f, \vec{\boldsymbol{F}}_N, \vec{\boldsymbol{F}}_T$, *etc.* instead of $m\vec{\boldsymbol{g}}, \vec{\boldsymbol{f}}, \vec{\boldsymbol{N}}, \vec{\boldsymbol{T}}$, *etc.* This is to reinforce the connection between a quantity (force), a single variable $(\vec{\boldsymbol{F}})$, and a unit.

- Average velocity is denoted $\vec{v}_{ave.}$ instead of $\bar{\vec{v}}$. I have found that using the subscript "ave." helps students remember that average velocity is different from initial and final velocity.
- The variable V is used for electric potential. Voltage (potential difference) is denoted by ΔV. Although $\Delta V = IR$ is different from how the equation looks in most physics texts, it is useful to teach circuits starting with electric potential, and it is useful to maintain the distinction between absolute electric potential (V) and potential difference (ΔV). This is also how the College Board represents electric potential *vs.* voltage on AP® Physics exams.
- Equations are typeset on one line when practical. While there are very good reasons for teaching $\vec{a} = \frac{\vec{F}_{net}}{m}$ rather than $\vec{F}_{net} = m\vec{a}$ and $I = \frac{\Delta V}{R}$ rather than $\Delta V = IR$, students' difficulty in solving for a variable in the denominator often causes more problems than does their lack of understanding of which are the independent and dependent variables.

Learning Progression

There are several categories of understandings and skills that simultaneously build on themselves throughout this course:

Content

The content topics are:

- Thermal Physics (Heat) & Thermodynamics
- Electricity & Magnetism
- Mechanical Waves
- Electromagnetic Waves, Light & Optics
- Quantum, Particle, Atomic & Nuclear Physics

Preliminary "unit zero" topics such as laboratory and mathematical skills are presented in the Physics 1 notes, and are not duplicated here.

Problem-Solving

This course builds on the problem-solving skills from Physics 1. The topics in this course require more high-level thinking to decide what the situation is for each problem, and which equation(s) apply.

Laboratory

This course continues the experimental design lessons learned in Physics 1. Because the topics require more specialized equipment, more time will be spent teaching students to use the equipment and giving them sufficient time to practice with it.

Scientific Discourse

In this course, the causal relationships between quantities are more complex than in Physics 1. Students need to continue to be given opportunities to explain these relationships throughout the course, both orally and in writing.

These notes would not have been possible without the assistance of many people. It would be impossible to include everyone, but I would particularly like to thank:

- Every student I have ever taught, for helping me learn how to teach, and how to explain and convey challenging concepts.
- The physics teachers I have worked with over the years who have generously shared their time, expertise, and materials. In particular, Mark Greenman, who has taught multiple courses on teaching physics and who, as the PhysTEC Teacher in Residence at Boston University, organizes a monthly meeting for Boston-area physics teachers to share laboratory activities and demonstrations; Barbara Watson, whose AP® Physics 1 and AP® Physics 2 Summer Institutes I attended, and with whom I have had numerous conversations about the teaching of physics, particularly at the AP® level; and Eva Sacharuk, who met with me weekly during my first year teaching physics to share numerous demonstrations, experiments and activities that she collected over her many decades in the classroom.
- Every teacher I have worked with, for their kind words, sympathetic listening, helpful advice and suggestions, and other contributions great and small that have helped me to enjoy and become competent at the profession of teaching.
- The department heads, principals and curriculum directors I have worked with, for mentoring me, encouraging me, allowing me to develop my own teaching style, and putting up with my experiments, activities and apparatus that place students physically at the center of a physics concept. In particular: Mark Greenman, Marilyn Hurwitz, Scott Gordon, Barbara Osterfield, Wendell Cerne, John Graceffa, Maura Walsh, Lauren Mezzetti, Jill Joyce, Tom Strangie, and Anastasia Mower.
- Everyone else who has shared their insights, stories, and experiences in physics, many of which are reflected in some way in these notes.

I am reminded of Sir Isaac Newton's famous quote, *"If I have seen further it is because I have stood on the shoulders of giants."*

About the Author

Jeff Bigler is a physics teacher at Lynn English High School in Lynn, Massachusetts. He has degrees from MIT in chemical engineering and biology, and is a National Board certified teacher in Science–Adolescence and Young Adulthood. He worked in biotech and IT prior to starting his teaching career in 2003. He has taught both physics and chemistry at all levels from conceptual to AP®.

He is married and has two adult daughters. His hobbies are music and Morris dancing.

Errata

As is the case in just about any large publication, these notes undoubtedly contain errors despite my efforts to find and correct them all.

Known errata for these notes are listed at:
https://www.mrbigler.com/Physics-2/Notes-Physics-2-errata.shtml

Contents

NGSS Standards/MA Curriculum Frameworks for Physics

Except where denoted with (MA), these standards are the same as the Next Generation Science (NGSS) Standards. Standards that are crossed out (~~like this~~) are covered in the Physics 1 notes. Note that both sets of notes may be necessary in order to cover all of the standards.

Standard	Topics	Chapters
HS-PS1-8	fission, fusion & radioactive decay: α, β & γ; energy released/absorbed	9
HS-PS2-1	~~Newton's 2nd ($F_{net} = ma$), motion graphs; ramps, friction, normal force, gravity,~~ magnetic force	Physics 1
HS-PS2-2	~~conservation of momentum~~	7
HS-PS2-3	~~lab: reduce impulse in a collision~~	—
HS-PS2-4	~~gravitation &~~ coulomb's law including relative changes	6
HS-PS2-5	electromagnetism: current produces magnetic field & vice-versa, including examples	8
HS-PS2-9(MA)	Ohm's Law, circuit diagrams, evaluate series & parallel circuits for ΔV, I or R.	7
HS-PS2-10(MA)	~~free-body diagrams, algebraic expressions & Newton's laws to predict acceleration for 1-D motion, including motion graphs~~	Physics 1
HS-PS3-1	conservation of energy including thermal, ~~kinetic, gravitational,~~ magnetic or electrical including ~~gravitational &~~ electric fields	5
HS-PS3-2	energy can be motion of particles or stored in fields. kinetic → thermal, evaporation/condensation, ~~gravitational potential energy~~, electric fields	4, 6
HS-PS3-3	lab: build a device that converts energy from one form to another.	Physics 1
HS-PS3-4a	zero law of thermodynamics (heat flow & thermal equilibrium)	5
HS-PS3-5	behavior of charges or magnets attracting & repelling	6, 8
HS-PS4-1	waves: $v = f\lambda$ & $T = 1/f$, EM waves traveling through space or a medium vs. mechanical waves in a medium	9, 10
HS-PS4-3	EM radiation is both wave & particle. Qualitative behavior of resonance, interference, diffraction, refraction, photoelectric effect and wave vs. particle model for both	10, 11
HS-PS4-5	Devices use waves and wave interactions with matter, such as solar cells, medical imaging, cell phones, wi-fi	10

MA Science Practices

Practice	Description
SP1	Asking questions.
SP2	Developing & using models.
SP3	Planning & carrying out investigations.
SP4	Analyzing & interpreting data.
SP5	Using mathematics & computational thinking.
SP6	Constructing explanations.
SP7	Engaging in argument from evidence.
SP8	Obtaining, evaluating and communicating information.

Introduction: Laboratory & Mathematics

Unit: Laboratory & Mathematics

Note that the topics of laboratory experiments, experimental design, uncertainty, solving word problems, vector math, and trigonometry are covered in the Physics 1 notes, and are not repeated here.

Topics covered in this chapter:

The purpose of this chapter is to teach prerequisite skills that will be used later in the course.

- *The AP® Physics Science Practices* lists & describes the scientific practices that are required by the College Board for AP® Physics 2.
- *Vectors*, *Vectors vs. Scalars in Physics*, and *Vector Multiplication* describe properties of vectors and how mathematics with vectors works.
- *Logarithms* is a review of the base 10 and natural logarithm functions.

Calculating uncertainty (instead of relying on significant figures) is a new and challenging skill that will be used in lab write-ups throughout the year.

Standards addressed in this chapter:

NGSS Standards/MA Curriculum Frameworks (2016):

This chapter addresses the following MA science and engineering practices:

Practice 1: Asking Questions and Defining Problems
Practice 2: Developing and Using Models
Practice 3: Planning and Carrying Out Investigations
Practice 4: Analyzing and Interpreting Data
Practice 6: Constructing Explanations and Designing Solutions
Practice 7: Engaging in Argument from Evidence
Practice 8: Obtaining, Evaluating, and Communicating Information

AP®

AP® Physics 2 Science Practices (2024):

This chapter addresses the following AP Physics 1 science practices:

SP 4.1 The student can justify the selection of the kind of data needed to answer a particular scientific question.

SP 4.2 The student can design a plan for collecting data to answer a particular scientific question.

SP 4.3 The student can collect data to answer a particular scientific question.

SP 4.4 The student can evaluate sources of data to answer a particular scientific question.

SP 5.1 The student can analyze data to identify patterns or relationships.

SP 5.2 The student can refine observations and measurements based on data analysis.

SP 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

Skills learned & applied in this chapter:

- Working with exponential and logarithmic functions.

Prerequisite Skills:

These are the mathematical understandings that are necessary for Physics 1 that are taught in the MA Curriculum Frameworks for Mathematics.

- Construct and use tables and graphs to interpret data sets.
- Solve simple algebraic expressions.
- Perform basic statistical procedures to analyze the center and spread of data.
- Measure with accuracy and precision (*e.g.,* length, volume, mass, temperature, time)
- Convert within a unit (*e.g.,* centimeters to meters).
- Use common prefixes such as milli-, centi-, and kilo-.
- Use scientific notation, where appropriate.
- Use ratio and proportion to solve problems.

Fluency in all of these understandings is a prerequisite for this course. Students who lack this fluency may have difficulty passing the class.

AP®

The AP® Physics Science Practices

Unit: Laboratory & Measurement

NGSS Standards/MA Curriculum Frameworks (2016): SP1, SP2, SP3, SP4, SP5, SP6, SP7

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): SP1, SP2, SP3, SP4, SP5, SP6, SP7

Mastery Objective(s): (Students will be able to…)

- Describe what the College Board and the State of Massachusetts want you to know about how science is done.

Tier 2 Vocabulary: data, claim, justify

Language Objectives:

- Explain the what the student is expected to do for each of the AP® Science Practices.

Tier 2 Vocabulary: practice, pose, model

Notes:

The College Board has described the scientific method in practical terms, dividing them into seven Science Practices that students are expected to learn in AP Physics 1.

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems.

A model is any mental concept that can explain and predict how something looks, works, is organized, or behaves. Atomic theory is an example of a model: matter is made of atoms, which are made of protons, neutrons, and electrons. The number, location, behavior and interactions of these sub-atomic particles explains and predicts how different types of matter behave.

1.1 The student can *create representations and models* of natural or man-made phenomena and systems in the domain.

1.2 The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.

1.3 The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.

1.4 The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.

1.5 The student can express key elements of natural phenomena across multiple representations in the domain.

AP®

Science Practice 2: The student can use mathematics appropriately.

Physics is the representation of mathematics in nature. It is impossible to understand physics without a solid understand of mathematics and how it relates to physics. For AP Physics 1, this means having an intuitive feel for how algebra works, and how it can be used to relate quantities or functions to each other. If you are the type of student who solves algebra problems via memorized procedures, you may struggle to develop the kind of mathematical understanding that is necessary in AP Physics 1.

2.1 The student can justify the selection of a mathematical routine to solve problems.

2.2 The student can *apply mathematical routines* to quantities that describe natural phenomena.

2.3 The student *can estimate numerically quantities* that describe natural phenomena.

Science Practice 3: The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

Ultimately, the answer to almost any scientific question is "maybe" or "it depends". Scientists pose questions to understand not just what happens, but the extent to which it happens, the causes, and the limits beyond which outside factors become dominant.

3.1 The student can pose scientific questions.

3.2 The student can refine scientific questions.

3.3 The student can evaluate scientific questions.

Science Practice 4: The student can plan and implement data collection strategies in relation to a particular scientific question.

Scientists do not "prove" things. Mathematicians and lawyers prove that something must be true. Scientists collect data in order to evaluate what happens under specific conditions, in order to determine what is likely true, based on the information available. Data collection is important, because the more and better the data, the more scientists can determine from it.

4.1 The student can *justify the selection of the kind of data* needed to answer a particular scientific question.

4.2 The student can *design a plan* for collecting data to answer a particular scientific question.

4.3 The student can *collect data* to answer a particular scientific question.

AP®

4.4 The student can *evaluate sources of data* to answer a particular scientific question.

Science Practice 5: The student can perform data analysis and evaluation of evidence.

Just as data collection is important, analyzing data and being able to draw meaningful conclusions is the other crucial step to understanding natural phenomena. Scientists need to be able to recognize patterns that actually exist within the data, and to be free from the bias that comes from expecting a particular result beforehand.

5.1 The student can *analyze data* to identify patterns or relationships.

5.2 The student can *refine observations and measurements* based on data analysis.

5.3 The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Science Practice 6: The student can work with scientific explanations and theories.

In science, there are no “correct” answers, only claims and explanations. A scientific claim is any statement that is believed to be true. In order to be accepted, a claim must be verifiable based on evidence, and any claim or explanation must be able to make successful predictions, which are also testable. Science does not prove claims to be universally true or false; science provides supporting evidence. Other scientists will accept or believe a claim provided that there is sufficient evidence to support it, and no evidence that directly contradicts it.

6.1 The student can justify claims with evidence.

6.2 The student can *construct explanations of phenomena based on evidence* produced through scientific practices.

6.3 The student can articulate the reasons that scientific explanations and theories are refined or replaced.

6.4 The student can make *claims and predictions about natural phenomena* based on scientific theories and models.

6.5 The student can evaluate alternative scientific explanations.

AP®

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across domains.

If a scientific principle is true in one domain, scientists must be able to consider that principle in other domains and apply their understanding from the one domain to the other. For example, conservation of momentum is believed by physicists to be universally true on every scale and in every domain, and it has implications in the contexts of laboratory-scale experiments, quantum mechanical behaviors at the atomic and sub-atomic levels, and special relativity.

7.1 The student can *connect phenomena and models* across spatial and temporal scales.

7.2 The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Vectors

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 1.1.A.1, 1.1.A.2, 1.1.A.3, 1.1.A.3.i, 1.1.A.3.ii, 1.1.B.1, 1.5.A, 1.5.A.1, 1.5.A.2, 1.5.A.3

Mastery Objective(s): (Students will be able to…)

- Identify the magnitude and direction of a vector.
- Combine vectors graphically and calculate the magnitude and direction.

Success Criteria:

- Magnitude is calculated correctly (Pythagorean theorem).
- Direction is correct: angle (using trigonometry) or direction (*e.g.,* "south", "to the right", "in the negative direction", *etc.*)

Language Objectives:

- Explain what a vector is and what its parts are.

Tier 2 Vocabulary: magnitude, direction

Notes:

<u>vector</u>: a quantity that has a direction as well as a magnitude (value/quantity).

E.g., if you are walking $1\,\frac{\text{m}}{\text{s}}$ to the north, the magnitude is $1\,\frac{\text{m}}{\text{s}}$ and the direction is north.

<u>scalar</u>: a quantity that has a value/quantity but does not have a direction. (A scalar is what you think of as a "regular" number, including its unit.)

<u>magnitude</u>: the part of a vector that is not the direction (*i.e.,* the value including its units). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if $\vec{\boldsymbol{F}}$ is 25 N to the east, then $\left\|\vec{\boldsymbol{F}}\right\| = 25\,\text{N}$. However, to make typesetting easier, it is common to use regular absolute value bars instead, *e.g.,* $\left|\vec{F}\right| = 25\,\text{N}$.

<u>resultant</u>: a vector that is the result of a mathematical operation (such as the addition of two vectors).

Variables that represent vectors are traditionally typeset in ***bold Italics***. Vector variables may also optionally have an arrow above the letter:

$$\boldsymbol{J}, \vec{\boldsymbol{F}}, \boldsymbol{v}$$

Variables that represent scalars are traditionally typeset in *plain Italics:*

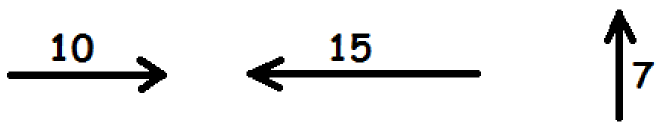
$$V, t, \lambda$$

Variable that represent only the magnitude of a vector (*e.g.,* in equations where the direction is not relevant) are typeset as if they were scalars:

For example, suppose $\vec{\boldsymbol{F}}$ is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.)

If we needed a variable to represent only the magnitude of 25 N, we would use the variable *F*.

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:



magnitude 10	magnitude 15	magnitude 7
direction: "to the right", 0°	direction: "to the left", +180°	direction: "up", +90°

The negative of a vector is a vector with the same magnitude in the opposite direction:

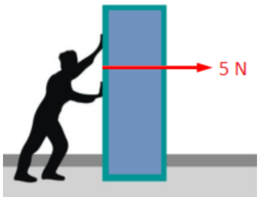
Note, however, that we use positive and negative numbers to represent the direction of a vector, but a negative value for a vector does not mean the same thing as a negative number in mathematics. In math, $-10 < 0 < +10$, because positive and negative numbers represent locations on a continuous number line.

However, a *velocity* of $-10\,\frac{\text{m}}{\text{s}}$ means "$10\,\frac{\text{m}}{\text{s}}$ in the negative direction". This means that $-10\,\frac{\text{m}}{\text{s}} > +5\,\frac{\text{m}}{\text{s}}$, because the first object is moving faster than the second ($10\,\frac{\text{m}}{\text{s}}$ *vs.* $5\,\frac{\text{m}}{\text{s}}$), even though the objects are moving in opposite directions.

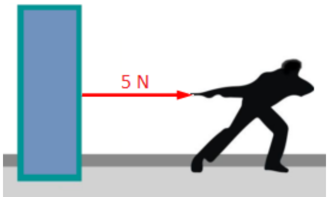
Translating Vectors

Vectors have a magnitude and direction but not a location. This means we can translate a vector (in the geometry sense, which means to move it without changing its size or orientation), and it's still the same vector quantity.

For example, consider a person pushing against a box with a force of 5 N to the right. We will define the positive direction to be to the right, which means we can call the force +5 N:

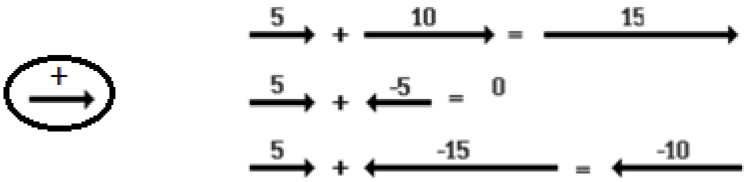


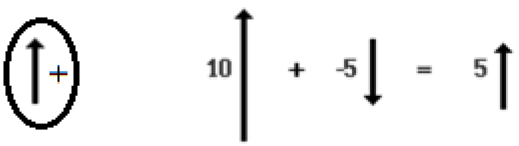
If the force is moved to the other side of the box, it's still 5 N to the right (+5 N), which means it's still the same vector:



Adding Vectors in One Dimension

If you are combining vectors in one dimension (*e.g,* horizontal), adding vectors is just adding positive and/or negative numbers:

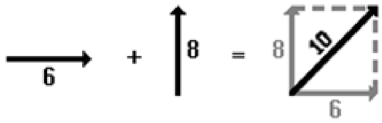




Adding vectors in Two Dimensions

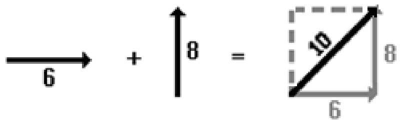
If the vectors are not in the same direction, we translate (slide) them until they meet, either tail-to-tail or tip-to-tail, and complete the parallelogram.

If the vectors are at right angles to one another, the parallelogram is a rectangle and we can use the Pythagorean theorem to find the magnitude of the resultant:



$(6^2 + 8^2 = 10^2)$

tail-to-tail



$(6^2 + 8^2 = 10^2)$

tip-to-tail

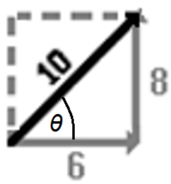
Note that the sum of these two vectors has a magnitude (length) of 10, not 14; "Adding" vectors means combining them using geometry.

The vector sum comes out the same whether you combine the vectors tail-to-tail or tip-to-tail. The decision of how to represent the vectors depends on the situation that you are modeling with them:

- Two forces pulling on the same object (think of two ropes connected to the same point) is best represented by drawing the vectors tail-to-tail.
- The displacement* of a walking path that starts in one direction and then turns is best represented by drawing the vectors tip-to-tail.

honors & AP®

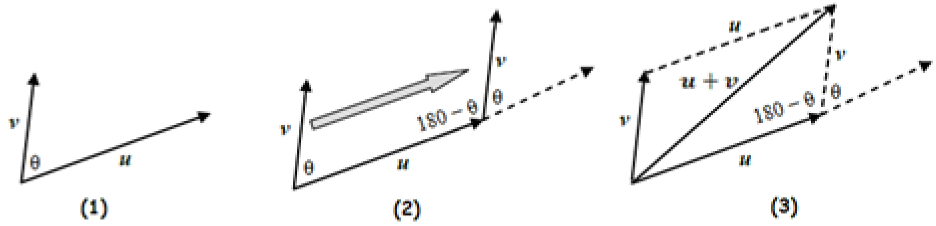
When working with vectors in multiple dimensions, we can use positive and negative numbers if the vectors are along the *x*-axis or the *y*-axis, but we need to use an angle to specify any other direction. To calculate the direction of the resultant of the vector operation above, we need to use trigonometry:



$$\tan\theta = \frac{8}{6} \quad \rightarrow \quad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.1°$$

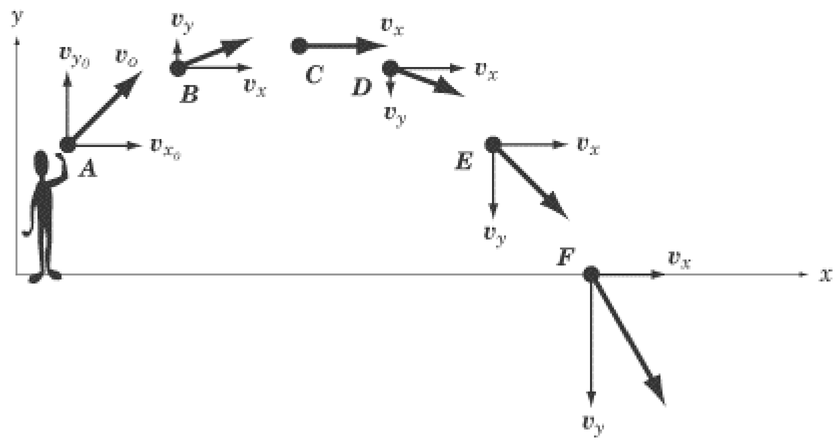
* Displacement is a vector quantity that represents the straight-line distance from one point to another. Displacement is covered in the next unit, Kinematics in One Dimension.

We would use exactly the same process to add vectors that are not perpendicular:



The trigonometry needed for these calculations requires the laws of sines and cosines. The calculations are not difficult, but this use of trigonometry is beyond the scope of this course.

One type of physics problem that commonly uses vectors is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, $\vec{\boldsymbol{v}}$, at angle θ, we can split the vector into a horizontal component, which we call $\vec{\boldsymbol{v}}_x$ and a vertical component, which we call $\vec{\boldsymbol{v}}_y$.



Notice that, in the case of projectile motion (such as throwing a ball), $\vec{\boldsymbol{v}}_x$ remains constant, but $\vec{\boldsymbol{v}}_y$ changes (because of the effects of gravity).

Because perpendicular vectors do not affect each other, we can apply equations to the two directions separately.

For example, in projectile motion (which you will learn about in detail in the **Error! Reference source not found.** topic starting **Error! Bookmark not defined.**), we usually use the equation $\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$, applying it separately in the *x*- and *y*-directions. This gives us two equations.

In the horizontal (*x*)-direction:

$$\vec{d}_x = \vec{v}_{o,x} t + \tfrac{1}{2}\cancelto{0}{\vec{a}_x} t^2$$
$$\vec{d}_x = \vec{v}_x t$$

In the vertical (*y*)-direction:

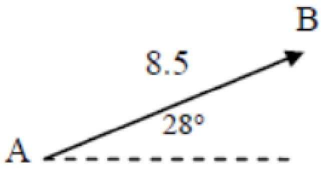
$$\vec{d}_y = \vec{v}_{o,y} t + \tfrac{1}{2}\vec{a}_y t^2$$
$$\vec{d}_y = \vec{v}_{o,y} t + \tfrac{1}{2}\vec{g} t^2$$

Note that each of the vector quantities ($\vec{d}, \vec{v}_o$ and $\vec{a}$) has independent *x*- and *y*-components. For example, $\vec{v}_{o,x}$ (the component of the initial velocity in the *x*-direction) is independent of $\vec{v}_{o,y}$ (the component of the initial velocity in the *x*-direction). This means *we treat them as completely separate variables*, and we can solve for one without affecting the other.

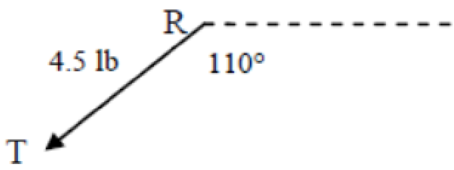
Homework Problems

Label the magnitude and direction (relative to horizontal) of each of the following:

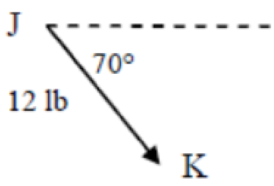
1. **(M)**



2. **(M)**

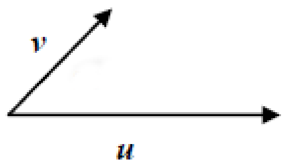


3. **(S)**

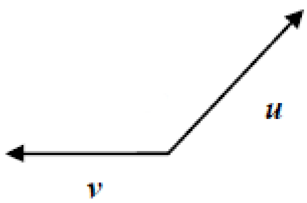


Sketch the resultant of each of the following.

4. **(M)**



5. **(M)**

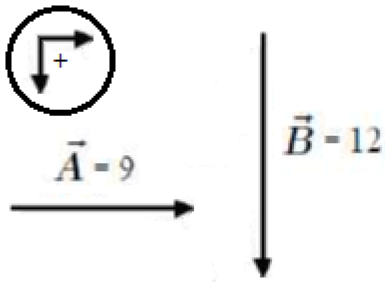


6. **(S)**

Consider the following vectors $\vec{\boldsymbol{A}} \& \vec{\boldsymbol{B}}$.

Vector $\vec{\boldsymbol{A}}$ has a magnitude of 9 and its direction is the positive horizontal direction (to the right).

Vector $\vec{\boldsymbol{B}}$ has a magnitude of 12 and its direction is the positive vertical direction (down).



7. **(M)** Sketch the resultant of $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$, and determine its magnitude and direction*.

8. **(S)** Sketch the resultant of $\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}$ (which is the same as $\vec{\boldsymbol{A}} + (-\vec{\boldsymbol{B}})$, and determine its magnitude and direction*.

* Finding the direction requires trigonometry. If your teacher skipped the right-angle trigonometry section, you only need to find the magnitude.

Vectors *vs.* Scalars in Physics

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 1.1.A.1, 1.1.A.3

Mastery Objective(s): (Students will be able to…)

- Identify vector *vs.* scalar quantities in physics.

Success Criteria:

- Quantity is correctly identified as a vector or a scalar.

Language Objectives:

- Explain why some quantities have a direction and others do not.

Tier 2 Vocabulary: magnitude, direction

Notes:

In physics, most numbers represent quantities that can be measured or calculated from measurements. Most of the time, there is no concept of a "deficit" of a measured quantity. For example, quantities like mass, energy, and power can only be nonnegative, because in classical mechanics there is no such thing as "anti-mass," "anti-energy," or "anti-power."

However, vector quantities have a direction as well as a magnitude, and direction can be positive or negative.

A rule of thumb that works *most* of the time in a high school physics class is:

Scalar quantities. These are usually positive, with a few notable exceptions (*e.g.,* work and electric charge).

Vector quantities. Vectors have a direction associated with them. For one-dimensional vectors, the direction is conveyed by defining a direction to be "positive". Vectors in the positive direction are expressed as positive numbers, and vectors in the opposite (negative) direction are expressed as negative numbers.

In some cases, you will need to split a vector into two component vectors, one vector in the x-direction, and a separate vector in the y-direction, in order to solve a problem. In these cases, you will need to choose which direction is positive and which direction is negative for *both* the x- and y-axes. Once you have done this, every vector quantity must be assigned a positive or negative value, according to the directions you have chosen.

Differences. The difference or change in a variable is indicated by the Greek letter Δ in front of the variable. Any difference can be positive or negative. However, note that a difference can either be a vector, indicating a change relative to the positive direction (*e.g.,* $\boldsymbol{\Delta x}$, which indicates a change in position), or scalar, indicating an increase or decrease (*e.g.,* ΔV, which indicates a change in volume).

Example:

Suppose you have a problem that involves throwing a ball straight upwards with a velocity of $15\,\frac{\text{m}}{\text{s}}$. Gravity is slowing the ball down with a downward acceleration of $10\,\frac{\text{m}}{\text{s}^2}$. You want to know how far the ball has traveled in 0.5 s.

Displacement, velocity, and acceleration are all vectors. The motion is happening in the y-direction, so we need to choose whether "up" or "down" is the positive direction. Suppose we choose "up" to be the positive direction. This means:

- When the ball is first thrown, it is moving upwards. This means its velocity is in the *positive* direction, so we would represent the initial velocity as $\vec{\mathbf{v}}_o = +15\,\frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the *negative* direction. We would therefore represent the acceleration as $\vec{\boldsymbol{a}} = -10\,\frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so its value is +0.5 s.

If we had to substitute the numbers into the formula:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \tfrac{1}{2}\vec{\boldsymbol{a}}t^2$$

we would do so as follows:

$$\vec{\boldsymbol{d}} = (+15)(0.5) + (\tfrac{1}{2})(-10)(0.5)^2$$

and we would find out that $\vec{\boldsymbol{d}} = +6.25\,\text{m}$.

The answer is *positive*. Earlier, we defined positive as "up", so the answer tells us that the displacement is upwards from the starting point.

What if, instead, we had chosen "down" to be the positive direction?

- When the ball is first thrown, it is moving upwards. This means its velocity is now in the *negative* direction, so we would represent the initial velocity as $\vec{v}_o = -15\,\frac{m}{s}$.
- Gravity is accelerating the ball downwards, which is the *positive* direction. We would therefore represent the acceleration as $\vec{a} = +10\,\frac{m}{s^2}$.
- Time is a scalar quantity, so its value is +0.5 s.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \tfrac{1}{2}\vec{a}t^2$$

we would do so as follows:

$$\vec{d} = (-15)(0.5) + (\tfrac{1}{2})(10)(0.5)^2$$

and we would find out that $\vec{d} = -6.25\,\text{m}$.

The answer is *negative*. However, remember that we defined "down" to be positive, which means "up" is the negative direction. This means the displacement is *upwards* from the starting point, as before.

In any problem you solve, the choice of which direction is positive *vs.* negative is arbitrary. The only requirement is that *every vector quantity in the problem* needs to be consistent with your choice.

Vector Multiplication

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): SP5

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): 1.1.B.1

Mastery Objective(s): (Students will be able to…)

- Correctly use and interpret the symbols "•" and "×" when multiplying vectors.
- Finding the dot product & cross product of two vectors.

Success Criteria:

- Magnitudes and directions are correct.

Language Objectives:

- Explain how to interpret the symbols "•" and "×" when multiplying vectors.

Tier 2 Vocabulary: magnitude, direction, dot, cross

Notes:

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = C$$

cross product: multiplication of two vectors that results in a new vector.

$$\vec{\boldsymbol{I}} \times \vec{\boldsymbol{J}} = \vec{\boldsymbol{K}}$$

tensor product: multiplication of two vectors that results in a tensor. $\vec{\boldsymbol{A}} \otimes \vec{\boldsymbol{B}}$ is a matrix of vectors that results from multiplying the respective components of each of the two vectors. It describes the effect of each component of the vector on each component of every other vector in the array. Tensors are beyond the scope of a high school physics course.

Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

If the two vectors have opposite directions, the equation needs a negative sign. For example, the force applied by a spring equals the spring constant (a scalar quantity) times the displacement:

$$\vec{\boldsymbol{F}}_s = -k\vec{\boldsymbol{x}}$$

The negative sign in the equation signifies that the force applied by the spring is in the opposite direction from the displacement.

The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the "dot product". Dot product multiplication of vectors is represented with a dot:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}}$$ *

The dot product of $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is:

$$\vec{\boldsymbol{A}} \bullet \vec{\boldsymbol{B}} = AB\cos\theta$$

where *A* is the magnitude of $\vec{\boldsymbol{A}}$, *B* is the magnitude of $\vec{\boldsymbol{B}}$, and θ is the angle between the two vectors $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$.

For example, in physics, *work* (a scalar quantity) is the dot product of the vectors *force* and *displacement* (distance):

$$W = \vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}} = Fd\cos\theta$$

* pronounced "A dot B"

The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{A}\times\vec{B}$$ *

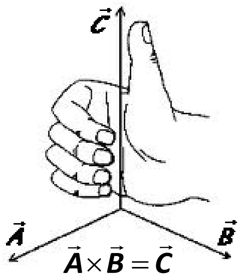
The magnitude of the cross product of vectors $\vec{A}$ and $\vec{B}$ that have an angle of θ between them is given by the formula:

$$\vec{A}\times\vec{B}=AB\sin\theta$$

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the "right hand rule":

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector.

Note that this means that the resultant vectors for $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ point in *opposite* directions, *i.e.,* the cross product of two vectors is *not* commutative!



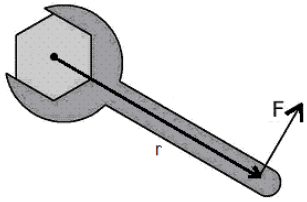
$$\vec{A}\times\vec{B}=\vec{C}$$
$$\vec{B}\times\vec{A}=-\vec{C}$$

On a two-dimensional piece of paper, a vector coming toward you (out of the page) is denoted by a set of ⊙ ⊙ ⊙ ⊙ ⊙ symbols, and a vector going away from you (into the page) is denoted by a set of ⊗ ⊗ ⊗ ⊗ ⊗ symbols.

Think of these symbols as representing an arrow inside a tube or pipe. The dot represents the tip of the arrow coming toward you, and the "X" represents the fletches (feathers) on the tail of the arrow going away from you.)

* pronounced "A cross B"

In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force $\vec{F}$ acting at a radius $\vec{r}$ is given by the equation:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$$

Because the direction of the force is usually perpendicular to the displacement, it is usually true that $\sin\theta = \sin 90° = 1$. This means the magnitude $rF\ \sin\theta = rF(1) = rF$. Using the right-hand rule, we determine that the *direction* of the resultant torque vector is coming out of the page.

(The force generated by the interaction between charges and magnetic fields, a topic covered in AP® Physics 2, is also a cross product.)

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

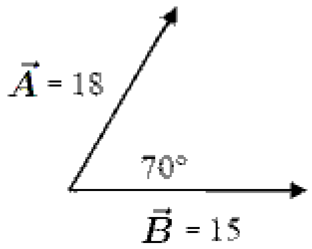
A:   $\sin\theta$

Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

Homework Problems

For the following vectors $\vec{A}$ & $\vec{B}$:



1. **(M)** Determine $\vec{A} \bullet \vec{B}$

2. **(M)** Determine $\vec{A} \times \vec{B}$ (both magnitude and direction)

Logarithms

Unit: Mathematics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Knowledge/Understanding:

- What logarithms represent and an intuitive understanding of logarithmic quantities.

Skills:

- Use logarithms to solve for a variable in an exponent.

Language Objectives:

- Understand the use of the terms "exponential" and "logarithm" and understand the vernacular use of "log" (otherwise a Tier 1 word) as an abbreviation for "logarithm".

Tier 2 Vocabulary: function

Notes:

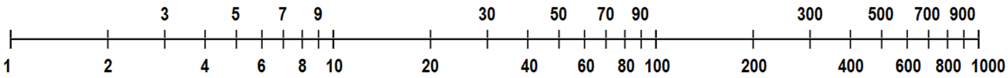
The logarithm may well be the least well-understood function encountered in high school mathematics.

The simplest logarithm to understand is the base-ten logarithm. You can think of the (base-ten) logarithm of a number as the number of zeroes after the number.

x		$\log_{10}(x)$
100 000	10^5	5
10 000	10^4	4
1 000	10^3	3
100	10^2	2
10	10^1	1
1	10^0	0
0.1	10^{-1}	−1
0.01	10^{-2}	−2
0.001	10^{-3}	−3
0.000 1	10^{-4}	−4
0.000 01	10^{-5}	−5

As you can see from the above table, the logarithm of a number turns a set of numbers that vary exponentially (powers of ten) into a set that vary linearly.

You can get a visual sense of the logarithm function from the logarithmic number line below:



Notice that the *distance* from 1 to 10 is the same as the *distance* from 10 to 100 and from 100 to 1000. In fact, the relative distance to every number on this number line is the logarithm of the number.

x	$\log_{10}(x)$	distance from beginning of number line
10^0	0	0
$10^{0.5} \approx 3.16$	0.5	½ cycle
$10^1 = 10$	1	1 cycle
$10^2 = 100$	2	2 cycles
$10^3 = 1000$	3	3 cycles

By inspection, you can see that the same is true for numbers that are not exact powers of ten. The logarithm function compresses correspondingly more as the numbers get larger.

The most useful mathematical property of logarithms is that they move an exponent into the linear part of the equation:

$$\log_{10}(10^3) = 3\log_{10}(10) = (3)(1) = 3$$

In fact, the logarithm function works the same way for any base, not just 10:

$$\log_2(2^7) = 7\log_2(2) = (7)(1) = 7$$

(In this case, the word "base" means the base of the exponent.) The general equation is:

$$\log_x(a^b) = b\log_x(a)$$

This is a powerful tool in solving for the exponent in an equation. This is, in fact, precisely the purpose of using logarithms in most mathematical equations.

Sample problem:

Q: Solve $3^x = 15$ for *x*.

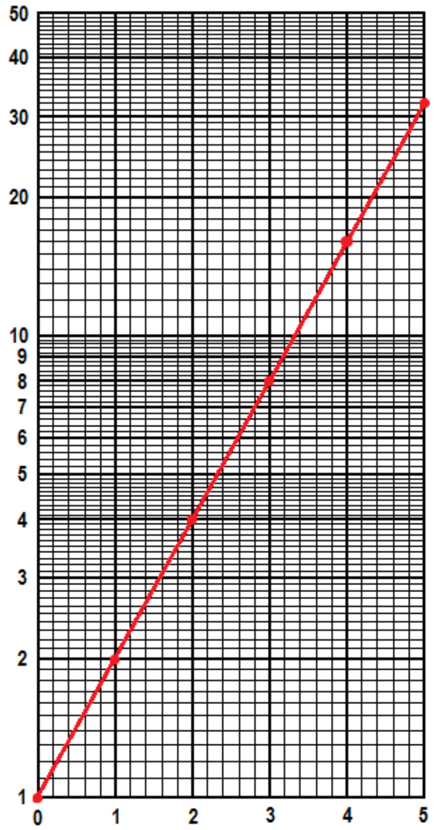
A: Take the logarithm (any base) of both sides. (Note that writing "log" without supplying a base implies that the base is 10.)

$$\log(3^x) = \log(15)$$
$$x\log(3) = \log(15)$$
$$(x)(0.477) = 1.176$$
$$x = \frac{1.176}{0.477} = 2.465$$

This is the correct answer, because $3^{2.465} = 15$

Logarithmic Graphs

A powerful tool that follows from this is using logarithmic graph paper to solve equations. If you plot an exponential function on semilogarithmic ("semi-log") graph paper (meaning graph paper that has a logarithmic scale on one axis but not the other), you get a straight line.



The graph at the right is the function $y = 2^x$. Notice where the following points appear on the graph:

Domain	Range
0	1
1	2
2	4
3	8
4	16
5	32

Notice also that you can use the graph to find intermediate values. For example, at x = 2.6, the graph shows that y = 6.06.

Natural Logarithms

The natural logarithm comes from calculus—it is the solution to the problem:

$$\int \frac{1}{x}\,dx = \log_e(x)$$

where the base of this logarithm, "*e*," is a constant (sometimes called "Euler's number") that is an irrational number equal to approximately 2.71828 18284 59045…

The natural logarithm is denoted "ln", so we would actually write:

$$\int \frac{1}{x}\,dx = \ln(x)$$

The number "*e*" is often called the exponential function. In an algebra-based physics class, the exponential function appears in some equations whose derivations come from calculus, notably some of the equations relating to resistor-capacitor (RC) circuits.

Finally, just as:

$$\log(10^x) = x \text{ and } 10^{\log(x)} = x$$

it is similarly true that:

$$\ln(e^x) = x \text{ and } e^{\ln(x)} = x$$

Introduction: Thermal Physics (Heat)

Unit: Thermal Physics (Heat)

Topics covered in this chapter:

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Heat & Temperature* describes the concept of heat as a form of energy and how heat energy is different from temperature.
- *Heat Transfer* describes how to calculate the rate of the transfer of heat energy from one object to another.
- *Specific Heat Capacity & Calorimetry* describes different substances' and objects' abilities to store heat energy.
- *Phase Diagrams* describes how to use a phase diagram to determine the state of matter of a substance at a given temperature and pressure.
- *Phases & Phase Changes* and *Heating Curves* addresses the additional calculations that apply when a substance goes through a phase change (such as melting or boiling).
- *Thermal Expansion* describes the calculation of the change in size of an object caused by heating or cooling.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Standards addressed in this chapter:

NGSS Standards/MA Curriculum Frameworks (2016):

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

9.1.B: Describe the temperature of a system in terms of the atomic motion within that system.

9.1.B.1: The temperature of a system is characterized by the average kinetic energy of the atoms within that system.

9.1.B.1.i: The Maxwell–Boltzmann distribution provides a graphical representation of the energies and speeds of atoms at a given temperature.

9.1.B.1.ii: The root-mean-square speed corresponding to the average kinetic energy for a particle of an ideal gas is related to the temperature of the gas by $K_{avg} = \frac{3}{2}k_B T = \frac{1}{2}mv_{rms}^2$

9.3.A: Describe the transfer of energy between two systems in thermal contact due totemperature differences of those two systems.

9.3.A.1: Two systems are in thermal contact if the systems may transfer energy by thermal processes.

9.3.A.1.i: Heating is the transfer of energy into a system by thermal processes.

9.3.A.1.ii: Cooling is the transfer of energy out of a system by thermal processes.

AP® only

9.3.A.2: The thermal processes by which energy may be transferred between systems at different temperatures are conduction, convection, and radiation.

9.3.A.3: Energy is transferred through thermal processes spontaneously from a higher-temperature system to a lower-temperature system.

9.3.A.3.i: In collisions between atoms from different systems, energy is most likely to be transferred from higher-energy atoms to lower-energy atoms.

9.3.A.3.ii: After many collisions of atoms from different systems, the most probable state is one in which both systems have the same temperature.

9.3.A.4: Thermal equilibrium results when no net energy is transferred by thermal processes between two systems in thermal contact with each other.

9.5.A: Describe the energy required to change the temperature of an object by a certain amount.

9.5.A.1: The amount of energy required to change the temperature of a material is related to the material's specific heat capacity.

9.5.A.2: The specific heat capacity of a material is an intrinsic property of that material that depends on the arrangement and interactions of the atoms that make up the material.

9.5.B: Describe the rate at which energy is transferred by conduction through a given material.

9.5.B.1: The rate at which energy is transferred by conduction through a given material is related to the thermal conductivity, the physical dimensions of the material, and the temperature difference across the material.

9.5.B.2: The thermal conductivity of a material is an intrinsic property of that material that depends on the arrangement and interactions of the atoms that make up the material.

15.4.A: Describe the electromagnetic radiation emitted by an object due to its temperature.

15.4.A.1: Matter will spontaneously convert some of its internal thermal energy into electromagnetic energy.

15.4.A.2: A blackbody is an idealized model of matter that absorbs all radiation that falls on the body. If the body is in equilibrium at a constant temperature, then it must in turn emit energy.

AP® only

15.4.A.3: A blackbody will emit a continuous spectrum that only depends on the body's temperature. The radiation emitted by a blackbody is often modeled by plotting intensity per unit wavelength as a function of wavelength.

15.4.A.3.i: The distribution of the intensity of a blackbody's spectrum as a function of temperature cannot be modeled using only classical physics concepts. A blackbody's spectrum is described by Planck's law, which assumes that the energy of light is quantized.

15.4.A.3.ii: The peak wavelength emitted by a blackbody (the wavelength at which the blackbody emits the greatest amount of radiation per unit wavelength) decreases with increasing temperature, as described by Wien's law.

15.4.A.3.iii: The rate at which energy is emitted (power) by a blackbody is proportional to the surface area of the body and to the temperature of the body raised to the fourth power, as described by the Stefan-Boltzmann law.

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Heat & Temperature

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS4-3a

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.1.B, 9.1.B.1, , 9.1.B.1.i, 9.1.B.1.ii

Mastery Objective(s): (Students will be able to…)

- Explain heat energy in macroscopic and microscopic terms.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain the difference between heat and temperature.

Tier 2 Vocabulary: heat, temperature

Labs, Activities & Demonstrations:

- Heat a small weight and large weight to slightly different temperatures.
- Fire syringe.
- Steam engine.
- Incandescent light bulb in water.
- Mixing (via molecular motion/convection) of hot *vs.* cold water (with food coloring).

Vocabulary:

heat: energy that can be transferred by moving atoms or molecules via transfer of momentum.

temperature: a measure of the average kinetic energy of the particles (atoms or molecules) of a system.

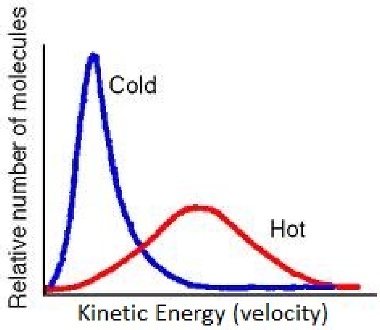
thermometer: a device that measures temperature, most often via thermal expansion and contraction of a liquid or solid.

Notes:

Heat is energy that is stored as the translational kinetic energy of the particles that make up an object or substance.

As you (should) recall from chemistry, particles (atoms or molecules) are always moving (even at absolute zero), and that energy can transfer via elastic collisions between the particles of one object or substance and the particles of another. (We will explore these concepts in more detail in the topic *Kinetic-Molecular Theory*, starting on page 92.)

Note that heat is the energy itself, whereas temperature is a measure of the quality of the heat—the average of the kinetic energies of the individual molecules:



Note that the particles of a substance have a range of kinetic energies, and the temperature is the average. Notice that when a substance is heated, the particles acquire a wider range of kinetic energies, with a higher average.

When objects are placed in contact, heat is transferred from each object to the other via the transfer of momentum that occurs when the individual molecules collide. Molecules that have more energy transfer more energy than they receive. Molecules that have less energy receive more energy than they transfer. This means three things:

1. Individual collisions transfer energy in both directions. The particles of a hot substance transfer energy to the cold substance, but the particles of the cold substance also transfer energy to the hot substance.

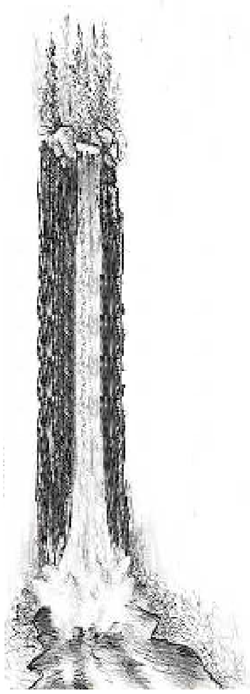
2. The net (overall) flow of energy is from objects with a higher temperature (more kinetic energy) to objects with a lower temperature (less kinetic energy). *I.e.,* more energy is transferred from the hot substance to the cold substance than *vice versa*.

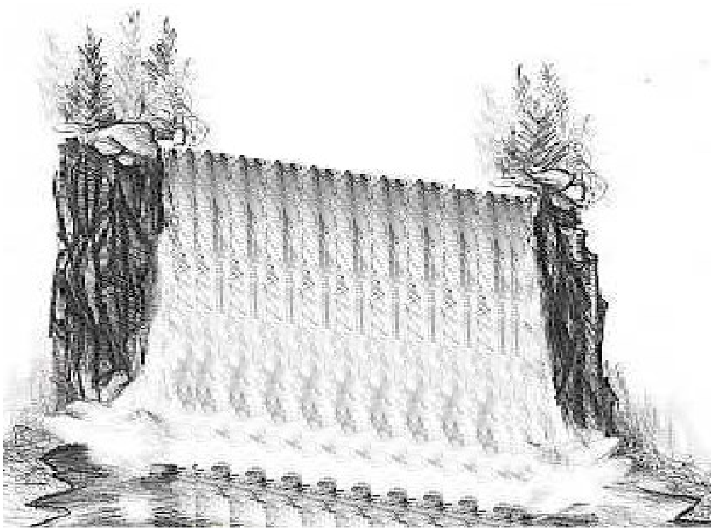
3. If you wait long enough, all of the molecules will have the same temperature (*i.e.,* the same average kinetic energy).

This means that the ***temperature*** of one object relative to another determines which direction the heat will flow, much like the way the elevation (vertical position) of one location relative to another determines which direction water will flow.

However, the total heat (energy) contained in an object depends on the mass as well as the temperature, in the same way that the total change in energy of the water going over a waterfall depends on the mass of the water as well as the height.

Consider two waterfalls, one of which is twice the height of the second, but the second of which has ten times as much water going over it as the first:





$\Delta U = mg(2h)$ $\Delta U = (10m)gh$

In the above pictures, each drop of water falling from the waterfall on the left has more gravitational potential energy, but more total energy goes over the waterfall on the right.

Similarly, each particle in an object at a higher temperature has more thermal energy than each particle in another object at a lower temperature.

If we built a waterway between the two falls, water could flow from the top of the first waterfall to the top of the second, but not *vice versa*.

Similarly, the net flow of heat is from a smaller object with higher temperature to a larger object with a lower temperature, but not *vice versa*.

Heat Flow

system: the region or collection of objects under being considered in a problem.

surroundings: everything that is outside of the system.

E.g., if a metal block is heated, we would most likely define the system to be the block, and the surroundings to be everything else.

We generally use the variable Q to represent heat in physics equations.

Heat flow is always represented in relation to the system.

Heat Flow	**Sign of *Q***	**System**	**Surroundings**
from the surroundings into the system	+ (positive)	gains heat (gets warmer)	lose heat (get colder)
from the system out to the surroundings	− (negative)	loses heat (gets colder)	gain heat (get hotter)

A positive value of Q means heat is flowing *into* the system. Because the heat is transferred from the molecules outside the system to the molecules in the system, the energy of the system increases, and the energy of the surroundings decreases.

A negative value of Q means heat is flowing *out of* the system. Because the heat is transferred from the molecules in the system to the molecules outside the system, the energy of the system decreases, and the energy of the surroundings increases.

This can be confusing. Suppose you set a glass of ice water on a table. When you pick up the glass, your hand gets colder because heat is flowing from your hand (which is part of the surroundings) into the system (the glass of ice water). This means the system (the glass of ice water) is gaining heat, and the surroundings (your hand, the table, *etc.*) are losing heat. The value of Q would be positive in this example.

In simple terms, you need to remember that your hand is part of the *surroundings*, not part of the system.

thermal equilibrium: when all of the particles in a system have the same average kinetic energy (temperature). When a system is at thermal equilibrium, no net heat is transferred. (*I.e.,* collisions between particles may still transfer energy, but the average temperature of the particles in the system—what we measure with a thermometer—is not changing.)

Heat Transfer

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-4a

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.3.A, 9.3.A.1, 9.3.A.1.i, 9.3.A.1.ii, 9.3.A.2, 9.3.A.3, 15.4.A, 15.4.A.1, 15.4.A.2, 15.4.A.3, 15.4.A.3.i, 15.4.A.3.ii, 15.4.A.3.iii

Mastery Objective(s): (Students will be able to…)

- Explain heat transfer by conduction, convection and radiation.
- Calculate heat transfer using Fourier's Law of Heat Conduction.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the mechanisms by which heat is transferred.

Tier 2 Vocabulary: conduction, radiation

Labs, Activities & Demonstrations:

- Radiometer & heat lamp.
- Almond & cheese stick.
- Flammable soap bubbles.
- Drop of food coloring in water *vs.* ice water

Notes:

Heat transfer is the flow of heat energy from one object to another. Heat transfer usually occurs through three distinct mechanisms: conduction, radiation, and convection.

conduction: transfer of heat through collisions of particles by objects that are ***in direct contact*** with each other. Conduction occurs when there is a net transfer of momentum from the molecules of an object with a higher temperature transfer to the molecules of an object with a lower temperature.

thermal conductivity (k): a measure of the amount of heat that a given length of a substance can conduct in a specific amount of time. Thermal conductivity is measured in units of $\frac{\text{J}}{\text{m·s·°C}}$ or $\frac{\text{W}}{\text{m·°C}}$.

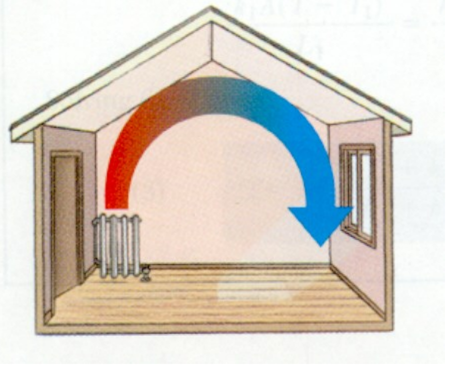
conductor: an object that allows heat to pass through itself easily; an object with high thermal conductivity.

insulator: an object that does not allow heat to pass through itself easily; a poor conductor of heat; an object with low thermal conductivity.

radiation: transfer of heat *through space* via electromagnetic waves (light, microwaves, *etc.*)

convection: transfer of heat *by motion of particles* that have a higher temperature exchanging places with particles that have a lower temperature. Convection usually occurs when air moves around a room.

Natural convection occurs when particles move because of differences in density. In a heated room, because cool air is more dense than warm air, the force of gravity is stronger on the cool air, and it is pulled harder toward the ground than the warm air. The cool air displaces the warm air, pushing it upwards out of the way.



In a room with a radiator, the radiator heats the air, which causes it to expand and be displaced upward by the cool air nearby. When the (less dense) warm air reaches the ceiling, it spreads out, and it continues to cool as it spreads. When the air reaches the opposite wall, it is forced downward toward the floor, across the floor, and back to the radiator.

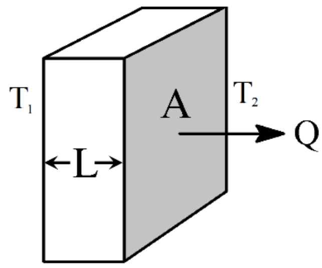
Forced convection can be achieved by moving heated or cooled air using a fan. Examples of this include ceiling fans and convection ovens. If your radiator does not warm your room enough in winter, you can use a fan to speed up the process of convection. (Make sure the fan is moving the air in the same direction that would happen from natural convection. Otherwise, the fan will be fighting against physics!)

Calculating Heat Transfer by Conduction

Heat transfer by conduction can be calculated using Fourier's Law of Heat Conduction:

$$P = \frac{Q}{t} = \pm kA\frac{\Delta T}{L}$$

where:



P = power (W)

Q = heat transferred (J)

t = time (s)

k = coëfficient of thermal conductivity $\left(\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)$

A = cross-sectional area (m^2)

ΔT = temperature difference (K or °C)

L = length (m)

The ± sign means that the value can be positive or negative, because the sign for Q is chosen based on whether the heat transfers into (+) or out of (−) the system.

Note that for insulation (the kind you have in the walls and attic of your home), you want the lowest possible thermal conductivity—you don't want the insulation to conduct the heat from the inside of your house to the outside! Because most people think that bigger numbers are better, the industry has created a measure of the effectiveness of insulation called the "R value". It is essentially the reciprocal of $\frac{k}{L}$, which means lower conductivity and more thickness gives better insulation.

Sample Problem:

Q: A piece of brass is 5.0 mm (0.0050 m) thick and has a cross-sectional area of 0.010 m^2. If the temperature on one side of the metal is 65°C and the temperature on the other side is 25°C, how much heat will be conducted through the metal in 30. s? The coefficient of thermal conductivity for brass is $120\,\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$.

A: $\frac{Q}{t} = kA\frac{\Delta T}{L}$

$$\frac{Q}{30} = (120)(0.010)\left(\frac{65-25}{0.0050}\right) = 9600$$

$$Q = 288\,000\text{ J} = 288\text{ kJ}$$

(Note that because the quantities of heat that we usually measure are large, values are often given in kilojoules or megajoules instead of joules.)

Calculating Heat Transfer by Radiation

Heat transfer by radiation is based on the temperature of a substance and its ability emit heat (emissivity). The equation is:

$$P = \frac{Q}{t} = \varepsilon \sigma A T^4$$

where:

P = power (W)

Q = heat (J)

t = time (s)

ε = emissivity (dimensionless; "blackbody" ≡ 1)

σ = Stefan-Boltzmann constant $\left(\sigma = 5.67\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)$

A = area (m^2)

T = temperature (K)

Note that because the equation contains T (rather than ΔT), the temperature needs to be in Kelvin.

emissivity (ε): a ratio of the amount of heat radiated by a substance to the amount of heat that would be radiated by a perfect "blackbody" of the same dimensions.

Emissivity is a dimensionless number (meaning that it has no units, because the units cancel), and is specific to the substance.

blackbody: an object that absorbs all of the heat energy that comes in contact with it (and reflects none of it).

Stefan-Boltzmann constant (σ): the constant that makes the above equation come out in watts. Note that the Stefan-Boltzmann constant is defined from other constants:

$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2}$, where k_B is the Boltzmann constant, h is Planck's constant, and c is the speed of light in a vacuum.

Wien's Displacement Law

In 1893, German physicist Wilhelm Wien discovered that radiation from a blackbody occurs in the form of electromagnetic radiation, over a range of wavelengths. The wavelength at which the maximum energy is radiated decreases as the temperature increases:

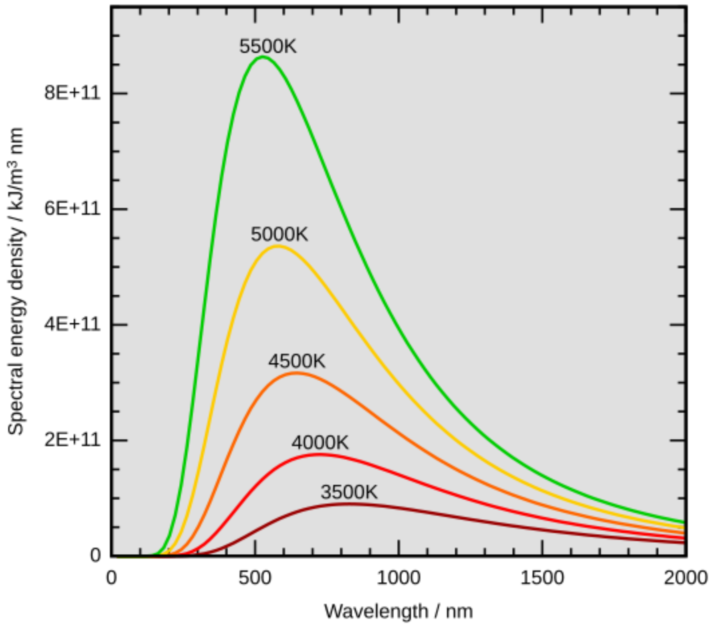


Image by 4C. Used with permission.

The wavelength of maximum radiation is described by the equation:

$$\lambda_{max} = \frac{b}{T}$$

where:

λ_{max} = wavelength of maximum radiated energy

b = Wien's displacement constant $= 2.897\,771\,955 \times 10^{-3}\,\text{m}\cdot\text{K}$

T = temperature (K)

honors (not AP®)

Wien's displacement law was superseded in 1900, when German physicist Max Planck derived a more general equation, now called Planck's radiation law. This law gives an equation for the spectral energy density (energy per unit volume per unit frequency):

$$u_{\nu}(\nu,T)=\frac{8\pi h\nu^3}{c^3}\cdot\frac{1}{e^{\left(h\nu/k_BT\right)}-1}$$

where:

$u_{\nu}(\nu,T)=$ spectral energy density (a function of wavelength & temperature)

$h=$ Planck's constant $=6.626\,070\,15\times10^{-34}\ \text{J}\cdot\text{s}$

$c=$ speed of light in a vacuum $=2.997\,924\,58\ \frac{\text{m}}{\text{s}}$

$k_B=$ Boltzmann constant $=1.380\,649\times10^{-23}\ \frac{\text{J}}{\text{K}}$

$T=$ temperature (K)

Homework Problems

You will need to look up coefficients of thermal conductivity in *Table K. Thermal Properties of Selected Materials* on page 473 of your reference tables.

1. **(S)** The surface of a hot plate is made of 12.0 mm (0.012 m) thick aluminum and has an area of 64 cm^2 (which equals 0.0064 m^2). If the heating coils maintain a temperature of 80.°C underneath the surface and the air temperature is 22°C, how much heat can be transferred through the plate in 60. s?

   Answer: 464 000 J* or 464 kJ

2. **(S)** A cast iron frying pan is 5.0 mm thick. If it contains boiling water (100°C), how much heat will be transferred into your hand if you place your hand against the bottom for two seconds?
   (Assume your hand has an area of 0.0040 m^2, and that body temperature is 37°C.)

   Answer: +8 064 J or +8.064 kJ
   (positive because the direction is stated as "*into* your hand")

3. **(M)** A plate of metal has thermal conductivity *k* and thickness *L*. One side has a temperature of T_h and the other side has a temperature of T_c, derive an expression for the cross-sectional area *A* that would be needed in order to transfer a certain amount of heat, *Q*, through the plate in time *t*.

   Answer: $A = \dfrac{QL}{kt\,(T_h - T_c)}$

* Note: Questions #1 and #3 do not specify the direction of heat transfer, so the answer could be either positive or negative.

4. **(M)** A glass window in a house has an area of 0.67 m^2 and a thickness of 2.4 mm $(2.4\times10^{-3}\text{ m})$. The temperature inside the house is 21 °C, and the outside temperature is 0 °C.

   a. **(M)** How much heat is lost through the window in 1 hour (3600 s) due to conduction?

      Use $1.0\,\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ for the thermal conductivity of the glass.

      Answer: −21 100 000 J = −21 100 kJ
      (negative because heat is *lost* through the window)

   b. **(M – honors; A – AP®)** How much heat is lost through the window in 1 hour (3600 s) due to radiation? (Assume the temperature of the entire glass is 21 °C for this problem.)

      Hint: Remember to convert the temperature to Kelvin.

      Answer: −940 000 J = −940 kJ
      (negative because heat is *lost* through the window)

   c. **(M – honors; A – AP®)** Which mode of heat transfer (conduction *vs.* radiation) accounts for the greater amount of heat loss?

Specific Heat Capacity* & Calorimetry†

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS4-3a

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.5.A, 9.5.A.1, 9.5.A.2

Mastery Objective(s): (Students will be able to…)

- Calculate the heat transferred when an object with a known specific heat capacity is heated.
- Perform calculations related to calorimetry.
- Describe what is happening at the molecular level when a system is in thermal equilibrium.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the specific heat capacity of a substance measures.
- Explain how heat is transferred between one substance and another.

Tier 2 Vocabulary: heat, specific heat capacity, “coffee cup” calorimeter

Labs, Activities & Demonstrations:

- Calorimetry lab.

Notes:

Different objects have different abilities to hold heat. For example, if you enjoy pizza, you may have noticed that the sauce holds much more heat (and burns your mouth much more readily) than the cheese or the crust.

The amount of heat that a given mass of a substance can hold is based on its specific heat capacity.

* Physicists call this quantity “specific heat”. Chemists call it “heat capacity”. The term “specific heat capacity” is used so that physicists and chemists can talk to one another and realize that they are discussing the same concept.

† Calorimetry is usually taught in chemistry. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

<u>specific heat capacity</u> (*C*): a measure of the amount of heat required per gram of a substance to produce a specific temperature change in the substance.

C_p: specific heat capacity, measured at constant pressure. For gases, this means the measurement was taken while allowing the gas to expand as it was heated.

C_v: specific heat capacity, measured at constant volume. For gases, this means the measurement was made in a sealed container, allowing the pressure to rise as the gas was heated.

For solids and liquids, $C_p \approx C_v$ because the pressure and volume change very little with heating. For gases, $C_p > C_v$ (always). For ideal gases, $C_p - C_v = R$, where R is a constant known as "the gas constant."

When there is a choice, C_p is more commonly used than C_v because it is easier to measure. When dealing with solids and liquids, most physicists just use C for specific heat capacity and don't worry about the distinction.

Calculating Heat from a Temperature Change

The amount of heat gained or lost when an object changes temperature is given by the equation:

$$Q = mC\Delta T = m\int_{T_1}^{T_2} C(T)\,dT$$

where:

Q = heat (J or kJ)
m = mass (g or kg)
C = specific heat capacity (usually $\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \equiv \frac{\text{J}}{\text{g}\cdot\text{K}}$)
$C(T)$ = specific heat capacity as a function of temperature
ΔT = temperature change (K or °C)*

Because problems involving heat often involve large amounts of energy, heat is often expressed in kilojoules (kJ) rather than joules.

Note that $1\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \equiv 1\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}} \equiv 1\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$ and $1\frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}} \equiv 1\frac{\text{kcal}}{\text{kg}\cdot{}^\circ\text{C}} = 4.18\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}} = 4.18\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

You need to be careful with the units. If the mass is given in kilograms (kg), your specific heat capacity will have units of $\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ and the heat energy will come out in kilojoules (kJ). If mass is given in grams, you will use units of $\frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$ and the heat energy will come out in joules (J).

* Because 1 K is the same size as 1 °C, the two units are equivalent for ΔT values. Note, however, that T in equations must be in kelvin, because a temperature of 0 in an equation must mean absolute zero.

Specific Heat Capacities of Some Substances

Substance	Specific Heat Capacity ($\frac{kJ}{kg \cdot K}$)	Substance	Specific Heat Capacity ($\frac{kJ}{kg \cdot K}$)
water at 20 °C	4.181	aluminum	0.897
ethylene glycol (anti-freeze)	2.460	glass	0.84
		iron	0.450
ice at −10 °C	2.080	copper	0.385
steam at 100 °C	2.11	brass	0.380
steam at 130 °C	1.99	silver	0.233
vegetable oil	2.00	lead	0.160
air	1.012	gold	0.129

Calorimetry

<u>calorimetry</u>: the measurement of heat flow

In a calorimetry experiment, heat flow is calculated by measuring the mass and temperature change of an object and applying the specific heat capacity equation.

<u>calorimeter</u>: an insulated container for performing calorimetry experiments.

<u>coffee cup calorimeter</u>: a calorimeter that is only an insulated container—it does not include a thermal mass (such as a mass of water). It is usually made of Styrofoam, and often resembles a coffee cup.

<u>bomb calorimeter</u>: a calorimeter for measuring the heat produced by a chemical reaction. A bomb calorimeter is a double-walled metal container with water between the layers of metal. The heat from the chemical reaction causes the temperature of the water to increase. Because the mass and specific heat capacity of the calorimeter (water plus metal) are known, the heat produced by the reaction can be calculated from the increase in temperature of the water.

It has a great name, but a bomb calorimeter doesn't involve actually blowing anything up. ☺

Solving Coffee Cup Calorimetry Problems

Most coffee cup calorimetry problems involve placing a hot object in contact with a colder one. Many of them involve placing a hot piece of metal into cold water.

To solve the problems, assume that both objects end up at the same temperature.

If we decide that heat gained (going into a substance) by each object that is getting hotter is positive, and heat lost (coming out of a substance) by every substance that is getting colder is negative, then the basic equation is:

$$\text{Heat Lost} + \text{Heat Gained} = \text{Change in Thermal Energy}$$
$$\sum Q_{lost} + \sum Q_{gained} = \Delta Q$$

If the calorimeter is insulated, then no heat is gained or lost by the entire system, which means $\Delta Q = 0$. This is often represented as $-\sum Q_{lost} = \sum Q_{gained}$

If we have two substances (#1 and #2), one of which is getting hotter and the other of which is getting colder, then our equation becomes:

$$\text{Heat Lost} + \text{Heat Gained} = \text{Change in Thermal Energy}$$
$$\sum Q_{lost} + \sum Q_{gained} = \Delta Q = 0$$
$$m_1 C_1 \Delta T_1 + m_2 C_2 \Delta T_2 = 0$$

In this example, ΔT_1 would be negative and ΔT_2 would be positive.

To solve a calorimetry problem, there are six quantities that you need: the two masses, the two specific heat capacities, and the two temperature changes. (You might be given initial and final temperatures for either or both, in which case you'll need to subtract. Remember that if the temperature increases, ΔT is positive, and if the temperature decreases, ΔT is negative.) The problem will usually give you all but one of these and you will need to find the missing one.

If you need to find the final temperature, use $\Delta T = T_f - T_i$ on each side. You will have both T_i numbers, so the only variable left will be T_f. (The algebra is straightforward, but ugly.)

Sample Problems:

Q: An 0.050 kg block of aluminum is heated and placed in a calorimeter containing 0.100 kg of water at 20. °C. If the final temperature of the water was 30. °C, to what temperature was the aluminum heated?

A: To solve the problem, we need to look up the specific heat capacities for aluminum and water in *Table K. Thermal Properties of Selected Materials* on page 473 of your Physics Reference Tables. The specific heat capacity of aluminum is $0.898 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$, and the specific heat capacity for water is $4.181 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$.

We also need to realize that we are looking for the initial temperature of the aluminum. ΔT is always ***final – initial***, which means $\Delta T_{\text{Al}} = 30 - T_{i,\text{Al}}$. (Because the aluminum starts out at a higher temperature, this will give us a negative number, which is what we want.)

$$\begin{aligned} m_{\text{Al}} C_{\text{Al}} \Delta T_{\text{Al}} \quad + \quad m_w C_w \Delta T_w &= 0 \\ (0.050)(0.897)(30 - T_i) + (0.100)(4.181)(30 - 20) &= 0 \\ 0.0449(30 - T_i) + \quad 4.181 &= 0 \\ 1.3455 - 0.0449 T_i + \quad 4.181 &= 0 \\ 5.5265 &= 0.0449 T_i \\ T_i = \frac{5.5265}{0.0449} &= 123.2\,^\circ\text{C} \end{aligned}$$

Q: An 0.025 kg block of copper at 95°C is dropped into a calorimeter containing 0.075 kg of water at 25°C. What is the final temperature?

A: We solve this problem the same way. The specific heat capacity for copper is $0.385 \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$, and $\Delta T_{\text{Cu}} = T_f - 95$ and $\Delta T_w = T_f - 25$. This means T_f will appear in two places. The algebra will be even uglier, but it's still a straightforward Algebra 1 problem:

$$\begin{aligned} m_{\text{Cu}} C_{\text{Cu}} \Delta T_{\text{Cu}} \quad + \quad m_w C_w \Delta T_w &= 0 \\ (0.025)(0.385)(T_f - 95_i) \; + \; (0.075)(4.181)(T_f - 25) &= 0 \\ 0.009625(T_f - 95) \quad + \quad 0.3138(T_f - 25) &= 0 \\ 0.009625\,T_f - (0.009625)(95) + 0.3136\,T_f - (0.3138)(25) &= 0 \\ 0.009625\,T_f - 0.9144 + 0.3138\,T_f - 7.845 &= 0 \\ 0.3234\,T_f &= 8.759 \\ T_f = \frac{8.759}{0.3234} &= 27\,^\circ\text{C} \end{aligned}$$

Homework Problems

You will need to look up specific heat capacities in *Table K. Thermal Properties of Selected Materials* on page 473 of your Physics Reference Tables.

1. **(S)** 375 kJ of heat is added to a 25.0 kg granite rock. If the temperature increases by 19.0 °C, what is the specific heat capacity of granite?

   Answer: $0.790\,\frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}}$

2. **(M)** A 0.040 kg block of copper at 95 °C is placed in 0.105 kg of water at an unknown temperature. After equilibrium is reached, the final temperature is 24 °C. What was the initial temperature of the water?

   Answer: 21.5 °C

3. **(S)** A sample of metal with a specific heat capacity of $0.50\,\frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}}$ is heated to 98 °C and then placed in an 0.055 kg sample of water at 22 °C. When equilibrium is reached, the final temperature is 35 °C. What was the mass of the metal?

   Answer: 0.0948 kg

4. **(S)** A 0.280 kg sample of a metal with a specific heat capacity of $0.430\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ is heated to 97.5 °C then placed in an 0.0452 kg sample of water at 31.2 °C. What is the final temperature of the metal and the water?

   Answer: 57 °C

5. **(M)** A sample of metal with mass m is heated to a temperature of T_m and placed into a mass of water M with temperature T_w. Once the system reaches equilibrium, the temperature of the water is T_f. Derive an expression for the specific heat capacity of the metal, C_m.

   Answer: $C_m = \dfrac{MC_w(T_f - T_w)}{m(T_m - T_f)}$

6. **(A)** You want to do an experiment to measure the conversion of gravitational potential energy to kinetic energy to heat by dropping 2.0 kg of copper off the roof of LEHS, a height of 14 m. How much will the temperature of the copper increase?
   (*Hint: Remember that potential energy is measured in* J *but specific heat capacity problems usually use* kJ.)

   Answer: 0.36°C

honors (not AP®)

Phase Diagrams*

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-11(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Identify the phase of a substance at any combination of temperature and pressure.
- Determine the melting and boiling points of a substance any pressure.

Success Criteria:

- Phases are correctly identified as solid, liquid, gas, supercritical fluid, *etc.,* in accordance with the temperature and pressure indicated on the phase diagram.
- Melting and boiling point temperatures are correctly identified for a substance from its phase diagram for a given pressure.
- The effects of a pressure or temperature change (*e.g.,* substance would melt, sublime, *etc.*) are correctly explained based on the phase diagram.

Language Objectives:

- Explain the regions of a phase diagram and the relationship between each region and the temperature and pressure of the substance..

Tier 2 Vocabulary: phase, curve, fusion, solid, liquid, gas, vapor

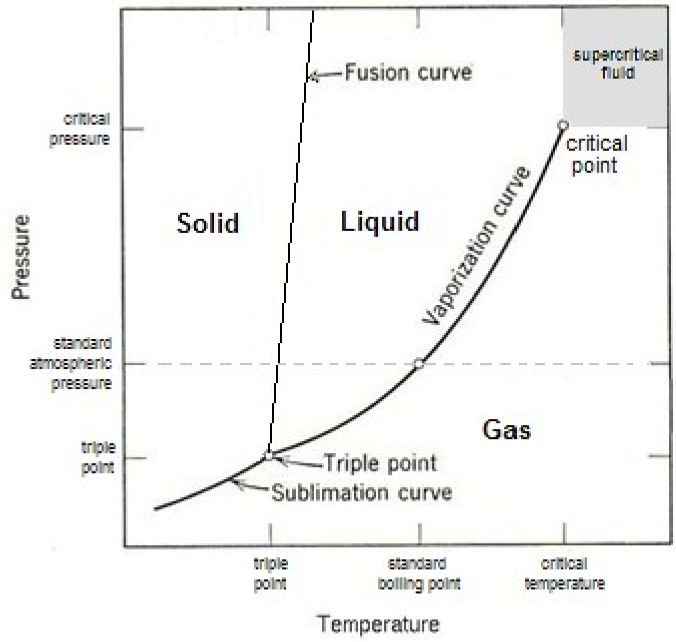
Notes:

The phase of a substance (solid, liquid, gas) depends on its temperature and pressure.

<u>phase diagram</u>: a graph showing the phase(s) present at different temperatures and pressures.

* Phase diagrams are usually taught in chemistry. However, they relate to the topics of phase changes and heating curves, which were moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

honors (not AP®)



fusion curve: the set of temperatures and pressures at which a substance melts/freezes.

vaporization curve: the set of temperatures & pressures at which a substance vaporizes/condenses.

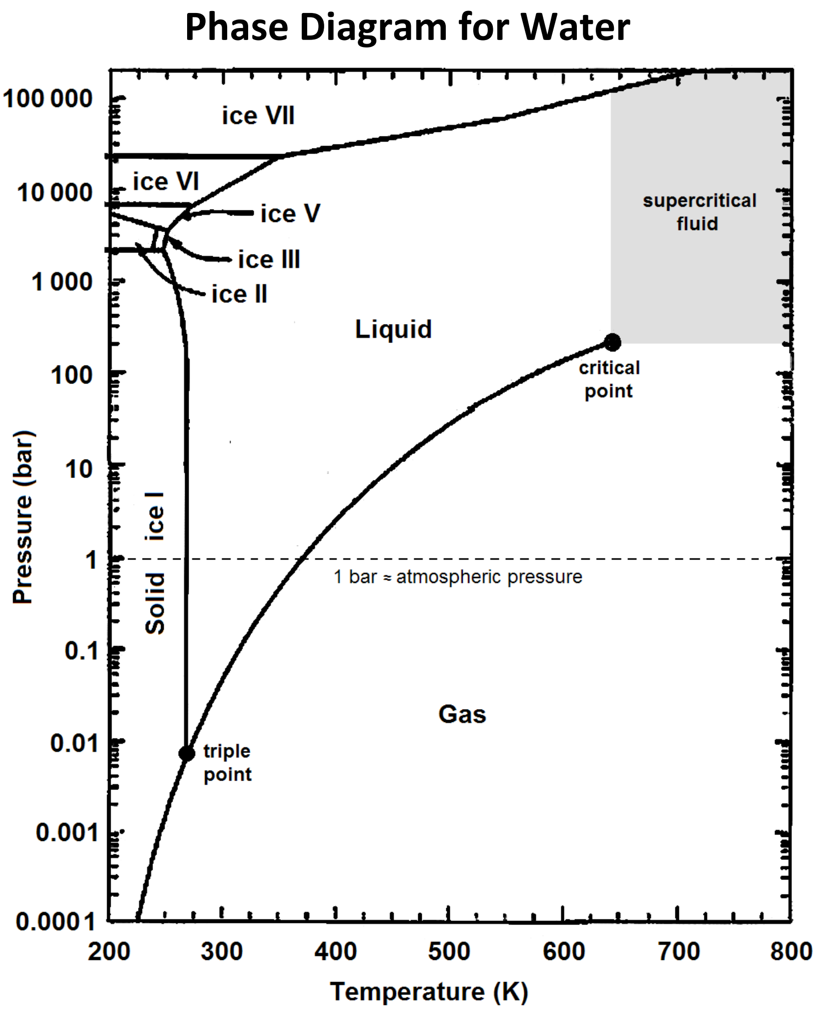
sublimation curve: the set of temperatures & pressures at which a substance sublimes/deposits.

triple point: the temperature and pressure at which a substance can exist simultaneously as a solid, liquid, and gas.

critical point: the highest temperature at which the substance can exist as a liquid. The critical point is the endpoint of the vaporization curve.

supercritical fluid: a substance whose temperature and pressure are above the critical point. The substance would be expected to be a liquid (due to the pressure), but the molecules have so much energy that the substance behaves more like a gas.

honors (not AP®)

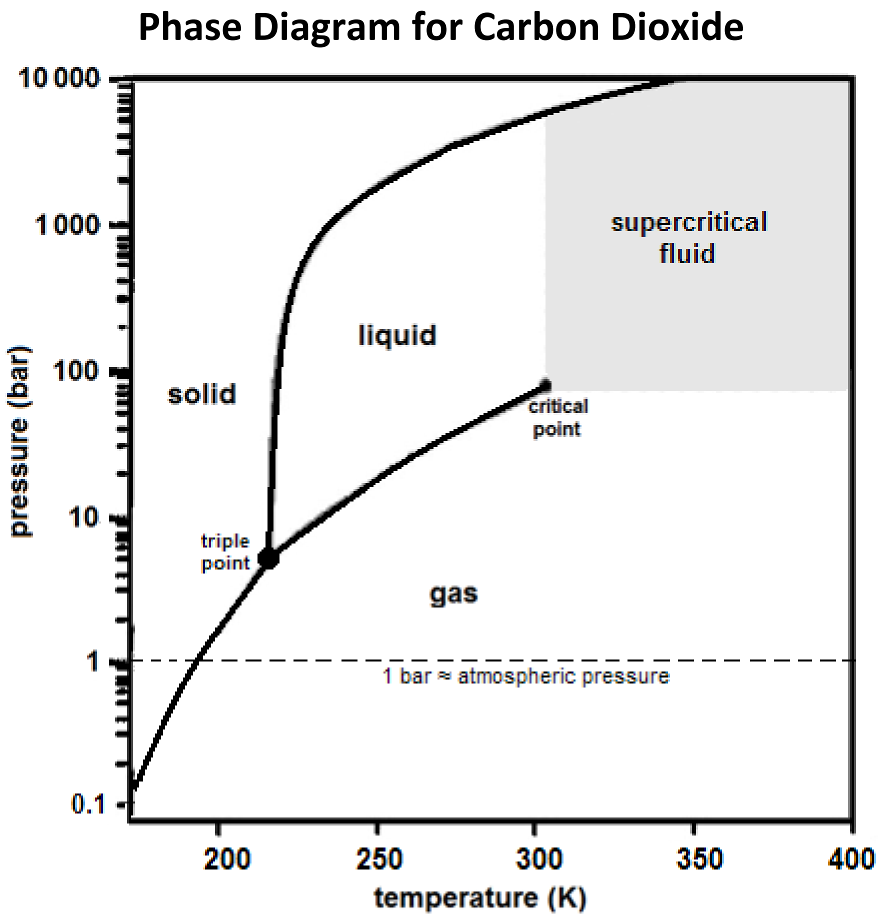


Note that pressure is on a logarithmic scale, and that standard atmospheric pressure is 1 bar ≈ 1 atm.

Note also that the temperature is in kelvin. To convert degrees Celsius to kelvin, add 273. (*e.g.*, 25 °C + 273 = 298 K.)

Notice that the slope of the fusion curve (melting/freezing line) is negative. This is because ice I is less dense than liquid water. At temperatures near the melting point and pressures less than about 2 000 bar, increasing the pressure will cause ice to melt. Water is one of the only known substances that exhibits this behavior.

honors (not AP®)



Notice that the pressure of the triple point for CO_2 is about 5 bar, which means CO_2 cannot be a liquid at atmospheric pressure. This is why dry ice (solid CO_2) sublimes directly from a solid to a gas.

honors (not AP®)

Homework Problems

Answer these questions based on the phase diagrams for water and carbon dioxide on the previous pages.

1. **(M)** Approximately what pressure would be necessary to boil water at a temperature of 350 K?

2. **(M)** What is the minimum pressure necessary for water to exist as a liquid at 350 K?

3. **(S)** At approximately what temperature would water boil if the pressure is 10 bar?

4. **(S)** What is the highest temperature at which carbon dioxide can exist as a liquid?

5. **(M)** At 1.0 bar of pressure, what is the temperature at which carbon dioxide sublimes?

6. **(S)** At room temperature (25 °C ≈ 300 K), what is the minimum pressure at which liquid carbon dioxide can exist?

7. **(M)** Describe the phase transitions and temperatures for water going from 200 K to 400 K at a pressure of 0.1 bar.

8. **(S)** Describe the phase transitions and temperatures for carbon dioxide going 200 K to 300 K at a pressure of 10 bar.

honors (not AP®)

Phases & Phase Changes*

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-3, HS-PS2-8(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Compare observable states of matter and phase transitions with behavior at the molecular level.

Success Criteria:

- Descriptions include connectedness and motion of molecules.
- Descriptions include comparative descriptions of molecular speed.
- Descriptions relate molecular motion and speed to temperature.

Language Objectives:

- Explain phase changes in terms of changes in molecular behavior.

Tier 2 Vocabulary: phase, solid, liquid, gas, vapor

Labs, Activities & Demonstrations:

- evaporation from boiling water on cloth

Notes:

macroscopic: objects or bulk properties of matter that we can observe directly.

microscopic: objects or properties of matter that are too small to observe directly.

Note that *macroscopic* properties of a substance are often determined by *microscopic* interactions between the individual molecules.†

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

solid: molecules are rigidly connected. A solid has a definite shape and a definite volume.

* Phase changes are generally taught in chemistry classes. However, because the calorimetry and heating curves topics were moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016, it is useful to review them here.

† In this section, the term "molecules" is used to refer to the particles that make up a substance. In chemistry, a molecule is a group of atoms that are covalently bonded together, and a substance can be made of individual atoms, molecules, crystals, or other types of particles. In these notes, the term "particles" is preferred, but "molecules" is used in this section because it conjures the impression of particles that are attached or bonded together in some way. This gives most students a reasonably correct picture of entities that are firmly attached to each other and cannot be pulled apart by physical means.

honors (not AP®)

liquid: molecules are loosely connected; bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of charged particles moving very rapidly.

phase change: when an object or substance changes from one phase to another through gaining or losing heat.

Breaking bonds requires energy. Forming bonds releases energy. This is true for the intermolecular bonds that hold a solid or liquid together as well as for chemical bonds.

As you probably know from experience, you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it).

- This is why evaporation causes cooling—because the system (the water) needs to absorb heat from its surroundings in order to make the change from a liquid to a gas (vapor).
- This is also why lids keep drinks hot. The lid is a barrier which significantly reduces the amount of evaporation.
- When you perspire, the water absorbs heat from you in order to evaporate, which cools you off.

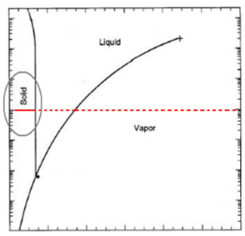
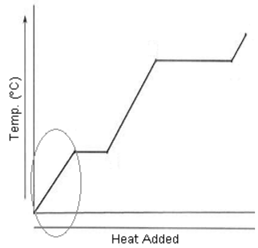
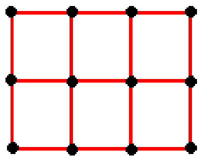
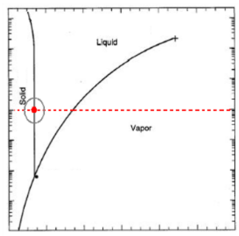
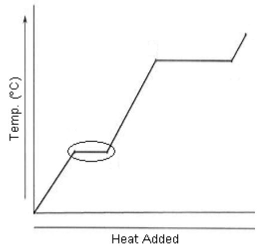
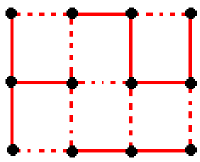
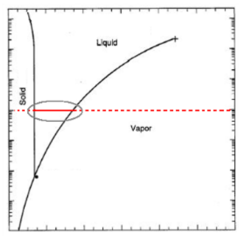
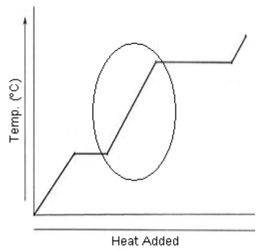
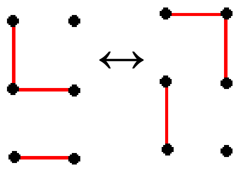
It is less obvious that energy is released when a gas condenses or a liquid freezes.

- Ice in your ice tray needs to give off heat in order to freeze. (Your freezer needs to remove that heat in order to make this happen.)
- Burns from steam are much more dangerous than burns from water, because the steam releases a large amount of heat (which is absorbed by your body) as it condenses.

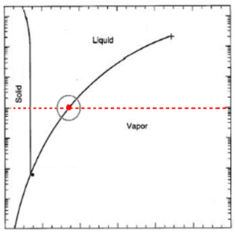
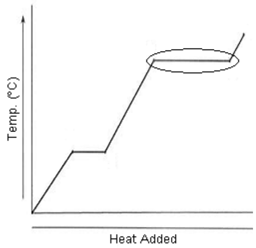
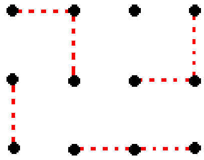
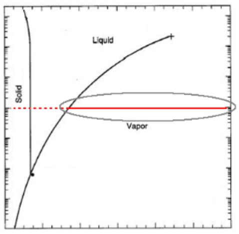
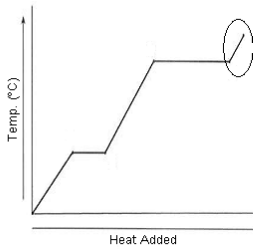
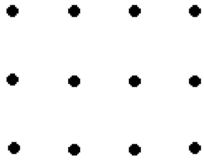
honors (not AP®)

States of Matter

The following table shows interactions between the molecules and some observable properties for solids, liquids and gases. (The table includes heating curves, which will be discussed in more detail later in the next section, *Heating Curves* starting on page 70. For now, understand that a heating curve shows how the temperature changes as heat is added. Notice in particular that the temperature stays constant during melting and boiling.)

state	phase diagram	heating curve	molecules
solid			rigidly bonded 
	adding energy makes molecules move faster; temperature increases		
melting			some bonds breaking 
	adding energy breaks some of the bonds; temperature remains constant		
liquid			bonds breaking & re-forming rapidly 
	adding energy makes molecules move faster; temperature increases		

honors (not AP®)

<table>
<tr><th>state</th><th>phase diagram</th><th>heating curve</th><th>molecules</th></tr>
<tr><td>boiling</td><td>
</td><td>
</td><td>all bonds breaking
</td></tr>
<tr><td colspan="4">adding energy breaks all remaining bonds;
temperature remains constant</td></tr>
<tr><td>vapor (gas)</td><td>
</td><td>
</td><td>molecules moving freely
</td></tr>
<tr><td colspan="4">adding energy makes molecules move faster;
temperature increases</td></tr>
</table>

Note that because liquids are continually forming and breaking bonds, if a liquid molecule at the surface breaks its bonds with other liquids, it can "escape" from the attractive forces of the other liquid molecules and become a vapor molecule. This is how evaporation happens at temperatures that are well below the boiling point of the liquid.

honors (not AP®)

Heating Curves*

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-4a

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Determine the amount of heat required for all of the phase changes that occur over a given temperature range.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: specific heat capacity, curve, phase

Labs, Activities & Demonstrations:

- Evaporation from washcloth.
- Fire & ice (latent heat of paraffin).

Notes:

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

solid: molecules are rigidly connected. A solid has a definite shape and volume.

liquid: molecules are loosely connected; bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of particles with rapidly changing charges.

* Heating curves are usually taught in chemistry. However, the topic was moved from chemistry to physics in the Massachusetts Curriculum Frameworks starting in 2016.

honors (not AP®)

<u>phase change</u>: when an object or substance changes from one phase to another through gaining or losing heat.

Breaking bonds requires energy. Forming bonds releases energy. This is true for the bonds that hold a solid or liquid together as well as for chemical bonds (regardless of what previous teachers may have told you!)

I.e., you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it). Energy is released when a gas condenses or a liquid freezes. (*E.g.,* ice in your ice tray needs to give off heat in order to freeze. Your freezer needs to remove that heat in order to make this happen.)

The reason evaporation causes cooling is because the system (the water) needs to absorb heat from its surroundings (*e.g.,* your body) in order to make the change from a liquid to a gas (vapor). When the water absorbs heat from you and evaporates, you have less heat, which means you have cooled off.

Calculating the Heat of Phase Changes

<u>heat of fusion</u> (ΔH_{fus}) (sometimes called "latent heat" or "latent heat of fusion"): the amount of heat required to melt one kilogram of a substance. This is also the heat released when one kilogram of a liquid substance freezes. For example, the heat of fusion of water is $334\,\frac{\text{J}}{\text{g}} \equiv 334\,\frac{\text{kJ}}{\text{kg}}$. The heat required to melt a sample of water is therefore:

$$Q = m\Delta H_{fus} = m(334\,\tfrac{\text{J}}{\text{g}})$$

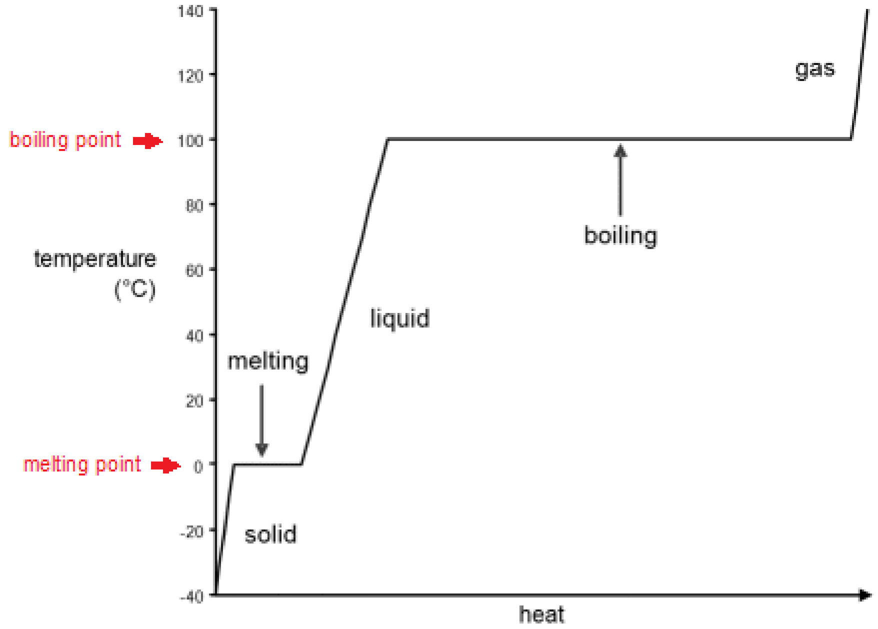
<u>heat of vaporization</u> (ΔH_{vap}): the amount of heat required to vaporize (boil) one kilogram of a substance. This is also the heat released when one kilogram of a gas condenses. For example, the heat of vaporization of water is $2260\,\frac{\text{J}}{\text{g}} \equiv 2260\,\frac{\text{kJ}}{\text{kg}}$. The heat required to boil a sample of water is therefore:

$$Q = m\Delta H_{vap} = m(2260\,\tfrac{\text{J}}{\text{g}})$$

(Again, pay attention to the units. If your mass is in kg, you will need to use the units $\frac{\text{kJ}}{\text{kg}}$ for your ΔH values; if your mass is in g, you will need to use the units $\frac{\text{J}}{\text{g}}$ for your ΔH values.)

honors (not AP®)

heating curve: a graph of temperature *vs.* heat added. The following is a heating curve for water:



In the "solid" portion of the curve, the sample is solid water (ice). As heat is added, the temperature increases. The specific heat capacity of ice is $2.11\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$, so the heat required is:

$$Q_{solid} = mC\Delta T = m(2.11\,\tfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(\Delta T)$$

In the "melting" portion of the curve, the sample is a mixture of ice and water. As heat is added, the ice melts, but the temperature remains at 0°C until all of the ice is melted. The heat of fusion of ice is $334\,\frac{\text{kJ}}{\text{kg}}$, so the heat required is:

$$Q_{melt} = m\Delta H_{fus} = m(334\,\tfrac{\text{kJ}}{\text{kg}})$$

In the "liquid" portion of the curve, the sample is liquid water. As heat is added, the temperature increases. The specific heat capacity of liquid water is $4.18\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$, so the heat required is:

$$Q_{liquid} = mC\Delta T = m(4.18\,\tfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}})(\Delta T)$$

honors (not AP®)

In the "boiling" portion of the curve, the sample is a mixture of water and water vapor (steam). As heat is added, the water boils, but the temperature remains at 100°C until all of the water has boiled. The heat of vaporization of water is $2260\,\frac{\text{kJ}}{\text{kg}}$, so the heat required is:

$$Q_{boil} = m\Delta H_{vap} = m(2260\,\tfrac{\text{kJ}}{\text{kg}})$$

In the "gas" portion of the curve, the sample is water vapor (steam). As heat is added, the temperature increases. The specific heat capacity of steam is approximately $2.08\,\frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}}$ (at 100°C; the specific heat capacity of steam decreases as the temperature increases), so the heat required is:

$$Q_{gas} = mC\Delta T = m(2.08\,\tfrac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}})(\Delta T)$$

Steps for Solving Heating Curve Problems

A heating curve problem is a problem in which a substance is heated across a temperature range that passes through the melting and/or boiling point of the substance, which means the problem includes heating or cooling steps and melting/freezing or boiling/condensing steps.

1. Sketch the heating curve for the substance over the temperature range in question. Be sure to include the melting and boiling steps as well as the heating steps.

2. From your sketch, determine whether the temperature range in the problem passes through the melting and/or boiling point of the substance.

3. Split the problem into:
   a. Heating (or cooling) steps within each temperature range.
   b. Melting or boiling (or freezing or condensing) steps.

4. Find the heat required for each step.
   a. For the heating/cooling steps, use the equation $Q = mC\Delta T$.
   b. For melting/freezing steps, use the equation $Q = m\Delta H_{fus}$.
   c. For boiling/condensing steps, use the equation $Q = m\Delta H_{vap}$.

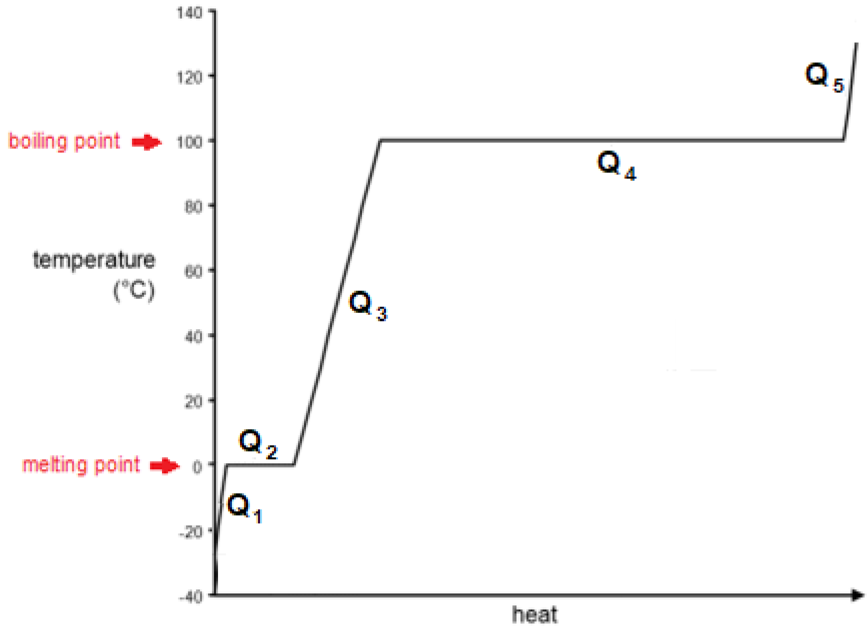
5. Add the values of Q from each step to find the total.

honors (not AP®)

Q: How much heat would it take to raise the temperature of 15.0 g of H_2O from −25.0 °C to +130.0 °C?

A: The H_2O starts out as ice. We need to:

1. Heat the ice from −25.0 °C to its melting point (0 °C).
2. Melt the ice.
3. Heat the water up to its boiling point (from 0 °C to 100 °C).
4. Boil the water.
5. Heat the steam from 100 °C to 130 °C.
6. Add up the heat for each step to find the total.



1. heat solid: $Q_1 = mC\Delta T = (15)(2.11)(25) = 791.25\text{ J}$
2. melt ice: $Q_2 = m\Delta H_{fus} = (15)(334) = 5\,010\text{ J}$
3. heat liquid: $Q_3 = mC\Delta T = (15)(4.181)(100) = 6\,270\text{ J}$
4. boil water: $Q_4 = m\Delta H_{vap} = (15)(2260) = 33\,900\text{ J}$
5. heat gas: $Q_5 = mC\Delta T = (15)(2.08)(30) = 936\text{ J}$
6. $Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$
   $Q = 791 + 5\,010 + 6\,270 + 33\,900 + 936 = 46\,910\text{ J}$

honors (not AP®)

Homework Problems

For the following problems, use data from the following table:

	C (sol.) $\left(\frac{kJ}{kg\cdot°C}\right)$	M.P. (°C)	ΔH_{fus} $\left(\frac{kJ}{kg}\right)$	C (liq) $\left(\frac{kJ}{kg\cdot°C}\right)$	B.P. (°C)	ΔH_{vap} $\left(\frac{kJ}{kg}\right)$	C_p (gas) $\left(\frac{kJ}{kg\cdot°C}\right)$
water	2.11	0	334	4.18	100	2260	2.08*
potassium	0.560	62	61.4	1.070	760	2025	0.671
mercury	0.142	−39	11.3	0.140	357	293	0.104
silver	0.217	962	111	0.318	2212	2360	—

*Note that because of the volume change from heating, the specific heat capacity of gases, C_p, increases with increasing temperature.

1. **(M)** A 0.0250 kg sample of water is heated from −40.0°C to 150. °C.

   a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the *y*-axis.

   b. Calculate the heat required for each step of the heating curve, and the total heat required.

   Answer: 80.01 kJ

honors (not AP®)

2. **(S)** A 0.085 kg sample of mercury is heated from 25°C to 500. °C.
   a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the *y*-axis.

   b. Calculate the heat required for each step of the heating curve, and the total heat required.

   Answer: 30.12 kJ

honors (not AP®)

3. **(S)** A 0.045 kg block of silver at a temperature of 22°C is heated with 20.0 kJ of energy. You want to determine the final temperature and what the physical state (solid, liquid, gas) of the silver at that temperature.

   a. Sketch the heating curve for silver. Label the starting temperature, melting point, and boiling point on the *y*-axis.

   b. Calculate the total heat required by calculating the heat for each step until the entire 20.0 kJ is accounted for.

   Answers: liquid, 1 369°C

honors (not AP®)

Thermal Expansion

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate changes in length & volume for solids, liquids and gases that are undergoing thermal expansion or contraction.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: expand, contract

Labs, Activities & Demonstrations:

- Balloon with string & heat gun.
- Brass ball & ring.
- Bi-metal strip.

Notes:

expand: to become larger

contract: to become smaller

thermal expansion: an increase in the length and/or volume of an object caused by a change in temperature.

When a substance is heated, the particles it is made of move farther and faster. This causes the particles to move farther apart, which causes the substance to expand.

Solids tend to keep their shape when they expand. (Liquids and gases do not have a definite shape to begin with.)

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0 °C to 4 °C. (Water expands as the temperature increases above 4 °C.)

honors (not AP®)

Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coëfficient of thermal expansion. The changes in length and volume are given by the equation:

$$\text{Length: } \Delta L = \alpha L_i \Delta T$$

$$\text{Volume: } \Delta V = \beta V_i \Delta T$$

where:

ΔL = change in length (m)
L_i = initial length (m)
α = linear coëfficient of thermal expansion (°C^{-1} or K^{-1})

ΔV = change in volume (m^3)
V_i = initial volume (m^3)
β = volumetric coëfficient of thermal expansion (°C^{-1} or K^{-1})

ΔT = temperature change (°C or K)

Values of α and β at 20°C for some solids and liquids:

Substance	α (°C^{-1})	β (°C^{-1})	Substance	α (°C^{-1})	β (°C^{-1})
aluminum	2.3×10^{-5}	6.9×10^{-5}	gold	1.4×10^{-5}	4.2×10^{-5}
copper	1.7×10^{-5}	5.1×10^{-5}	iron	1.18×10^{-5}	3.33×10^{-5}
brass	1.9×10^{-5}	5.6×10^{-5}	lead	2.9×10^{-5}	8.7×10^{-5}
diamond	1×10^{-6}	3×10^{-6}	mercury	6.1×10^{-5}	1.82×10^{-4}
ethanol		7.5×10^{-4}	silver	1.8×10^{-5}	5.4×10^{-5}
glass	8.5×10^{-6}	2.55×10^{-6}	water (liq.)	6.9×10^{-5}	2.07×10^{-4}

honors (not AP®)

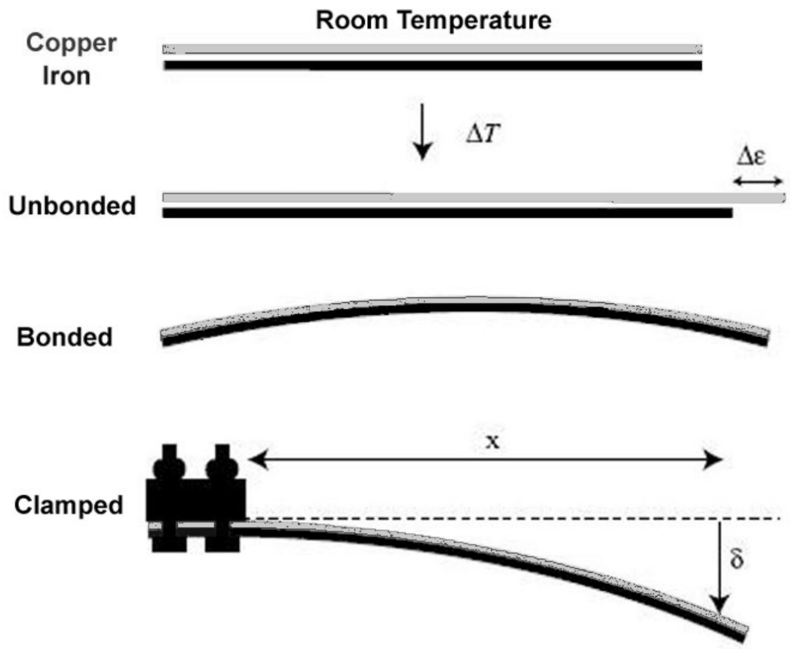
expansion joint: a space deliberately placed between two objects to allow room for the objects to expand without coming into contact with each other.

Bridges often have expansion joints in order to leave room for sections of the bridge to expand or contract without damaging the bridge or the roadway.

Railroad rails are sometimes welded together in order to create a smoother ride, which enables high-speed trains to use them. Unfortunately, if expansion joints are not placed at frequent enough intervals, thermal expansion can cause the rails to bend and buckle, resulting in derailments:

honors (not AP®)

bimetal strip: a strip made from two metals with different coëfficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.



Sample Problems:

Q: Find the change in length of an 0.40 m brass rod that is heated from 25 °C to 980 °C.

A: For brass, $\alpha = 1.9\times10^{-5}\ ^{\circ}\text{C}^{-1}$.

$$\Delta L = \alpha L_i \Delta T$$
$$\Delta L = (1.9\times10^{-5})(0.40)(955)$$
$$\Delta L = 0.0073\text{ m}$$

honors (not AP®)

Q: A typical mercury thermometer contains about 0.22 cm^3 (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25 °C to 50. °C.

A: For mercury, $\beta = 1.82\times10^{-4}\ ^{\circ}C^{-1}$.

$$\Delta V = \beta V_i \Delta T$$
$$\Delta V = (1.82\times10^{-4})(0.22)(25)$$
$$\Delta V = 0.000\,91\ \text{cm}^3$$

If the distance from the 25 °C to the 50 °C mark is about 3.0 cm, we could use this information to figure out the bore (diameter of the column of mercury) of the thermometer:

$$V = \pi r^2 h$$
$$0.000\,91 = (3.14)r^2(3.0)$$
$$r^2 = \frac{0.000\,91}{(3.14)(3.0)} = 9.66\times10^{-5}$$
$$r = \sqrt{9.66\times10^{-5}} = 0.0098\ \text{cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is (2)(0.0098) = 0.0197 cm, which is about 0.20 mm.

Homework Problems

honors (not AP®)

You will need to look up coëfficients of thermal expansion in *Table K. Thermal Properties of Selected Materials* on page 473 of your Physics Reference Tables.

1. **(S)** A brass rod is 27.50 cm long at 25 °C. How long would the rod be if it were heated to 750. °C in a flame?

   Answer: 27.88 cm

2. **(M)** A steel bridge is 625 m long when the temperature is 0 °C.

   a. If the bridge did not have any expansion joints, how much longer would the bridge be on a hot summer day when the temperature is 35 °C?
   (Use the linear coëfficient of expansion for iron.)

      Answer: 0.258 m

   b. Why do bridges need expansion joints?

3. **(M)** A 15.00 cm long bimetal strip is aluminum on one side and copper on the other. If the two metals are the same length at 20.0 °C, how long will each be at 800. °C?

   Answers: aluminum: 15.269 cm; copper: 15.199 cm

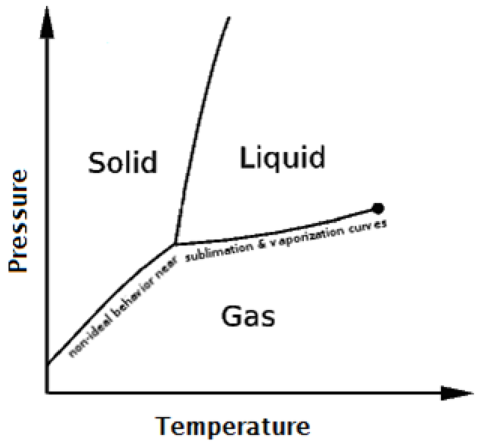
4. **(S)** A glass volumetric flask is filled with water exactly to the 250.00 mL line at 50. °C. What volume will the water occupy after it cools down to 20. °C?

   Answer: 248.45 mL

honors (not AP®)

Thermal Expansion of Gases

<u>ideal gas</u>: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.,* gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)



For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles' Law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1 and T_1 are the initial volume and temperature, and V_2 and T_2 are the final volume and temperature, respectively. Volume can be any volume unit (as long as it is the same on both sides), but temperature *<u>must be</u>* in Kelvin.

Sample Problem:

Q: If a 250 mL container of air is heated from 25 °C to 95 °C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

T_1 = 25 °C + 273 = 298 K T_2 = 95 °C + 273 = 368 K

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{250}{298} = \frac{V_2}{368}$$

$$V_f = 308.7 \approx 310 \text{ mL}$$

Because we used mL for V_1, the value of V_2 is therefore also in mL.

honors (not AP®)

Homework Problems

1. **(S)** A sample of argon gas was cooled, and its volume went from 380. mL to 250. mL. If its final temperature was −45.0 °C, what was its original temperature?

   Answer: 347 K or 74 °C

2. **(M)** A balloon contains 250. mL of air at 50 °C. If the air in the balloon is cooled to 20.0 °C, what will be the new volume of the air?

   Answer: 226.8mL

Introduction: Thermodynamics

Unit: Thermodynamics

Topics covered in this chapter:

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Kinetic-Molecular Theory* explains the implications of the theory that gases are made of large numbers of independently-moving particles.
- *Gas Laws* and *The Ideal Gas Law* describe and explain relationships between pressure, volume, temperature and the number of particles in a sample of gas.
- *Energy Conversion* describes conversion between heat and other forms of energy.
- *Thermodynamics* describes the transfer of energy into or out of a sample of gas.
- *Pressure-Volume (PV) Diagrams* and *Heat Engines* describe the relationship between changes in pressure and volume, heat, and work done on or by a gas.
- *Efficiency* describes the relationship between the work obtained from changes to a sample of gas and the maximum amount of energy that is theoretically available.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

9.1.A: Describe the pressure a gas exerts on its container in terms of atomic motion within that gas.

9.1.A.1: Atoms in a gas collide with and exert forces on other atoms in the gas and with the container in which the gas is contained.

9.1.A.1.i: Collisions involving pairs of atoms or an atom and a fixed object can be described and analyzed using conservation of momentum principles.

9.1.A.1.ii: The pressure exerted by a gas on a surface is the ratio of the sum of the magnitudes of the perpendicular components of the forces exerted by the gas's atoms on the surface to the area of the surface.

9.1.A.1.iii: Pressure exists throughout the gas itself, not just at the boundary between the gas and the container.

9.2.A: Describe the properties of an ideal gas.

9.2.A.1: The classical model of an ideal gas assumes that the instantaneous velocities of atoms are random, the volumes of the atoms are negligible compared to the total volume occupied by the gas, the atoms collide elastically, and the only appreciable forces on the atoms are those that occur during collisions.

AP® only

9.2.A.2: An ideal gas is one in which the relationships between pressure, volume, the number of moles or number of atoms, and temperature of a gas can be modeled using the equation $PV = nRT = Nk_BT$

9.2.A.3: Graphs modeling the pressure, temperature, and volume of gases can be used to describe or determine properties of that gas.

9.2.A.4: A temperature at which an ideal gas has zero pressure can be extrapolated from a graph of pressure as a function of temperature.

9.3.A: Describe the transfer of energy between two systems in thermal contact due to temperature differences of those two systems.

9.3.A.1: Two systems are in thermal contact if the systems may transfer energy by thermal processes.

9.3.A.1.i: Heating is the transfer of energy into a system by thermal processes.

9.3.A.1.ii: Cooling is the transfer of energy out of a system by thermal processes.

9.3.A.3: Energy is transferred through thermal processes spontaneously from a higher-temperature system to a lower-temperature system.

9.3.A.3.i: In collisions between atoms from different systems, energy is most likely to be transferred from higher-energy atoms to lower-energy atoms.

9.3.A.3.ii: After many collisions of atoms from different systems, the most probable state is one in which both systems have the same temperature.

9.3.A.4: Thermal equilibrium results when no net energy is transferred by thermal processes between two systems in thermal contact with each other.

9.4.A: Describe the internal energy of a system.

9.4.A.1: The internal energy of a system is the sum of the kinetic energy of the objects that make up the system and the potential energy of the configuration of those objects.

9.4.A.1.i: The atoms in an ideal gas do not interact with each other via conservative forces, and the internal structure is not considered. Therefore, an ideal gas does not have internal potential energy.

9.4.A.1.ii: The internal energy of an ideal monatomic gas is the sum of the kinetic energies of the constituent atoms in the gas.

AP® only

9.4.A.2: Changes to a system's internal energy can result in changes to the internal structure and internal behavior of that system without changing the motion of the system's center of mass.

9.4.B: Describe the behavior of a system using thermodynamic processes.

9.4.B.1: The first law of thermodynamics is a restatement of conservation of energy that accounts for energy transferred into or out of a system by work, heating, or cooling.

9.4.B.1.i: For an isolated system, the total energy is constant.

9.4.B.1.ii: For a closed system, the change in internal energy is the sum of energy transferred to or from the system by heating, or work done on the system.

9.4.B.1.iii: The work done on a system by a constant or average external pressure that changes the volume of that system (for example, a piston compressing a gas in a container) is defined as $W = -P\Delta V$.

9.4.B.2: Pressure-volume graphs (also known as PV diagrams) are representations used to represent thermodynamic processes.

9.4.B.2.i: Lines of constant temperature on a PV diagram are called isotherms.

9.4.B.2.ii: The absolute value of the work done on a gas when the gas expands or compresses is equal to the area underneath the curve of a plot of pressure vs. volume for the gas.

9.4.B.3: Special cases of thermal processes depend on the relationship between the configuration of the system, the nature of the work done on the system, and the system's surroundings. These include constant volume (isovolumetric), constant temperature (isothermal), and constant pressure (isobaric), as well as processes where no energy is transferred to or from the system through thermal processes (adiabatic).

9.6.A: Describe the change in entropy for a given system over time.

9.6.A.1: The second law of thermodynamics states that the total entropy of an isolated system can never decrease and is constant only when all processes the system undergoes are reversible.

9.6.A.2: Entropy can be qualitatively described as the tendency of energy to spread or the unavailability of some of the system's energy to do work.

9.6.A.2.i: Localized energy will tend to disperse and spread out.

9.6.A.2.ii: Entropy is a state function and therefore only depends on the current state or configuration of a system, not how the system reached that state.

AP® only

9.6.A.2.iii: Maximum entropy occurs when a system is in thermodynamic equilibrium.

9.6.A.3: The change in a system's entropy is determined by the system's interactions with its surroundings.

9.6.A.3.i: Closed systems spontaneously move toward thermodynamic equilibrium.

9.6.A.3.ii: The entropy of a closed system never decreases, but the entropy of an open system can decrease because energy can be transferred into or out of the system.

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Kinetic-Molecular Theory

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.1.A, 9.1.A.1, 9.1.A.1.i, 9.1.A.ii, 9.1.B, 9.1.B.1, 9.1.B.1.i, 9.1.B.1.ii

Mastery Objective(s): (Students will be able to…)

- Explain how each aspect of Kinetic-Molecular Theory applies to gases.

Success Criteria:

- Descriptions account for behavior at the molecular level.
- Descriptions account for measurable properties, *e.g.,* temperature, pressure, volume, *etc.*

Language Objectives:

- Explain how gas molecules behave and how their behavior relates to properties we can measure.

Tier 2 Vocabulary: kinetic, gas, ideal, real

Notes:

In chemistry you learned about matter, including its composition, structure, and changes that it can undergo. In physics, we are interested in matter to the extent that it can be used to bring objects or energy in contact with each other and transfer forces, energy or momentum from one object or collection of objects to another. This chapter is about gases and using properties of gases to convert between mechanical and thermal energy.

Properties of Different States of Matter

State	Description	Uses
solid	Particles rigidly bonded. Bonds difficult to break. (Definite shape & definite volume)	Construction materials where structure is important. Conduction of heat and/or electricity. Storage of heat as thermal mass.
liquid	Particles loosely bonded and have limited movement. Bonds continuously breaking & reforming. (Definite volume, but indefinite shape.)	Chemical reactions & heat transfer where continual mixing of materials is needed. Storage of heat as thermal mass.
gas	Particles not bonded and able to move freely. (Indefinite shape & volume.)	Heat and materials transfer in large spaces. ***Conversion of energy between heat and mechanical work.***

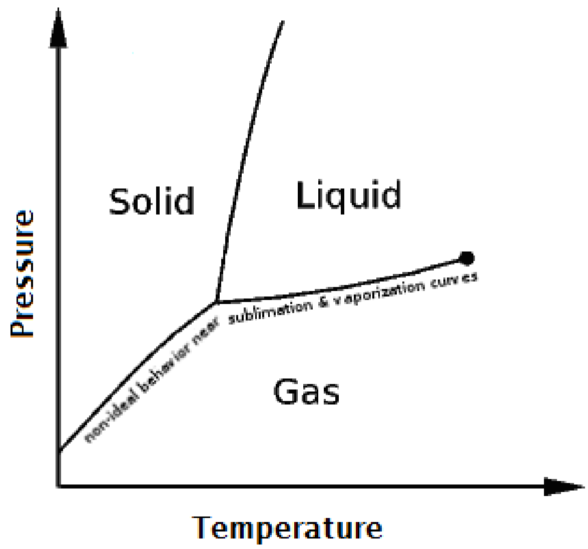
Kinetic-Molecular Theory (KMT)

Kinetic-Molecular Theory (KMT) is a theory, developed by James C. Maxwell and Ludwig Boltzmann, that predicts the behavior of gases by modeling them as moving molecules. ("kinetic" = "moving".) The theory states that:

- Gases are made of very large numbers of molecules*
- Molecules are constantly moving (obeying Newton's laws of motion), and their speeds are constant
- Molecules are very far apart compared with their diameter
- Molecules collide with each other and walls of container in elastic collisions
- Molecules behaving according to KMT are not reacting† or exerting any other forces (attractive or repulsive) on each other.

ideal gas: a gas whose molecules behave according to KMT. Most gases are ideal under *some* conditions (but not all). In general, gases behave ideally when they are not close to the solid or liquid regions of the phase diagram for the substance.

real gas: a gas whose molecules do ***not*** behave according to KMT. This can occur with all gases, most commonly at temperatures and pressures that are close to the solid or liquid regions of the phase diagram for the substance.



* "Particle" is more correct, but the theory is called Kinetic-***Molecular*** Theory. In this chapter we will use the terms "particle" and "molecule" interchangeably, with apologies to chemists.

† Of course, reactions can occur, but chemical reactions are part of collision theory, which is separate from KMT.

Measurable Properties of Gases

All gases have the following properties that can be measured:

Property	Variable	S.I. Unit	Description
amount	*N*	—	amount of gas (particles)
	n	mole (mol)	amount of gas (moles) (1 mol = 6.02×10^{23} particles)
volume	*V*	cubic meter (m^3)	space that the gas takes up
temperature	*T*	kelvin (K)	ability to transfer heat through collisions with other molecules (average kinetic energy of the particles)
pressure	*P*	pascal (Pa)	average force on the walls of the container due to collisions between the molecules and the walls

Notes about calculations:

- Moles are based on the definition that $1\,\text{mole} = 6.022\,140\,76 \times 10^{23}\ \text{particles}$. 1 mole was originally the number of carbon atoms in exactly 12 grams of carbon-12, such that the molar mass of a substance is the same number of grams as the average atomic mass of one atom in atomic mass units. This definition persisted, despite the fact that the base mass unit of the MKS system is the kilogram.
- Temperature must be absolute, which means you *must* use Kelvin. A temperature of 0 in a gas laws calculation can only mean absolute zero.
- Pressures must be absolute. (For example, you can't use a tire gauge because it measures "gauge pressure," which is the difference between atmospheric pressure and the pressure inside the tire.) A pressure of 0 in a gas laws calculation can only mean that there are no molecules colliding with the walls.

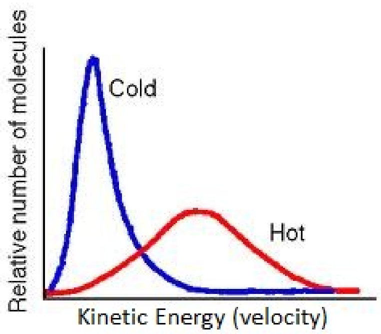
Other Common Units

- **Volume** can be measured in liters (L) or milliliters (mL).
  1 m^3 = 1 000 L and 1 L = 1 000 mL
- **Pressure** can be measured in many different units.
  1 atm = 101 325 Pa = 14.696 psi = 760 mm Hg = 29.92 in. Hg
  1 bar = 100 000 Pa = 100 kPa ≈ 1 atm

Temperature and Particle Motion

Particles of all substances, whether solids, liquids, or gases, are in constant motion. As described in *Phases & Phase Changes* starting on page 66, the particles of solids and liquids are bonded to one or more other particles. Gas particles, however, are not bonded to other particles. As stated above, these particles move at high speeds in straight lines until they collide with other particles or the walls of the container.

As described in *Heat & Temperature*, starting on page 41, temperature is related to the average kinetic energy of the particles. The kinetic energies follow the Maxwell-Boltzmann probability distribution, which is characterized by a longer tail on the right side. (The equation requires multivariable calculus and is beyond the scope of this course.)



The relationship between the kinetic energy and the average velocity of the molecules is characterized by the equation:

$$K_{avg} = \tfrac{3}{2} k_B T = \tfrac{1}{2} m v_{rms}^2$$

where:

K_{avg} = average kinetic energy of the particles (J)
k_B = Boltzmann constant $= 1.38\times10^{-23}\ \tfrac{\text{J}}{\text{K}}$
T = absolute temperature (K)
m = mass of a particle (kg)
v_{rms} = root mean square velocity ($\tfrac{\text{m}}{\text{s}}$)

The "root mean square velocity" simply means that v^2 is an average, which we call the "mean square velocity". v is the square root of this quantity, so we call it the "root mean square velocity".

Root mean square velocity is also discussed in *Thermodynamics*, starting on page 113.

Gas Laws

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.2.A, 9.2.A.1, 9.2.A.2, 9.2.A.3, 9.2.A.4

Mastery Objective(s): (Students will be able to…)

- Qualitatively describe the relationship between any two of the quantities: *number of particles*, *temperature*, *pressure*, and *volume* in terms of Kinetic Molecular Theory (KMT).
- Quantitatively determine the *number of particles*, *temperature*, *pressure*, or *volume* in a before & after problem in which one or more of these quantities is changing.

Success Criteria:

- Descriptions relate behavior at the molecular level to behavior at the macroscopic level.
- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Understand and correctly use the terms “pressure,” “volume,” and “temperature,” and “ideal gas.”
- Explain the placement of each quantity in the ideal gas law.

Tier 2 Vocabulary: ideal, law

Labs, Activities & Demonstrations:

- Vacuum pump (pressure & volume) with:
  - balloon (air vs. water)
  - shaving cream
- Absolute zero apparatus (pressure & temperature)
- Balloon with tape (temperature & volume)
- Can crush (pressure, volume & temperature)

Notes:

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost. (See the section on *Kinetic-Molecular Theory*, starting on page 92.)

Avogadro's Principle

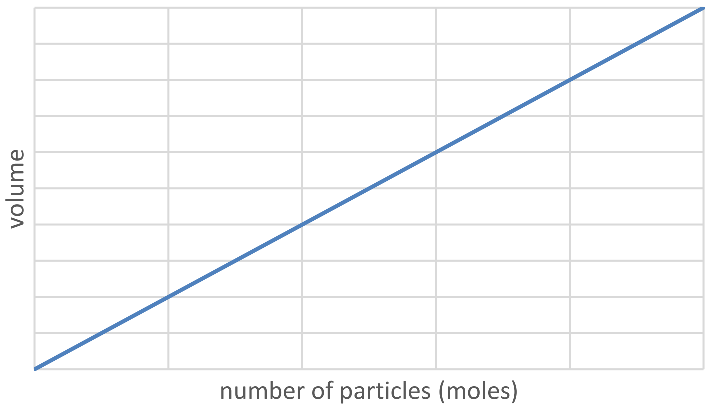
In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of particles.

Demonstration	Outcome	What the molecules are doing	Conclusion
put more (moles of) air into a balloon $n\uparrow$	the volume of the balloon got larger $V\uparrow$	crowding each other → pushing each other farther away	n and V are directly proportional. $\frac{V}{n} = \text{constant}$

If the pressure and temperature are constant, then for an ideal gas:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \text{and} \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}\,*$$

(particles) (moles)



Although Avogadro's principle was originally stated in terms of particles (N) of gas, it is almost always more convenient to work with moles (n).

* Avogadro's principle is usually stated $\frac{n_1}{V_1} = \frac{n_2}{V_2}$. I have inverted it in these notes so that the quantities in the numerator and denominator are the same as the quantities in the numerator and denominator of the combined gas law.

(Note that by convention, gas laws use subscripts "1" and "2" instead of "o" for initial no subscript for final.)

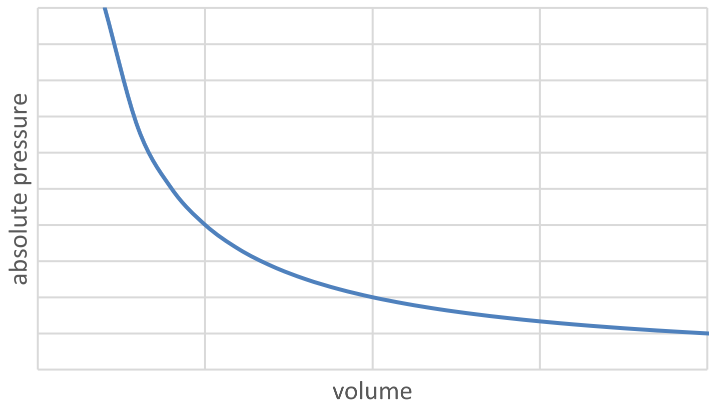
Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional.

Demonstration	Outcome	What the molecules are doing	Conclusion
decrease pressure by putting a balloon in a vacuum chamber $P\downarrow$	the volume of the air inside the balloon increased $V\uparrow$	expanding the space = more surface area → less force per unit area (less pressure)	*P* and *V* are inversely proportional. PV = constant

Therefore, if the temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1V_1 = P_2V_2$$



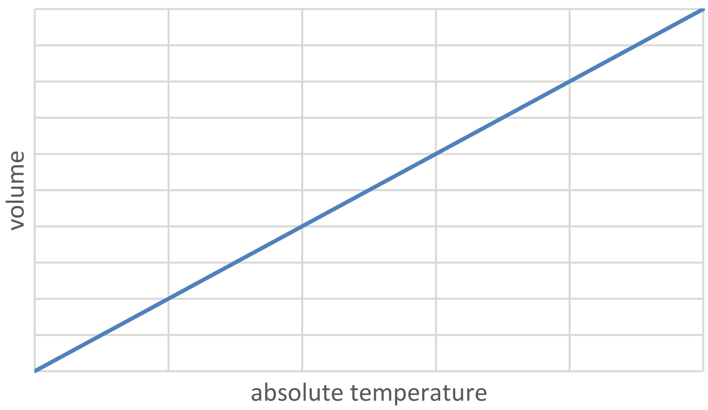
Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional.

Demonstration	Outcome	What the molecules are doing	Conclusion
place masking tape around balloon and heat with hot air gun $T\uparrow$	the volume of the air got larger and expanded the balloon except where the tape pinched it $V\uparrow$	moving more slowly → pushing each other less far away	*V* and *T* are directly proportional. $\frac{V}{T} = \text{constant}$

If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



Note that if a plot of temperature *vs.* volume is extrapolated to a volume of zero, the *x*-intercept will be absolute zero.

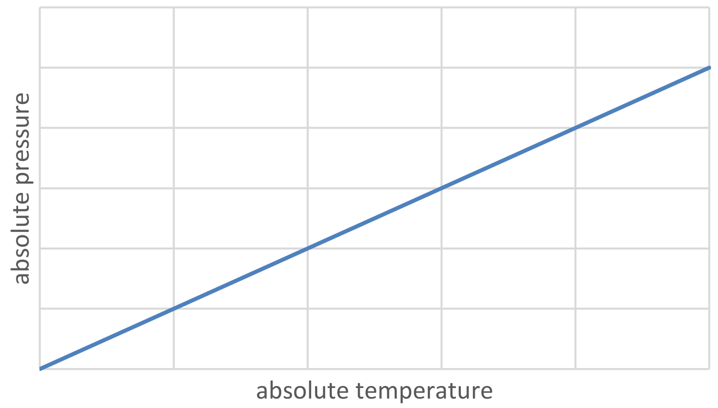
Gay-Lussac's Law

In 1702, French physicist Guillaume Amontons discovered that there is a relationship between the pressure and temperature of a gas. However, precise thermometers were not invented until after Amontons' discovery so it wasn't until 1808, more than a century later, that French chemist Joseph Louis Gay-Lussac confirmed this law mathematically. The pressure law is most often attributed to Gay-Lussac, though some texts refer to it as Amontons' Law.

Demonstration	**Outcome**	**What the molecules are doing**	**Conclusion**
increase temperature by heating a metal sphere full of air $T\uparrow$	the pressure of the air increased $P\uparrow$	moving faster → colliding with more force	P and T are directly proportional. $\frac{P}{T} = \text{constant}$

If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$



Note that at absolute zero, gas molecules do not exert any pressure on the walls of the container.

The Combined Gas Law

We can combine each of the above principles. When we do this (keeping *P* and *V* in the numerator and *n* (or *N*) and *T* in the denominator for consistency), we get following relationship for an ideal gas:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} = \text{constant} \qquad \frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \text{constant}$$

using moles using particles

Note, however, that in most situations where we want to calculate properties of a gas, the number of moles or particles remains constant. This means $n_1 = n_2$ or $N_1 = N_2$, and we can cancel it from the equation. This gives:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

The above equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

When using the combined gas law, any quantity that is not changing may be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.)

For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel *T* from both sides of the combined gas law, you get:

$$\frac{P_1 V_1}{\cancel{T_1}} = \frac{P_2 V_2}{\cancel{T_2}}$$ which simplifies to $P_1 V_1 = P_2 V_2$ (Boyle's Law)

Solving Problems Using the Combined Gas Law

You can use this method to solve any "before/after" gas law problem:

1. Determine which variables you have
2. Determine which values are ***initial*** (#1) *vs.* ***final*** (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures ***must*** be in Kelvin!)

Note: because quantities appear on both sides of the equation, it is not necessary to use S.I. units when solving problems using the combined gas law. It is, however, important to ***use the same units for the same quantity on both sides of the equation***.

Sample problem:

Q: A gas has a temperature of 25 °C and a pressure of 1.5 bar. If the gas is heated to 35 °C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25 °C and 35 °C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):	**Time 2 ("after")**:
$P_1 = 1.5$ bar	$P_2 = P_2$
$T_1 = 25\text{ °C} + 273 = 298\text{ K}$	$T_2 = 35\text{ °C} + 273 = 308\text{ K}$

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Gay-Lussac's Law)}$$

4. Plug in our values and solve:

$$\frac{1.5\,\text{bar}}{298\,\text{K}} = \frac{P_2}{308\,\text{K}} \quad \rightarrow \quad \boxed{P_2 = 1.55\text{ bar}}$$

Homework Problems

Solve these problems using one of the gas laws in this section. Remember to convert temperatures to Kelvin!

1. **(M)** A sample of oxygen gas occupies a volume of 250. mL at a pressure of 740. torr. What volume will it occupy at 800. torr?

   Answer: 231.25 mL

2. **(M)** A sample of O_2 is at a temperature of 40.0 °C and occupies a volume of 2.30 L. To what temperature should it be raised to occupy a volume of 6.50 L?

   Answer: 612 °C

3. **(S)** H_2 gas was cooled from 150. °C to 50. °C. Its new pressure is 750 torr. What was its original pressure?

   Answer: 980 torr

4. **(S)** A 2.00 L container of N_2 had a pressure of 3.20 atm. What volume would be necessary to decrease the pressure to 98.0 kPa?

   (*Hint: notice that the pressures are in different units. You will need to convert one of them so that both pressures are in either* atm *or* kPa.)

   Answer: 6.62 L

5. **(S)** A sample of air has a volume of 60.0 mL at S.T.P. What volume will the sample have at 55.0 °C and 745 torr?

   Answer: 73.5 mL

6. **(M)** N_2 gas is enclosed in a tightly stoppered 500. mL flask at 20.0 °C and 1 atm. The flask, which is rated for a maximum pressure of 3.00 atm, is heated to 680. °C. Will the flask explode?

   Answer: P_2 = 3.25 atm. Yes, the flask will explode.

7. A scuba diver's 10. L air tank is filled to a pressure of 210 bar at a dockside temperature of 32.0 °C. When the diver is breathing the air underwater, the water temperature is 8.0 °C, and the pressure is 2.1 bar.

   a. **(M)** What volume of air does the diver use?

      Answer: 921 L

   b. **(S)** If the diver uses air at the rate of 8.0 L/min, how long will the diver's air last?

      Answer: 115 min

Ideal Gas Law

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.2.A, 9.2.A.1, 9.2.A.2, 9.2.A.3, 9.2.A.4

Mastery Objective(s): (Students will be able to…)

- Describe the relationship between any two variables in the ideal gas law.
- Apply the ideal gas law to problems involving a sample of gas.

Success Criteria:

- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Explain the placement of each quantity in the ideal gas law.

Tier 2 Vocabulary: ideal, law

Notes:

ideal gas: a gas that behaves according to Kinetic-Molecular Theory (KMT).

When we developed the combined gas law, before we cancelled the number of moles or particles, we had the equations:

$$\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2} = \frac{PV}{nT} = R \text{ (constant)} \qquad \frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \frac{PV}{NT} = k_B \text{ (constant)}$$

using moles using particles

where n is the number of moles of gas, and N is the number of gas particles. One mole is 6.02×10^{23} particles, which means $N = (6.02 \times 10^{23})n$

Because P, V, n and T are all of the quantities needed to specify the conditions of an ideal gas, this expression must be true for *any ideal gas* under *any conditions*. If V is in m^3, P is in Pa, n is in moles, and T is in Kelvin, then:

$$R = 8.31 \frac{\text{J}}{\text{mol·K}} \quad \text{and} \quad k_B = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$$

R is called "the gas constant," and k_B is Boltzmann's constant.

We can rearrange $\frac{PV}{nT} = R$ and $\frac{PV}{NT} = k_B$ to get the ideal gas law in its familiar form:

$$PV = nRT \quad \text{and} \quad PV = Nk_B T$$

Other Values of R

The purpose of the gas constant R is to convert the quantity nT from units of mol·K to units of pressure × volume. This constant can have different values, depending on the units that it needs to cancel:

$$R = 8.31\,\frac{\text{J}}{\text{mol·K}} \equiv 8.31\,\frac{\text{m}^3\text{·Pa}}{\text{mol·K}} \equiv 8.31\,\frac{\text{L·kPa}}{\text{mol·K}} \equiv 8.31\times 10^{-3}\,\frac{\text{kJ}}{\text{mol·K}}$$

$$R = 0.0821\,\frac{\text{L·atm}}{\text{mol·K}} \qquad R = 62.4\,\frac{\text{L·torr}}{\text{mol·K}} \qquad R = 1.987\,\frac{\text{cal}}{\text{mol·K}} \equiv 1.987\,\frac{\text{BTU}}{\text{lb-mol·}^\circ\text{R}}$$

Use of non-S.I. units, such as atm or torr, is more common in chemistry. In this course, we will use the S.I. units of m^3 for volume and Pa for pressure. The unit Pa·m^3 is equivalent to a joule.

Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of moles (or particles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

Sample Problem:

A 3.50 (*n*) mol sample of an ideal gas has a pressure of 120 000 (*P*) Pa and a temperature of 35 °C. What is its volume?
$T \rightarrow K$ (*V*)

Answer:

Note that because pressure is given in pascals (Pa), we need to use the value of the gas constant that also uses Pa: $R = 8.31\,\frac{\text{m}^3\cdot\text{Pa}}{\text{mol}\cdot\text{K}}$

P = 120 000 Pa
V = V
n = 3.50 mol
$R = 8.31\,\frac{\text{m}^3\cdot\text{Pa}}{\text{mol}\cdot\text{K}}$
T = 35 °C + 273 = 308 K

Then we substitute these numbers into the ideal gas law and solve:

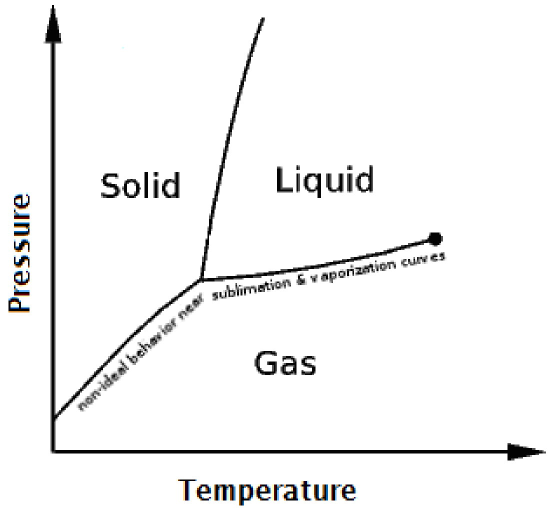
$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{(3.50)(8.31)(308)}{120\,000} = 0.0747\,\text{m}^3$$

honors (not AP®)

Real Gases

As stated previously, when the temperature and pressure of a gas are close to the solid or liquid regions of the phase diagram for the substance, gases start to exhibit non-ideal behaviors. Recall the following definition of a real gas:



real gas: a gas whose molecules do ***not*** behave according to kinetic-molecular theory (KMT). This occurs most commonly at temperatures and pressures that are close to the solid or liquid regions of the phase diagram for the substance.

In the late 19th century, the Dutch physicist Johannes van der Waals published a correction to the ideal gas law that can be applied to real gases.

The van der Waals Equation applies correction factors to the pressure and volume terms in the equation:

$$\left(P + a\frac{n^2}{V^2}\right)(V - nb) = nRT$$

in this equation, the constants *a* and *b* are properties specific to a gas and must be looked up or determined experimentally.

The corrected pressure term ($P + a\frac{n^2}{V^2}$ instead of P) is because molecules attract each other slightly at low pressures, but repel each other when they are forced close together. This repulsion acts like additional pressure.

The corrected volume term ($V - nb$ instead of V) term is because the ideal gas law assumes that the molecules are far enough apart that we do not need to consider the volumes of the molecules themselves as part of the volume of their container. As the molecules are brought closer together, we have to subtract the space taken up by *n* moles of molecules from the available volume.

You will not need to solve problems using the van der Waals equation in this course.

Homework Problems

Use the ideal gas law to solve the following problems. Be sure to choose the appropriate value for the gas constant and to convert temperatures to Kelvin.

1. **(M)** A sample of 1.00 moles of oxygen at 50.0 °C and 98.6 kPa occupies what volume?

   Answer: 27.2 L

2. **(S)** If a steel cylinder with a volume of 1.50 L contains 10.0 moles of oxygen, under what pressure is the oxygen if the temperature is 27.0 °C?

   Answer: 164 atm = 125 000 torr = 16 600 kPa

3. **(S)** In a gas thermometer, the pressure of 0.500 L of helium is 113.30 kPa at a temperature of −137 °C. How many moles of gas are in the sample?

   Answer: 0.050 mol

4. **(M)** A sample of 4.25 mol of hydrogen at 20.0 °C occupies a volume of 25.0 L. Under what pressure is this sample?

   Answer: 4.09 atm = 3 108 torr = 414 kPa

Energy Conversion

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Describe the conversion of energy between heat and other forms.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Describe and explain an example of conversion of heat into mechanical work.

Tier 2 Vocabulary: heat, energy

Labs, Activities & Demonstrations:

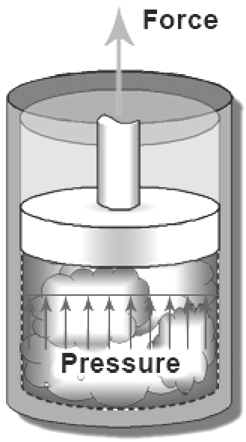
- steam engine
- fire syringe
- metal spheres & paper

Notes:

The law of conservation of energy states that total energy is always conserved, but that energy can be converted from one form to another.

We have already seen this in mechanics with the conversion between gravitational potential energy and kinetic energy.

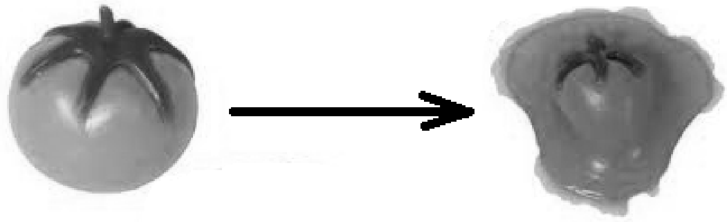
Heat is energy. Like other forms of energy, it can do work. For example, in a steam engine, heat is used to boil water in a sealed container. As more water boils, there is more gas in the boiler, which makes the pressure increase. If the gas can only expand by pushing against something (like a piston), the force from the pressure can do work by moving the piston and whatever it's connected to.



In mechanics, recall that collisions can be elastic or inelastic. In an elastic collision, kinetic energy is conserved; in an inelastic collision, some of the kinetic energy is converted to other forms, mostly heat.

We can use the law of conservation of energy to estimate the amount of energy converted to heat in a completely inelastic collision.

Consider a 0.150 kg tomato hitting the wall at a velocity of $20.0\,\frac{\text{m}}{\text{s}}$.



After the collision, the velocity of the tomato and the wall are both zero. This means the kinetic energy of the tomato after the collision is zero. Because energy must be conserved, this means all of the kinetic energy from the tomato must have been converted to heat.

$$E_k = \tfrac{1}{2}mv^2$$
$$E_k = (\tfrac{1}{2})(0.150)(20.0)^2 = 30.0\text{ J}$$

Now consider the same tomato with a mass of 0.150 kg and a velocity of $20.0\,\frac{\text{m}}{\text{s}}$ hitting a 1.00 kg block of wood that is initially at rest. This is still an inelastic collision, but now the wood is free to move, which means it has kinetic energy after the collision.

To solve this problem, we need to use conservation of momentum to find the velocity of the tomato + wood after the collision, and then use the velocity before and after to calculate the change in kinetic energy.

Before the collision:

$$\vec{p} = m_t\vec{v}_t + \cancelto{0}{m_w\vec{v}_w}$$
$$\vec{p} = (0.150)(+20.0) + 0 = +3.00\text{ N}\cdot\text{s}$$

$$K = \tfrac{1}{2}m_t v_t^2 + \cancelto{0}{\tfrac{1}{2}m_w v_w^2}$$
$$K = (\tfrac{1}{2})(0.150)(20.0)^2 + 0 = 30.0\text{ J}$$

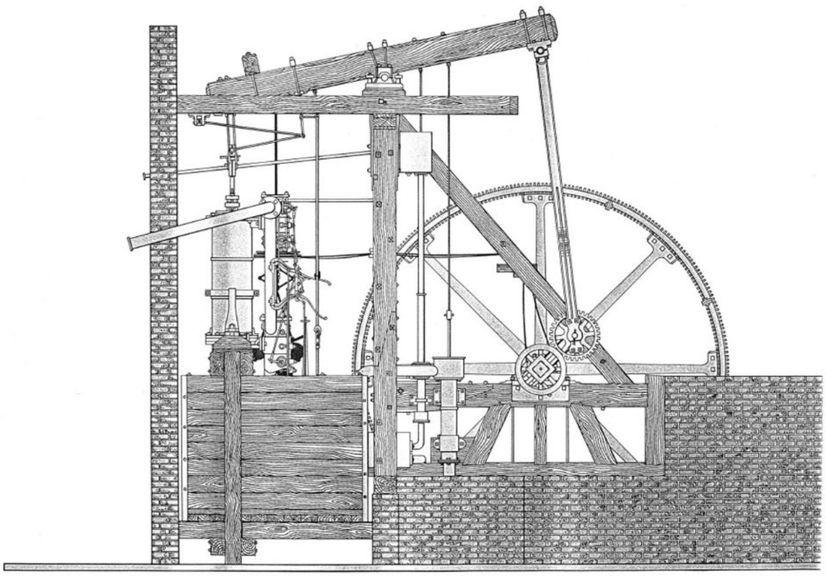
After the collision:

$$\vec{p} = (m_t + m_w)\vec{v}$$
$$+3.00 = (0.150 + 1.00)\vec{v} = 1.15\vec{v}$$
$$\vec{v} = +2.61\,\tfrac{\text{m}}{\text{s}}$$

$$K = \tfrac{1}{2}mv^2$$
$$K = (\tfrac{1}{2})(1.15)(2.61)^2 = 3.91\text{ J}$$

This means there is 30.0 – 3.91 = 26.1 J of kinetic energy that is "missing" after the collision. This "missing" energy is mostly converted to heat. If you could measure the temperature of the tomato and the wood extremely accurately before and after the collision, you would find that both would be slightly warmer as a result of the "missing" 26.1 J of energy.

The first instance of a machine using heat to do work was in 1698, when Thomas Savery patented a steam-driven water pump. In 1769, Scottish engineer James Watt and investor John Roebuck patented a steam engine that could be used for a variety of purposes, including running sawmills, cotton mills, and anything else that required a large amount of force. Watt built his first prototype steam engine in 1788.



James Watt's prototype steam engine, 1788

The invention of the steam engine was a significant factor in the spread of the the industrial revolution, and all of the societal changes that went with it.

Thermodynamics is the study of heat energy and its conversion to other forms of energy. In chemistry, thermodynamics is thermal energy that drives chemical reactions. In physics, thermodynamics is thermal energy that can be converted to mechanical work (which you may recall from physics 1, is a force applied over a distance).

Thermodynamics

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.3.A, 9.3.A.4, 9.4.A, 9.4.A.1, 9.4.A.1.i, 9.4.A.1.ii, 9.4.A.2, 9.4.B, 9.4.B.1, 9.4.B.1.i, 9.4.B.1.ii, 9.4.B.1.iii, 9.6.A, 9.6.A.1, 9.6.A.2, 9.6.A.2.i, 9.6.A.2.iii, 9.6.A.3, 9.6.A.3.i, 9.6.A.3.ii

Mastery Objective(s): (Students will be able to…)

- Calculate kinetic energy, internal energy and work done by the particles of a gas.

Success Criteria:

- Solutions have the correct quantities substituted for the correct variables.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Describe the different types of energy (kinetic, internal, work) and explain what they measure.

Tier 2 Vocabulary: internal, energy, work

Labs, Activities & Demonstrations:

- heat exchange dice game
- dice distribution game
- entropy (microstates) percentile dice game

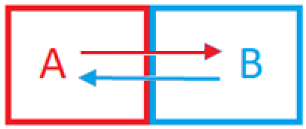
Notes:

thermodynamics: the study of heat-related (thermal) energy changes (dynamics)

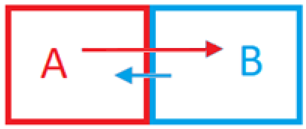
Thermodynamics is an application of the law of conservation of energy. In Physics 1, we studied changes between gravitational potential energy and kinetic energy. Thermodynamic changes involve the same principle; the details and the equations, however, are quite different.

As was the case with gas laws, the topic of thermodynamics is studied by both chemists and physicists. Chemists tend to be more concerned with the heat produced and consumed by chemical changes and reactions. Physicists tend to be more concerned with the conversion between thermal energy (regardless of how it is produced) and other forms of energy, particularly mechanical.

<u>thermal equilibrium</u>: if two systems “A” and “B” are in thermal equilibrium, heat is transferred from A to B at the same rate as heat is transferred from B to A.



Thermal equilibrium: equal amounts of heat transferred in each direction.



Not equilibrium: different amounts of heat transferred in each direction.

<u>temperature</u>: a measure of the average kinetic energy of the particles in a substance. (K)

$$K_{ave.} = \tfrac{1}{2} m v_{ave.}^2 = \tfrac{3}{2} k_B T$$

where:

$K_{ave.}$ = average kinetic energy (J)
m = mass of a particle (kg)
M = molar mass (mass of one mole of particles) ($\frac{\text{kg}}{\text{mol}}$)*
$v_{ave.}$ = average velocity of a particle ($\frac{\text{m}}{\text{s}}$)
k_B = Boltzmann's constant = $1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$
T = temperature (K)

The factor of $\frac{3}{2}$ is because there is an equal probability of a collision between particles in the *x*-, *y*-, and *z*-directions, and $(3)(\frac{1}{2}) = \frac{3}{2}$.

<u>root mean square velocity</u> (v_{rms}): the geometric mean (average) velocity of a particle. ($\frac{\text{m}}{\text{s}}$) The rms velocity is derived by solving $\frac{1}{2} m v_{ave.}^2 = \frac{3}{2} k_B T$ for the average velocity:

$$v_{rms} = \sqrt{v_{ave.}^2} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}}$$

* Chemists express molar mass in $\frac{\text{g}}{\text{mol}}$ rather than $\frac{\text{kg}}{\text{mol}}$. Because we use the MKS system in physics, we need to express molar mass in $\frac{\text{kg}}{\text{mol}}$.

internal energy (U*): the total thermal energy of a system due to the kinetic energy of its particles.

If the kinetic energy of a single particle is $K_{ave.} = \frac{3}{2}k_BT$, then the total kinetic energy in a system that has N particles would be:

$$U = NK_{ave.} = \frac{3}{2}Nk_BT$$

Because it is generally unwieldy to perform calculations for systems with large numbers of particles, it is more convenient to use moles. Substituting nR for Nk_B gives the equation for the internal energy of a system that has n moles of particles:

$$U = \frac{3}{2}nRT$$

Similarly, a *change* in internal energy (ΔU) is related to the corresponding *change* in temperature (ΔT):

$$\Delta U = \frac{3}{2}nR\Delta T \quad (= \frac{3}{2}Nk_B\Delta T)$$

heat (Q): thermal energy transferred into or out of a system. (J)

work (W): mechanical energy (such as the application of a force over a distance) transferred into or out of a system. (J)

The work that a gas can do comes from its ability to move an object by applying a force on it as it expands. If the pressure is constant:

$$W = Fd = F\Delta x$$

$$P = \frac{F}{A} \quad \rightarrow \quad F = PA$$

$$\therefore\ W = (PA)\Delta x$$

$$\Delta V = A\Delta x$$

$$\therefore\ W = P\Delta V$$

If a gas does work by expanding, the energy is transferred from the gas (the system) to the object that the gas is pushing against (the surroundings). This means that when the volume increases (ΔV is positive), energy is leaving the system (W is negative). Conversely, if work is done to compress a gas, energy is entering the system in order to compress the gas (W is positive), and the volume decreases (ΔV is negative). This means that W and $P\Delta V$ must have opposite signs, which gives the equation:

$$W = -P\Delta V$$

assuming that pressure is constant.

* Chemistry textbooks often use the variable E instead of U.

If pressure is not constant, then $W=-\Delta(PV)$, which means you would need to calculate *PV* at each point, taking the limit as the distance between the data points shrinks to zero, and add them up. In calculus, this is the integral: $W=-\int P\,\mathrm{d}V$, where *P* is a function of *V*.

In an algebra-based course, we will limit ourselves to problems where the pressure is constant, or where the pressure change is linear and you can use the average pressure, giving:

$$W=-P_{ave.}\Delta V$$

entropy (*S*): "unusable" thermal energy in a system. Energy in the form of entropy is unavailable because it has "escaped" or "spread out". (Entropy will be discussed further in the Second Law of Thermodynamics.)

Laws of Thermodynamics

The laws of thermodynamics describe the behavior of systems with respect to changes in heat energy.

For historical reasons, the laws are numbered from 0−3* instead of 1−4, because the 0 law was added after the others, and the laws are often referred to by their number.

0. If a system is at thermal equilibrium, every component of the system has the same temperature. ("You have to play the game.")
1. Heat always flows from a region of higher internal energy to a region of lower internal energy. Because internal energy is directly proportional to temperature, this is equivalent to saying that heat flows from a region of higher temperature to a region of lower temperature. This means you can't get more heat out of a system than you put in. ("You can't win.")
2. In almost every change, some energy is irretrievably lost to the surroundings. Entropy is a measure of this "lost" energy. The entropy of the universe is always increasing, which means on any practical scale, you will always get out less energy than you put in. ("You can't break even.")
3. Conservation of energy always applies. In any closed system, the total energy (internal energy + entropy + work) remains constant. If energy was "lost," it turned into an increase in entropy. ("You can't get out of the game.")

* There is one type of person in the world: those who start counting from zero and those who start counting from one.

Zeroth Law (or Zero Law)

The zeroth law says that if you have multiple systems in thermal equilibrium (the heat transferred from "A" to "B" is equal to the heat transferred from "B" to "A"), then the systems must have the same temperature. The consequences of this are:

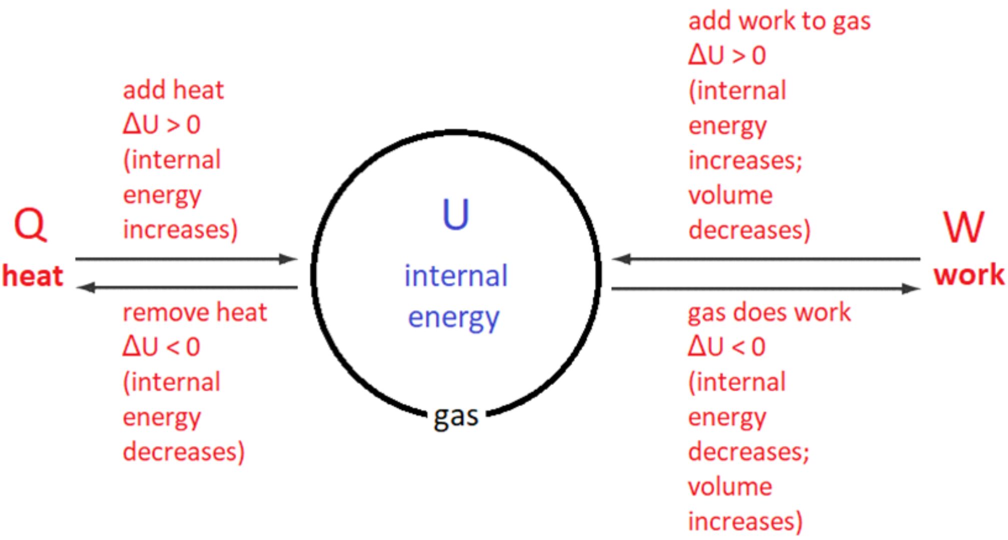
- If we have three (or more) systems "A," "B," and "C," and A is in thermal equilibrium with B, and B is in thermal equilibrium with C, this means that A, B, and C must all have the same temperature, and A is therefore in thermal equilibrium with C. (This is akin to the transitive property of equality in mathematics.)
- If an object with a higher temperature (a "hotter" object) is in contact with an object with a lower temperature (a "colder" object), heat will flow from the object with higher temperature to the object with lower temperature until the temperatures are the same (the objects are in thermal equilibrium).

First Law

Consider an isolated system—*i.e.,* a system where heat and other forms of energy cannot enter nor leave the system. This system is doing no work, but it has internal energy.

Internal energy is similar to potential energy—it is a property of a system that is not doing work currently, but has the potential to do work in the future.

According to the First Law, the internal energy of a system increases $(\Delta U > 0)$ if energy is added in the form of heat $(Q > 0)$ or work $(W > 0)$. The internal energy decreases if energy is removed from the system, by removing heat $(Q < 0)$ or by using the internal energy of the system (the gas) to do work on the surroundings ($W < 0$ because work is going out of the system).



In equation form, the First Law looks like this:

$$\Delta U = Q + W^{*}$$

The First Law is simply the law of conservation of energy—the change in internal energy comes from the heat added to or removed from the system combined with the work done on or by the system.

Combining the First Law with the definition of internal energy gives:

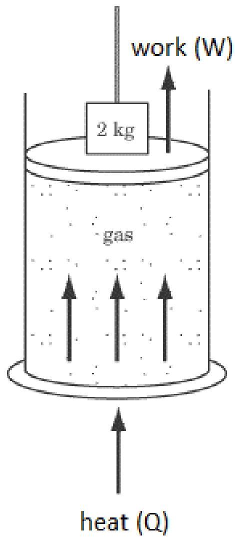
$$\Delta U = \tfrac{3}{2} nR\Delta T = Q + W$$

* Some textbooks define work exclusively as work done ***by*** the system ***on*** the surroundings (*i.e.,* energy ***leaving*** the system). Using this definition would reverse the sign of *W* in the equation, giving:

$$\Delta U = Q - W$$

Sample Problem:

Q: A cylinder containing 1.8 mol of an ideal gas with a temperature of 275 K has a piston with a weight on top. The combined mass of the piston plus the weight is 2.0 kg, and the cross-sectional area of the piston is 0.01 m^2. The volume of the gas in the cylinder is 0.033 m^3.



Heat is added, and the volume of the gas increases to 0.040 m^3. How much heat was added to the gas?

A: When the gas is heated, the following occur:

1. In order to increase the volume, the gas has to expand, which means the temperature needs to increase. We know how much the volume increased, and the pressure remains constant (the piston pushes the same amount throughout the process). We can use $\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$ to find the final temperature of the gas.

$$\frac{\cancel{P_1}V_1}{T_1}=\frac{\cancel{P_2}V_2}{T_2} \rightarrow \frac{0.033}{275}=\frac{0.040}{T_2} \rightarrow T_2=333\,\text{K}$$

2. The increase in temperature means the internal energy of the gas increased. We can use $\Delta U=\frac{3}{2}nR\Delta T$ to find out how much the internal energy of the gas increased (ΔU). The equation $\Delta U=Q+W$ tells us that some of the heat energy needed to be used for the increase in internal energy, and the rest of it was used to do the work of raising the piston.

$$\Delta U=\tfrac{3}{2}nR\Delta T \rightarrow \Delta U=(\tfrac{3}{2})(1.8)(8.31)(333-275)=1301\,\text{J}$$

3. We can calculate the work used to raise the piston from the work equation from physics 1: $W=mg\Delta h$ (Note that $\Delta V=A\Delta h$.)

$$W=-mg\Delta h=-mg\frac{\Delta V}{A}=-(2)(10)\frac{0.007}{0.01}=-14\,\text{J}$$

The work is negative because the energy is going out of the system (remember that the system is the gas) and into the surroundings.

4. Once we have ΔU and W, we can find Q by applying the first law:

$$\Delta U=Q+W$$
$$1301=Q+(-14)$$
$$1315\,\text{J}=Q$$

Alternatively, we could calculate the work using $W = -P\Delta V$, but we would need to use gauge pressure rather than absolute pressure. (See the explanation below.)

The first two steps are the same as above.

1. This step is the same as step #1 above—we need to find the temperature change necessary to produce the change in volume.

2. This step is the same as step #2 above—we need to calculate the change in internal energy of the gas caused by the change in temperature.

3. Instead of calculating the work using equations from physics 1, we can use $W = -P\Delta V$. However, pressure needs to be the amount of pressure that is doing the work, which is the difference in pressure between the inside of the piston and the outside of the piston. (This would be the gauge pressure inside the cylinder.) We can calculate this using the pressure equation from fluids in physics 1:

$$P = \frac{F}{A} = \frac{(2)(10)}{0.01} = 2\,000\text{ Pa}$$

   This is the pressure at which the gas needs to do work.

   Once we have the pressure, the work is given by:

$$W = -P\Delta V$$
$$\Delta V = 0.040 - 0.033 = 0.007\text{ m}^3$$
$$W = -(2000)(0.007) = -14\text{ J}$$

4. Once we have ΔU and W, we can find Q by applying the first law:

$$\Delta U = Q + W$$
$$1301 = Q + (-14)$$
$$1315\text{ J} = Q$$

Second Law

The Second law tells us that heat energy cannot flow from a colder system to a hotter one unless work is done on the system. This is why your coffee gets cold and your ice cream melts.

One consequence of this law is that no machine can work at 100% efficiency; all machines generate some heat, and some of that heat is always lost to the surroundings.

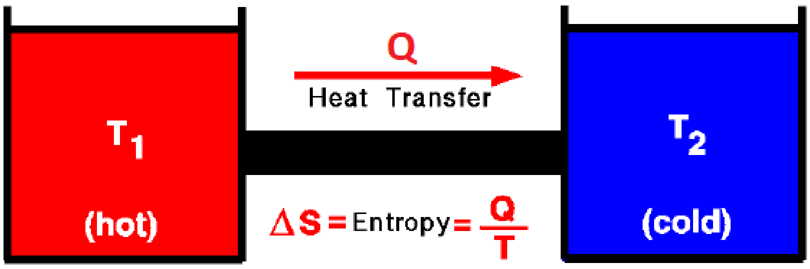
Entropy

Because energy must be conserved, we have to account for energy that still exists, but has "gotten lost" ("escaped" or "spread out") and is no longer usable by the system. Energy that has spread out and cannot be recovered is called entropy.

For example, when an egg falls to the floor and breaks, gravitational potential energy is converted to a combination of internal energy (the measurable increase in the temperature of the egg), and entropy (heat energy that is radiated to the environment and "lost"). Over time, the internal energy in the egg is also radiated to the environment and "lost" as the egg cools off. Ultimately, all of the gravitational potential energy ends up converted to entropy, which is the heat energy that is dissipated and cannot be recovered.

Entropy is sometimes called "disorder" or "randomness", but in the thermodynamic sense this is not correct. The entropy of your room is a *thermodynamic* property of the heat energy in your room, not a commentary on the amount of dirty laundry on the floor!

Whenever heat is transferred from an object with a higher temperature to an object with a lower temperature, the heat "spreads out" as it warms the colder object. The amount of energy that goes into this "spread" is called entropy.



If the heat could be transferred in a way that is completely reversible (which is impossible and would take an infinite amount of time), then you would be able to recover the energy that was converted to entropy when transferring it back to the hotter object. We call this fictitious heat "reversible heat," denoted Q_{rev}.

honors (not AP®)

$$\Delta S = \frac{Q_{rev}}{T}$$

Real energy transfers that take place in finite amounts of time can never recover all of the energy that was turned into entropy. This means the actual increase in entropy is always more than the amount that would occur in a reversible process:

$$\Delta S = \frac{Q_{rev}}{T} \geq \frac{Q}{T}$$

In other words, the energy that is lost to entropy by transfer of an amount of heat Q would be exactly $\frac{Q}{T}$ for a completely reversible process (*i.e.,* $Q = Q_{rev}$), and more than that for any real process.

Because actual heat transfer in a finite amount of time cannot be completely reversible, some heat is lost to the surroundings and the actual entropy change is always greater than the actual heat change at a given temperature. The concept of a reversible process is an idealization that represents the maximum amount of work that could theoretically be extracted from the process.

Consequences of the Second Law

- The total entropy of an isolated system can never decrease, and is constant only when all of the thermodynamic processes are reversible.
- Localized energy will tend to disperse (spread out). When this happens, thermal energy is lost to the surroundings, and the entropy of the system increases.
- Entropy is a state function, which means the entropy of a system does not depend on the energy pathway the system followed.
- Closed systems spontaneously move toward thermodynamic equilibrium (*i.e.,* all parts of the system have the same temperature). When a system has reached thermodynamic equilibrium, the system's entropy is maximized.
- Because the universe is a closed system, the entropy of the universe is always increasing.
- The entropy of an open system can be decreased through a transfer of energy into the system from the surroundings.

In physics, there is a hierarchy of thinking. Conservation of energy, conservation of momentum, and the Second Law are at the top of the hierarchy. Just as special relativity tells us that time, distance and mass all need to be changeable in order to maintain conservation of energy and momentum, the Second Law explains why time cannot move backwards—to do so would require a decrease in the entropy of the universe.

Third Law

The Third Law tells us that in an isolated system, the total energy of the system must be constant. (An isolated system is a system for which it is not possible to exchange energy with the surroundings.) This makes intuitive sense; because energy must be conserved, if no energy can be added or taken away, then the total energy cannot change.

Thermodynamic Quantities and Equations

Because energy is complex and exists in so many forms, there are many thermodynamic quantities that can be calculated in order to quantify the energy of different portions of a system. The following is a list of some of the more familiar ones:

Selected Thermodynamic Quantities

Variable	Name	Description
Q	heat	Thermal energy (heat) transferred into or out of a system due to a difference in temperature.
W	work	Mechanical energy transferred into or out of a system through the action of a force applied over a distance. $W = \vec{F} \bullet \vec{d} = -P\Delta V$
U*	internal energy	Total thermal (non-chemical) energy contained within the particles of a system because of their kinetic energy. $U = \frac{3}{2}nRT$ $\Delta U = Q + W = \frac{3}{2}nR\Delta T$
S	entropy	Energy that is "lost" (inaccessible) because it has spread to the surroundings or has spread to separate microstates and cannot be utilized by the particles of the system.
A	Helmholtz free energy	Useful work that could theoretically be obtained from a system. $A = U - TS$ $\Delta A = \Delta U - T\Delta S$
H	enthalpy	Heat energy available in a chemical reaction. $H = U + PV$ $\Delta H = \Delta U + P\Delta V = \Delta U - W$
G	Gibbs free energy	Total energy available in a chemical reaction. $G = H - TS$ $\Delta G = \Delta H - T\Delta S$

Because this is a physics course, we will leave enthalpy and Gibbs free energy to the chemists.

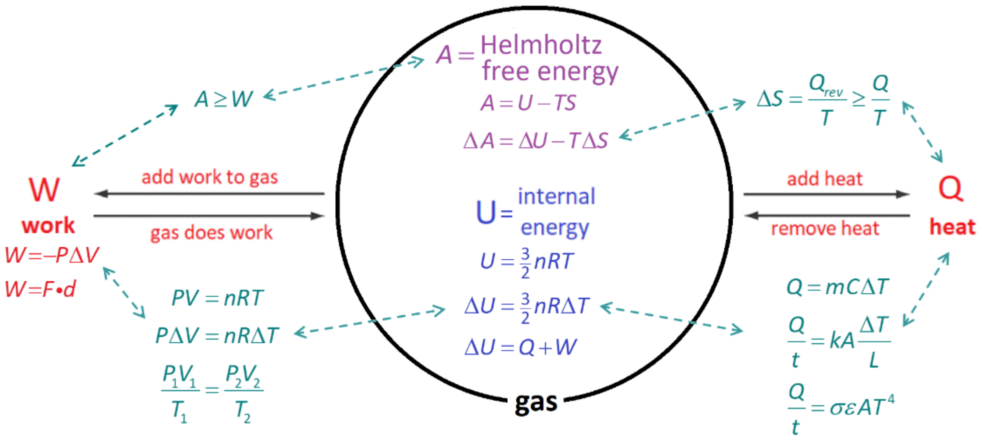
* Some chemistry textbooks use the variable *E* instead of *U*.

Thermodynamics Equations Used in This Course

Most of the thermodynamics problems encountered in this course are applications of the following equations:

Equation	Quantities that are Changing
$U = \frac{3}{2}nRT$ $\Delta U = \frac{3}{2}nR\Delta T$	internal energy *vs.* temperature
$\Delta U = Q + W$	internal energy *vs.* heat & work
$PV = nRT$ $P\Delta V = nR\Delta T$	pressure & volume *vs.* temperature
$W = -P\Delta V$ *	work *vs.* volume

What makes thermodynamics challenging is that there are many relationships between the quantities in these equations, as shown in the following thermodynamics equation map:



It is often necessary to combine equations. For example:

$$\Delta U = \frac{3}{2}nR\Delta T = Q + W$$

$$-W = P\Delta V = nR\Delta T$$

(Note that we moved the negative sign from $W = -P\Delta V$ to the other side of the equation.)

* In an algebra-based course, we need to restrict ourselves to problems in which the pressure remains constant during volume changes. In a calculus-based course, this equation would be $W = -\int P\,dV$, where *P* is a function of *V*.

The problems that you will encounter will involve a change in a measurable state variable (pressure, volume and/or temperature). To solve these problems, you will need to:

1. Determine what the change involves:
   - heat transfer (Q)
   - work (W) resulting from a change in volume (ΔV).
   - a change in internal energy (ΔU) resulting from a change in temperature (ΔT).

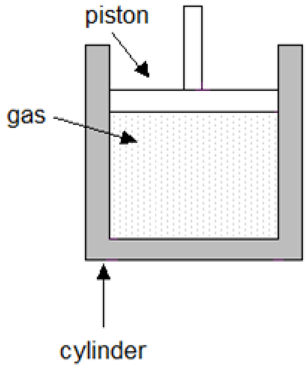
   (There can be more than one of these happening at the same time.)

2. If necessary, determine initial and/or final values of these state variables in relation to other variables using equations such as:

   - $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
   - $PV = nRT$

3. Apply algebraic combinations of these equations to find each of the necessary quantities to answer the question.

Homework Problems



Each of the problems below consists of a gas in a cylinder, as shown in the diagram to the right.

The gas in the cylinder has an initial volume of 0.01 m^3 and an initial temperature of 300 K. The piston has an area of 0.2 m^2, moves freely (with negligible friction), and has a weight of 250 N. Atmospheric pressure outside the cylinder is 100 000 Pa.

You may assume that the cylinder is perfectly insulated, which means the amount of heat that escapes from the cylinder in each situation is negligible.

1. **(M)** What is the initial pressure inside the cylinder? (*This is a fluids problem.*)

   Answer: 101 250 Pa

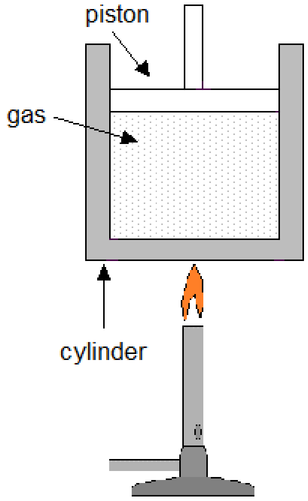
2. **(M)** How many moles of gas are in the cylinder?

   Answer: 0.406 mol

3. **(M)** What is the internal energy of the gas in the cylinder?

   Answer: 1 519 J

4. **(M)** A Bunsen burner is placed below the cylinder and is turned on. Heat is added to the gas (with the pressure remaining constant) until the volume increases to 0.012 m^3.



   a. What is the new temperature of the gas?

      Answer: 360 K

   b. What is the new internal energy of the gas?

      Answer: 1822.5 J

   c. How much work was done to raise the piston? (If you use $W = P\Delta V$, remember to use gauge pressure by subtracting the atmospheric pressure that is pushing down on the piston.)

      Answer: 2.5 J

   d. How much total heat was added to the gas?

      Answer: 306.25 J

5. **(M)** Suppose instead that the piston from problem #4 above was fixed in position and was not allowed to move, so the volume remains constant at 0.01 m^3 while the 306.25 J of heat from question 4d was added.

   a. What is the new temperature of the gas?

      Answer: 360.5 K

   b. What is the new pressure of the gas?

      Answer: 121 667 Pa

   c. If the piston is then released and allowed to move freely, what will the new pressure be inside the cylinder?

      Answer: 101 250 Pa (the same as in problem #1 above)

Pressure-Volume (PV) Diagrams

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.4.B, 9.4.B.2, 9.4.B.2.i, 9.4.B.ii, 9.4.B.3

Mastery Objective(s): (Students will be able to…)

- Determine changes in heat, work, internal energy and entropy from a pressure-volume (PV) diagram.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain what is physically happening to a gas for each section of a PV diagram.

Tier 2 Vocabulary: internal, energy, heat, work

Notes:

P-V diagram: a graph that shows changes in pressure *vs.* changes in volume.

Recall that:

$$W = -\int P\,dV$$

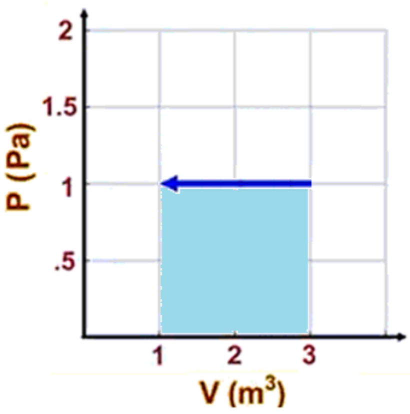
On a graph, the integral is the area "under the curve" (meaning the area between the curve and the *x*-axis).

Therefore, if we plot a graph of pressure *vs.* volume with pressure on the *y*-axis and volume on the *x*-axis, the integral would therefore be represented by the area between the curve (pressure) and the *x*-axis.

This means that *the work done by a thermodynamic change equals the area under a P-V graph*.*

* While the above explanation requires calculus, as stated earlier we will limit ourselves to areas that can be calculated using simple geometry equations. Note that some of these will result in situations that would not be realistically achievable in the "real world".

In the following example, suppose that a gas is compressed from 3 m^3 to 1 m^3 at a pressure of 1 Pa. (A pressure of 1 Pa is much smaller than you would encounter in any real problem; these numbers were chosen to keep the math simple.)



The pressure is P = 1 Pa, and the change in volume is ΔV = −2 m^3. Because pressure is constant, we can use $W = -P\Delta V = -(1)(-2) = +2\text{ J}$.

$P\Delta V$ is the area under the graph. Because it is a rectangular region, the area is the base of the rectangle times the height. The base is 2 m^3 and the height is 1 Pa, which gives an area of 2 J.

Note that the arrow showing the change points to the *left*, which indicates that the volume is *decreasing*. Because work must be put *into* the gas in order to compress it, this means that the work done ***on*** the gas will be positive.[*] This is where the negative sign comes from. $W = -P\Delta V$ means that:

- if work is done on the gas (work is positive), the gas is compressed and the change in volume is therefore negative.
- If work is done by the gas on the surroundings (work is negative), the gas expands and the change in volume is therefore positive.

We will look at the effects of changes in pressure *vs.* volume in four types of pressure-volume changes:

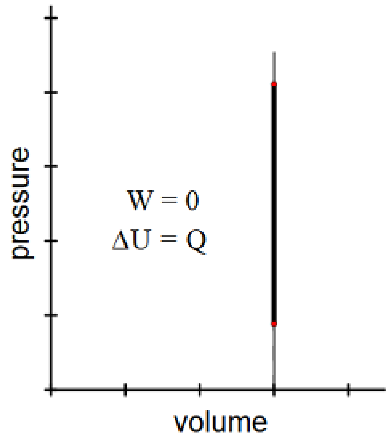
- isochoric (constant volume)
- isothermal (constant temperature)
- adiabatic (no heat loss)
- isobaric (constant pressure).

[*] Unless explicitly stated otherwise, positive work means work done ***on*** the gas, meaning that energy is added to the gas and the internal energy of the gas increases.

Isochoric

From Greek "iso" (same) and "khoros" (volume). An isochoric change is one in which volume remains constant, but pressure and temperature may vary.

An example is any rigid, closed container, such as a thermometer.



$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Because the volume is not changing, there is no way for the gas to displace anything. (Recall from Physics 1 that $W = \vec{F} \bullet \vec{d}$.) If there is no displacement, there is no work, which means $W = 0$.

$$\Delta U = Q + \cancelto{0}{W} = \frac{3}{2} nR\Delta T$$

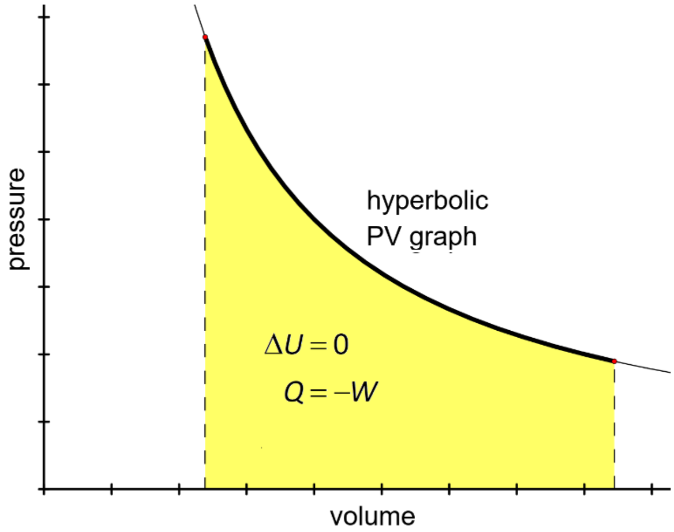
$$\Delta U = Q = \frac{3}{2} nR\Delta T$$

Another way to think of a constant volume change is that if you add heat to a rigid container of gas, none of the energy can be converted to work, so all of it must be converted to an increase in internal energy (*i.e.,* an increase in temperature).

Isothermal

Constant temperature. From Greek "iso" (same) and "thermotita" (heat). An isothermal change is one in which temperature remains constant, but pressure and volume may vary.

An example is any "slow" process, such as breathing out through a wide open mouth.



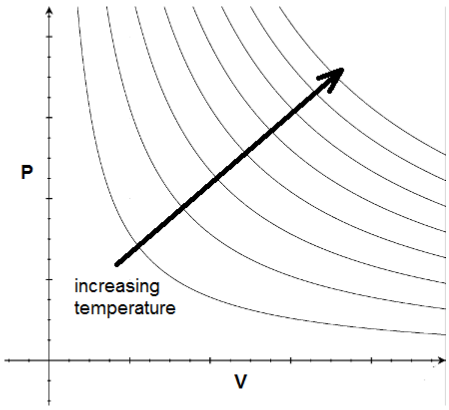
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow P_1V_1 = P_2V_2$$

Because $\Delta T = 0$ (definition of isothermal), this means

$$\Delta U = \frac{3}{2}nR\Delta T = 0$$

Further, because:

$$\Delta U = Q + W = 0$$
$$Q = -W$$



The P-V curve for an isothermal process is called an <u>isotherm</u>. Note that isotherms are hyperbolas, *i.e.,* they are solutions to the equation *PV* = constant. You may recall that allowing pressure and volume to change while keeping temperature constant is represented by Boyle's Law: $P_1V_1 = P_2V_2$

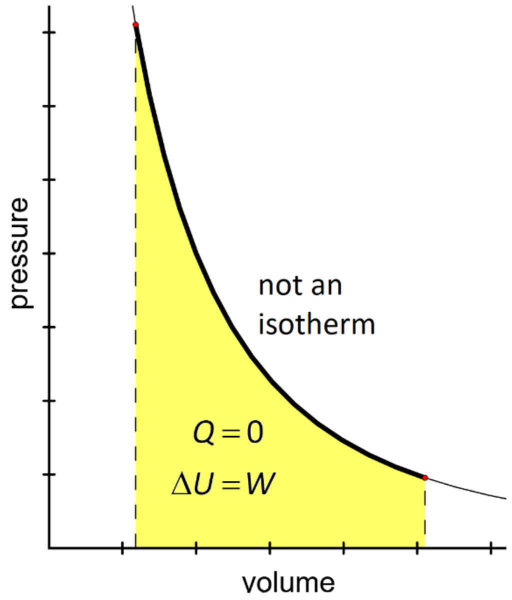
As temperature increases, the isotherm moves farther away from the origin.

Adiabatic

An adiabatic process is one in which there is no heat exchange with the environment. From Greek "a" (not) + "dia" (through) + "batos" (passable).

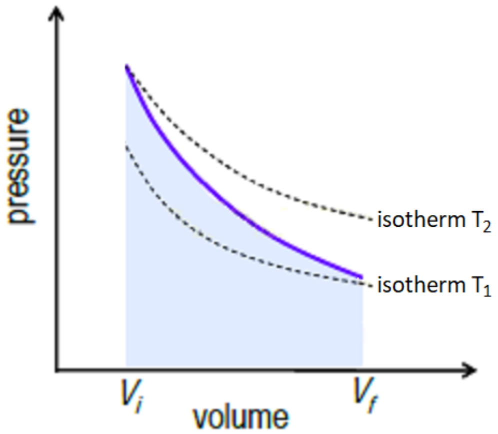
An example is any "fast" process, such as forcing air out through pursed lips or a bicycle tire pump.

Because the definition of an adiabatic process is one for which Q = 0, this means:



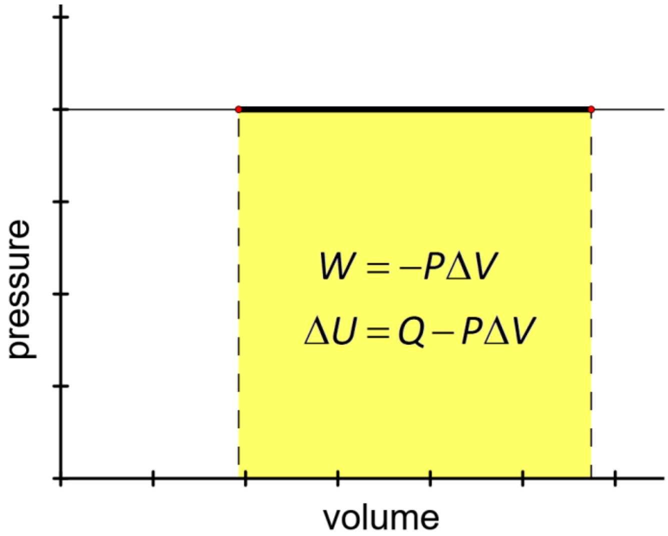
$$Q=0$$
$$\Delta U=\overset{0}{\cancel{Q}}+W=\tfrac{3}{2}nR\Delta T$$
$$\Delta U=W=\tfrac{3}{2}nR\Delta T$$

Note that adiabatic expansion (sudden increase in volume without time for heat transfer) results in a decrease in temperature, and adiabatic compression (sudden decrease in volume without time for heat transfer) results in an increase in temperature.



Isobaric

From Greek "iso" (same) and "baros" (weight). An isobaric change is one in which pressure remains constant, but volume and temperature may vary.



Some examples include a weighted piston, a flexible container in earth's atmosphere, or a hot air balloon.

$$\frac{\cancel{P_1} V_1}{T_1} = \frac{\cancel{P_2} V_2}{T_2} \rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Isobaric changes involve changes in both Q and W, because to change the volume of a gas while keeping pressure constant, you need to add or remove heat, but the resulting change in volume means that work is being done.

$$W = -\Delta(PV) = -P\Delta V$$
$$\Delta U = Q + W$$
$$\Delta U = Q + (-P\Delta V)$$
$$\Delta U = Q - P\Delta V$$

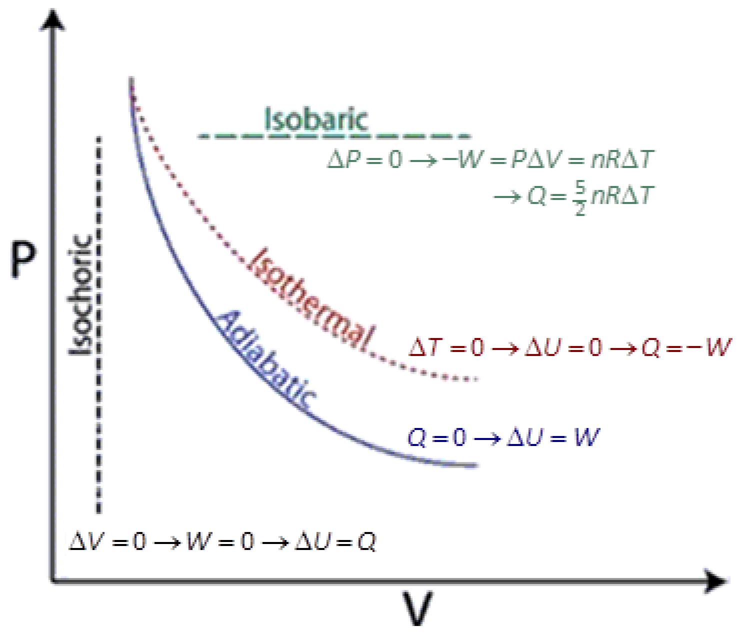
Adding $P\Delta V$ to both sides gives $Q = \Delta U + P\Delta V$.

Now, because $\Delta U = \frac{3}{2}nR\Delta T$ and $P\Delta V = nR\Delta T = \frac{2}{2}nR\Delta T$, that means:

$$Q = \Delta U + P\Delta V$$
$$\downarrow \qquad \downarrow$$
$$Q = \frac{3}{2}nR\Delta T + \frac{2}{2}nR\Delta T$$
$$Q = \frac{5}{2}nR\Delta T$$

This makes sense, because some of the heat is used to do the work of expanding the gas $(P\Delta V = nR\Delta T)$, and some of the heat is used to increase the temperature. $(\Delta U = \frac{3}{2}nR\Delta T)$.

If we wanted to compare all four processes on the same PV diagram, they would look like this:



Positive *vs.* Negative Work

In thermodynamics problems, whether work is represented by a positive or negative number depends on how the problem is stated.

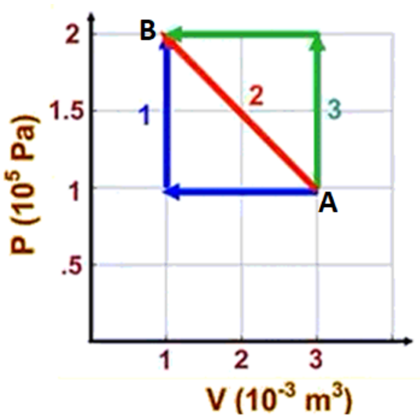
Work done ***on*** the gas: a positive number means work is coming from the surroundings into the gas.

Work done ***by*** the gas: a positive number means work is going from the gas out to the surroundings.

If the problem does not specify otherwise, the convention is to use a positive number to indicate work done on (*i.e.,* going into) the gas.

Sample Problem

Q: Calculate the work done as the pressure and volume of a gas are taken from point A to point B along each of paths 1, 2, and 3.



A: Process #1 is first isobaric (constant pressure), then isochoric (constant volume).

For the isobaric part of the process:

$$W = -P\Delta V$$
$$W = -(1\times10^{5})(1\times10^{-3} - 3\times10^{-3})$$
$$W = -(1\times10^{5})(-2\times10^{-3})$$
$$W = 2\times10^{2} = 200\,\text{J}$$

For the isochoric process, there is no change in volume, which means the gas does no work (because it cannot push against anything). Therefore *W* = 0.

The total work for process #1 is therefore 200 J.

Notice that the work is equal to the area under the PV graph, which is a rectangular area with a base of 2×10^{-3} m^3 and a height of 1×10^5 Pa.

$$W = (1\times10^{5})(2\times10^{-3}) = 200\text{ J}$$

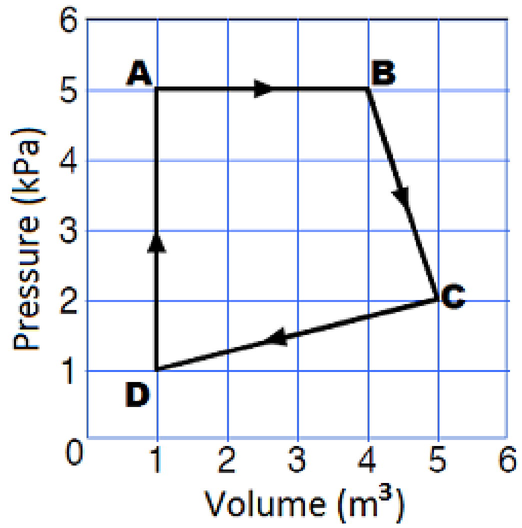
Note that because the arrow points to the left, this means the *volume is decreasing*. That means *work is being done on the gas*, which means the *work is represented by a positive number*. (We have to make this determination any time we use the graph to calculate the work.)

For process #2, the area is the 200 J square that we calculated for process #1 plus the area of the triangle above it, which is $\frac{1}{2}bh = \frac{1}{2}(2\times10^{-3})(1\times10^{5}) = 100\text{ J}$. Therefore, 200 J + 100 J = 300 J.

For process #3, the area under the curve is $W = (2\times10^{5})(2\times10^{-3}) = 400\text{ J}$.

Homework Problems*

Problems #1–8 refer to the following PV diagram, in which 2 moles of gas undergo the pressure and volume changes represented by the path from point A to B to C to D and back to A.



1. **(M)** Which thermodynamic process takes place along the path from point A to point B?

2. **(M)** Which thermodynamic process takes place along the path from point D to point A?

3. **(M)** How much work is done ***by the gas*** as it undergoes a change along the curve from point B → C? (Remember to use a positive number for work done on the gas by the surroundings, and a negative number for work done by the gas on the surroundings.)

   Answer: +3500 J

* These problems are from a worksheet by Tony Wayne.

4. **(S)** How much work is done ***on** the gas* as it undergoes a change along the curve from point C → D?

   Answer: +6000 J

5. **(S)** How much net work is done ***by** the gas* on the surroundings as it undergoes a change along the curve from point A → B → C → D → A?

   Answer: +12 500 J

6. **(S)** What is the temperature of the 2 moles of gas at point A?

   Answer: 300.8 K

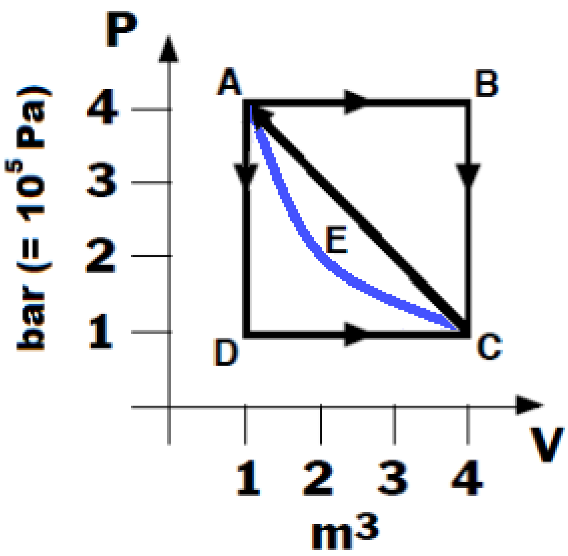
7. **(M)** What is the change in internal energy of the gas during the process from point D → A?

   Answer: 6000 J

8. **(M)** How much work is done on or by the gas during the process from point D → A?

   Answer: zero

Problems #9–13 refer to the following diagram:



9. **(S)** For which process(es) is $Q=\frac{5}{2}nR\Delta T$? Show calculations to justify your answer.

Answer: A → B and D → C

10. **(M)** For which process(es) is no work done? Explain.

Answer: A → D and B → C

11. **(M)** Which thermodynamic process takes place along path E?

12. **(M)** Calculate the heat exchanged in process A → B? Is heat added or released? Explain.

Answer: 3×10^{6} J; heat is added because the temperature increases.

13. **(M)** Does path A → D → C → E → A require more or less work than path A → D → C → A? Explain.

14. **(S)** Calculate the work done ***by** the gas* in processes A → B → C → A and A → D → C → A.

Answer: A → B → C → A: 450 000 J
A → D → C → A: −450 000 J

honors (not AP®)

Heat Engines

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the energy produced or used by a heat engine.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain what is happening to a gas through each of the steps of a heat engine cycle.

Tier 2 Vocabulary: internal, energy, heat, engine, work

Labs, Activities & Demonstrations:

- Stirling engine

Notes:

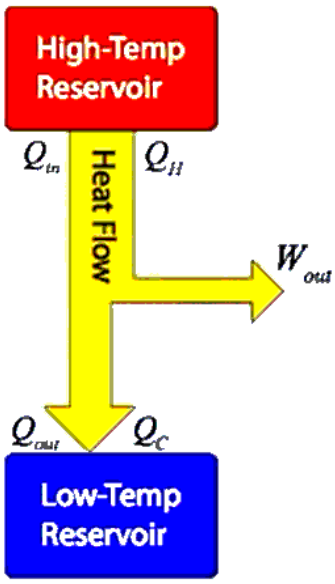
heat engine: a device that turns heat energy into mechanical work.

A heat engine operates by taking heat from a hot place (heat source), converting some of that heat into work, and dumping the rest of the heat into a cooler reservoir (heat sink).

A large number of the machines we use—most notably cars—employ heat engines.

honors (not AP®)

The basic principle of heat engine is the first law of thermodynamics (heat flows from a region of higher temperature to a region of lower temperature). Because heat is a form of energy, some of that energy can be harnessed to do work.

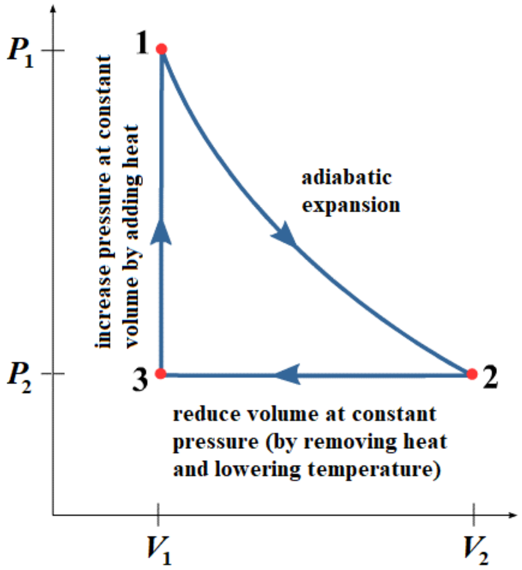


The law of conservation of energy tells us that all of the energy that we put into the heat engine must go somewhere. Therefore, the work done plus the heat that comes out must equal the heat we put in.

This means:

$$Q_{in} = Q_{out} + W_{out}$$

The above picture is easier to understand in the context of a PV diagram.



1. Starting from point 1 (V_1, P_1), the gas is expanded adiabatically (without losing heat) to point 2 (V_2, P_2). The area under the graph from point 1–2 represents the work that is done by the gas (W_{out}) as it expands and cools.

2. Ultimately, we need to get the gas back to point 1 so we can begin the cycle over again, but in a way that costs less work than we got out. There are many ways to accomplish this. In this example, the next step is to reduce the volume isobarically, by removing heat (Q_{out}). (This is what the low temperature reservoir is for.) This results in a decrease in temperature as well as a decrease in volume, which gets us to point 3 (V_1, P_2).

3. Now we increase the pressure to get from point 3 (V_1, P_2) to point 1 (V_1, P_1) by adding heat to the gas without letting the volume change. We do this by bringing the gas back to the high temperature reservoir so it can absorb the heat (Q_{in}).

honors (not AP®)

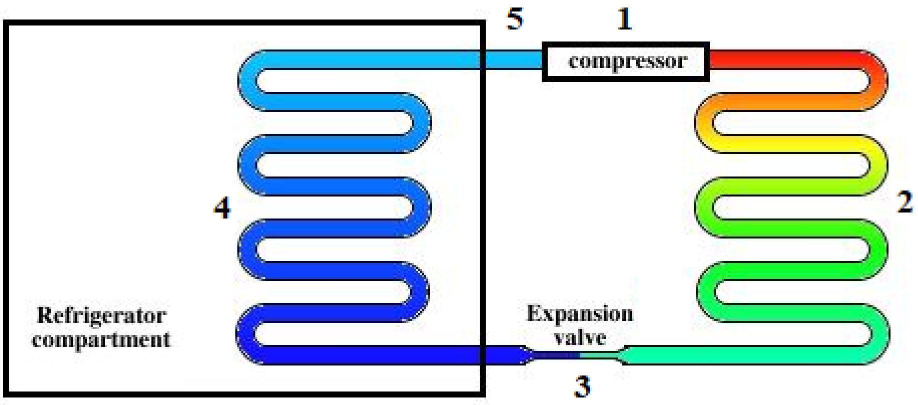
Heat Pumps

A heat pump is a device, similar to a heat engine, that "pumps" heat from one place to another. A refrigerator and an air conditioner are examples of heat pumps. A refrigerator uses a fluid ("refrigerant") to transfer (or "pump") heat from the inside of the refrigerator to the outside of the refrigerator (and into your kitchen).

This is why you can't cool off the kitchen by leaving the refrigerator door open—even if you had a 100 % efficient refrigerator (most refrigerators are actually only 20–40 % efficient), all of the heat that you pumped out of the refrigerator is still in the kitchen!

A refrigerator works by the following process.

1. Work is put in to compress a refrigerant (gas). In most cases, the gas is compressed until it turns into a liquid, which means additional energy is stored in the phase change. This increases the temperature of the refrigerant to about 70 °C.
2. The refrigerant (now a liquid) passes through cooling coils on the back of the refrigerator. The liquid is cooled through convection by the air in the kitchen to about 25 °C
3. The refrigerant (still a liquid) is pumped to the inside of the refrigerator and allowed to expand to a gas adiabatically. Work comes out of the gas, and the temperature drops to about −20 °C.
4. Heat is transferred via convection from the contents (the food) to the refrigerant.
5. The refrigerant (still a gas) is pumped out of the refrigerator, which brings us back to step 1.

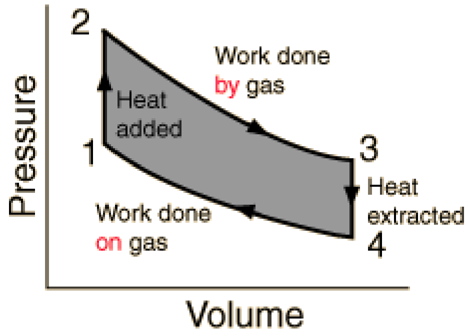


A heater can operate under the same principle, by putting the cooling coils inside the room and having the expansion (which cools the refrigerant) occur outside. Individual rooms in homes are sometimes heated and cooled by reversible heat pumps called "mini-splits".

honors (not AP®)

Heat Engines and PV Diagrams

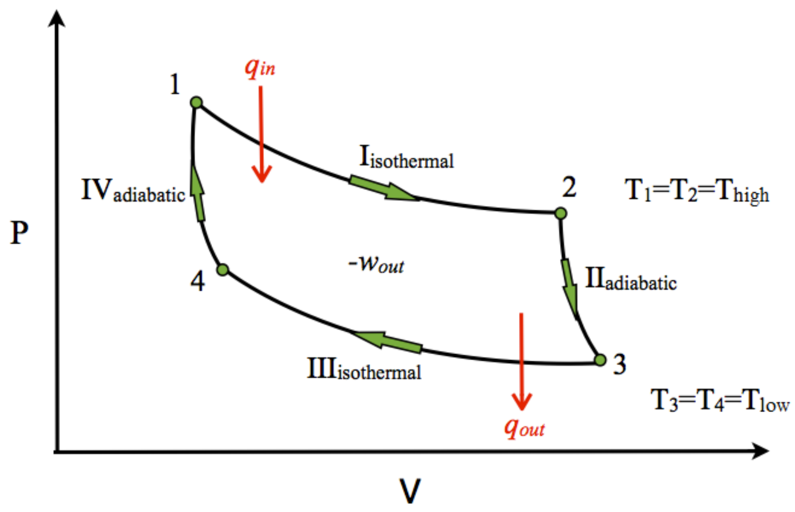
On a PV diagram, a heat engine is any closed loop or cycle:



Recall that on a PV diagram, a curve that moves from left to right represents work done *by* the gas on the surroundings. (Work is leaving the system, so $\Delta W < 0$.) A curve that moves from right to left represents work done *on* the gas by the surroundings. (Work is entering the system so $\Delta W > 0$.)

A heat engine is a clockwise cycle, which means more work is done going to the right than to the left, which means there is a net flow of work out of the system (*i.e.,* the heat is being used to do work). A refrigerator is a counterclockwise cycle, in which more work is put in and more heat is taken out.

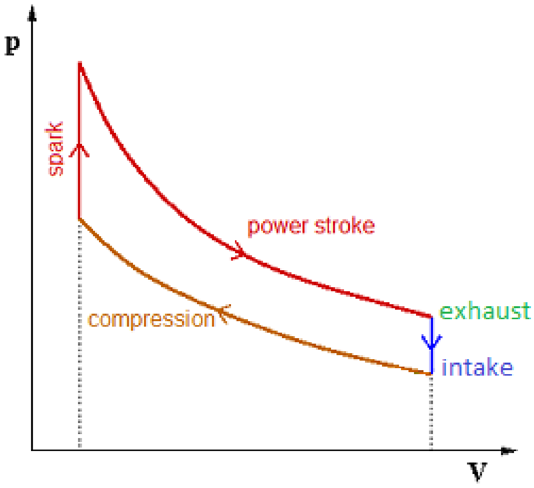
The Carnot cycle, named after the French physicist Nicolas Carnot, is the most efficient type of heat engine. The Carnot cycle, which uses only adiabatic (no heat loss) and isothermal processes, is the basis for heat pumps (including refrigerators and air conditioners) :



honors (not AP®)

The internal combustion engine in a car is also a type of heat engine. The engine is called a "four-stroke" engine, because the piston makes four strokes (a back or forth motion) in one complete cycle. The four strokes are:

1. The piston moves down (intake), sucking a mixture of gasoline and air into the cylinder.
2. The piston moves up (compression), compressing the gases in the cylinder.
3. The spark plug creates a spark, which combusts the gases. This increases the temperature in the cylinder to approximately 250°C.
4. The gas expands (power stroke), which is the work that the engine provides to make the car go.



5. The piston raises again, forcing the exhaust gases out of the cylinder (exhaust).

Note that, at the end of the cycle, the gas is hotter than its original temperature. The hot gas from the cylinder is dumped out the exhaust pipe, and fresh (cool) gas and fuel is added. This is why the blue intake arrow on the right moves downward—the intake is at a lower temperature (lower isotherm).

The energy to move the piston for the intake and exhaust strokes is provided by the power strokes of the other pistons.

This cycle—constant temperature compression, constant volume heating (spark), constant temperature expansion (power), and constant volume gas exchange (exhaust) is called the Otto cycle, named after Nikolaus August Otto, who used this type of heat engine to build the first commercially successful internal combustion engine.

honors (not AP®)

Efficiency

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the efficiency of a thermodynamic process.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain how the efficiency of a process relates to the energy it uses and the work it produces.

Tier 2 Vocabulary: energy, heat, work

Notes:

efficiency (η): the ratio of the energy consumed by a device or process to the energy output by the device or process.

Assume that a heat engine starts with a certain temperature, which means a certain internal energy (U). The engine takes heat from a heat source at the incoming temperature T_{in}, does work (W), and exhausts heat at the higher temperature T_{out}. Assuming the internal energy of the machine itself stays constant, this means $\Delta U = 0$. Therefore, from the First law:

$$\Delta U = 0 = \Delta Q - \Delta W$$
$$0 = Q_{in} - Q_{out} - \Delta W$$
$$\Delta W = Q_{in} - Q_{out}$$

A 100% efficient heat engine would turn all of the heat into work, and would exhaust no heat ($Q_{out} = 0$, which would mean $\Delta W = Q_{in}$). Of course, real engines cannot do this, so we define efficiency, *e*, as the ratio of work out to heat in, *i.e.*:

$$e = \frac{\Delta W}{\Delta Q_{in}} = \frac{\Delta Q_{in} - \Delta Q_{out}}{\Delta Q_{in}} = \frac{\Delta Q_{in}}{\Delta Q_{in}} - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}}$$

Because the engine is doing work, $\Delta W > 0$, which means $0 \le e \le 1$. Furthermore, a consequence of the Second law is that some energy is always lost to the surroundings (entropy), which means $Q_{out} > 0$ and therefore $0 \le e < 1$.

honors (not AP®)

Sample Problem

Q: 80. J of heat is injected into a heat engine, causing it to do work. The engine then exhausts 20. J of heat into a cool reservoir. What is the efficiency of the engine?

A: $Q_{in} = 80\,\text{J}$ and $Q_{out} = 20\,\text{J}$. Therefore:

$$e = 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{20}{80} = 1 - 0.25 = 0.75$$

Because efficiency is usually expressed as a percentage, we would say that the engine is 75% efficient.

The following table gives energy conversion efficiencies for common devices and processes. In all of these cases, the "lost" energy is converted to heat that is given off to the surroundings.

Energy Conversion Efficiency

Device/Process	Typical Efficiency
gas generator	up to 40%
coal/gas-fired power plant	45%
combined cycle power plant	60%
hydroelectric power plant	up to 90%
wind turbine	up to 59%
solar cell	6–40%; usually 15%
hydrogen fuel cell	up to 85%
internal combustion engine	25%
electric motor, small (10–200 W)	50–90%
electric motor, large (> 200 W)	70–99%
photosynthesis in plants	up to 6%
human muscle	14–27%
refrigerator	20%
refrigerator, energy-saving	40–50%
light bulb, incandescent	0.7–5%
light bulb, fluorescent	8–16%
light bulb, LED	4–15%
electric heater	100%
firearm	30%

honors (not AP®)

You may notice that an electric heater is 100% efficient, because all of its energy is converted to heat. However, this does not mean that electric heat is necessarily the best choice for your home, because the power plant that generated the electricity is probably only 45% efficient.

Heating Efficiency

Heating efficiency is calculated in a similar way. The difference is that the energy produced by the heater *is* Q_{out}, which means:

$$\eta = \frac{Q_{out}}{Q_{in}} = \frac{\text{usable heat out}}{\text{total energy in}}$$

"Usable heat out" means heat that is not lost to the environment. For example, if the boiler or furnace in your house is 70% efficient, that means 70% of the energy from the gas or oil that it burned was used to heat the steam, hot water or hot air that was used to heat your house. The other 30% of the energy heated the air in the boiler or furnace, and that heat was lost to the surroundings when the hot air went up the chimney.

Older boilers and furnaces (pre-1990s) were typically 70% efficient. Newer boilers and furnaces are around 80% efficient, and high-efficiency boilers and furnaces that use heat exchangers to collect the heat from the exhaust air before it goes up the chimney can be 90−97% efficient.

Efficiency of a Heat Pump

Carnot's theorem states that the maximum possible efficiency of a heat pump is related to the ratio of the temperature of the heat transfer fluid (liquid or gas) when it enters the heat pump to the temperature when it exits:

$$\eta \le 1 - \frac{T_{in}}{T_{out}}$$

Note that the Carnot equation is really the same as the efficiency equation in the previous section. Recall that for heating or cooling a substance:

$$Q = mC\Delta T = mC(T_{out} - T_{in})$$

The refrigerant is the same substance, which means mC is the same for the input as for the output, and it drops out of the equation.

honors (not AP®)

It is a little counter-intuitive that a higher temperature difference means the heat pump is more efficient, but you should think about this in terms of heat transfer. (See the section on Heat Transfer starting on page 45.) Recall from Fourier's Law of Conduction that a higher temperature difference means a higher rate of heat transfer from one side to the other. In other words, the more heat you pump into the refrigerant, the higher its temperature will be when it leaves the system, and therefore the more efficiently the pump is moving heat. Conversely, if $T_{out} = T_{in}$, then the heat pump cannot transfer any heat, and the efficiency is zero.

Sample Problem

Q: Refrigerant enters a heat pump at 20. °C (293 K) and exits at 300. °C (573 K). What is the Carnot efficiency of this heat pump?

A: Carnot's equation states that:

$$\eta = 1 - \frac{T_{in}}{T_{out}} = 1 - \frac{293}{573}$$

$$\eta = 1 - 0.51 = 0.49$$

I.e., this heat pump is 49% efficient.

Introduction: Electric Force, Field & Potential

Unit: Electric Force, Field & Potential

Topics covered in this chapter:

This chapter discusses static electric charges, how they behave, and how they relate to each other.

- *Electric Charge* and *Coulomb's Law* describe the behavior of individual charged particles and their effects on each other.
- *Electric Fields* describes the behavior of an electric force field on charged particles.
- *Electric Field Vectors* and *Equipotential Lines & Maps* describe ways of representing electric fields in two dimensions.

Standards addressed in this chapter:

NGSS Standards/MA Curriculum Frameworks (2016):

HS-PS2-4. Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

HS-PS3-1. Use algebraic expressions and the principle of energy conservation to calculate the change in energy of one component of a system when the change in energy of the other component(s) of the system, as well as the total energy of the system including any energy entering or leaving the system, is known. Identify any transformations from one form of energy to another, including thermal, kinetic, gravitational, magnetic, or electrical energy, in the system.

HS-PS3-2. Develop and use a model to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles and objects or energy stored in fields.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

10.1.A: Describe the electric force that results from the interactions between charged objects or systems.

10.1.A.1: Charge is a fundamental property of all matter.

10.1.A.1.i: Charge is described as positive or negative.

10.1.A.1.ii: The magnitude of the charge of a single electron or proton, the elementary charge *e*, can be considered to be the smallest indivisible amount of charge.

10.1.A.1.iii: The charge of an electron is $-e$, the charge of a proton is $+e$, and a neutron has no electric charge.

10.1.A.1.iv: A point charge is a model in which the physical size of a charged object or system is negligible in the context of the situation being analyzed.

10.1.A.2: Coulomb's law describes the electrostatic force between two charged objects as directly proportional to the magnitude of each of the charges and inversely proportional to the square of the distance between the objects.

10.1.A.3: The direction of the electrostatic force depends on the signs of the charges of the interacting objects and is parallel to the line of separation between the objects.

10.1.A.3.i: Two objects with charges of the same sign exert repulsive forces on each other.

10.1.A.3.ii: Two objects with charges of opposite signs exert attractive forces on each other.

10.1.A.4: Electric forces are responsible for some of the macroscopic properties of objects in everyday experiences. However, the large number of particle interactions that occur make it more convenient to treat everyday forces in terms of nonfundamental forces called contact forces, such as normal force, friction, and tension.

10.1.B: Describe the electric and gravitational forces that result from interactions between charged objects with mass.

10.1.B.1: Electrostatic forces can be attractive or repulsive, while gravitational forces are always attractive.

AP® only

10.1.B.2: For any two objects that have mass and electric charge, the magnitude of the gravitational force is usually much smaller than the magnitude of the electrostatic force.

10.1.B.3: Gravitational forces dominate at larger scales even though they are weaker than electrostatic forces, because systems at large scales tend to be electrically neutral.

10.1.C: Describe the electric permittivity of a material or medium.

10.1.C.1: Electric permittivity is a measurement of the degree to which a material or medium is polarized in the presence of an electric field.

10.1.C.2: Electric polarization can be modeled as the induced rearrangement of electrons by an external electric field, resulting in a separation of positive and negative charges within a material or medium.

10.1.C.3: Free space has a constant value of electric permittivity, ε_o, that appears in physical relationships.

10.1.C.4: The permittivity of matter has a value different from that of free space that arises from the matter's composition and arrangement.

10.1.C.4.i: In a given material, electric permittivity is determined by the ease with which electrons can change configurations within the material.

10.1.C.4.ii: Conductors are made from electrically conducting materials in which charge carriers move easily; insulators are made from electrically nonconducting materials in which charge carriers cannot move easily.

10.2.A: Describe the behavior of a system using conservation of charge.

10.2.A.1: The net charge or charge distribution of a system can change in response to the presence of, or changes in, the net charge or charge distribution of other systems.

10.2.A.1.i: The net charge of a system can change due to friction or contact between systems.

10.2.A.1.ii: Induced charge separation occurs when the electrostatic force between two systems alters the distribution of charges within the systems, resulting in the polarization of one or both systems.

10.2.A.1.iii: Induced charge separation can occur in neutral systems.

10.2.A.2: Any change to a system's net charge is due to a transfer of charge between the system and its surroundings.

10.2.A.2.i: The charging of a system typically involves the transfer of electrons to and from the system.

10.2.A.2.ii: The net charge of a system will be constant unless there is a transfer of charge to or from the system.

AP® only

10.2.A.3: Grounding involves electrically connecting a charged system to a much larger and approximately neutral system (*e.g.,* Earth).

10.3.A: Describe the electric field produced by a charged object or configuration of point charges.

10.3.A.1: Electric fields may originate from charged objects.

10.3.A.2: The electric field at a given point is the ratio of the electric force exerted on a test charge at that point to the charge of the test charge.

10.3.A.2.i: A test charge is a point charge of small enough magnitude such that its presence does not significantly affect an electric field in its vicinity.

10.3.A.2.ii: An electric field points away from isolated positive charges and toward isolated negative charges.

10.3.A.2.iii: The electric force exerted on a positive test charge by an electric field is in the same direction as the electric field.

10.3.A.3: The electric field is a vector quantity and can be represented in space using vector field maps

10.3.A.3.i: The net electric field at a given location is the vector sum of individual electric fields created by nearby charged objects.

10.3.A.3.ii: Electric field maps use vectors to depict the magnitude and direction of the electric field at many locations within a given region.

10.3.A.3.iii: Electric field line diagrams are simplified models of electric field maps and can be used to determine the relative magnitude and direction of the electric field at any position in the diagram.

10.3.B: Describe the electric field generated by charged conductors or insulators.

10.3.B.1: While in electrostatic equilibrium, the excess charge of a solid conductor is distributed on the surface of the conductor, and the electric field within the conductor is zero.

10.3.B.1.i: At the surface of a charged conductor, the electric field is perpendicular to the surface.

10.3.B.1.ii: The electric field outside an isolated sphere with spherically symmetric charge distribution is the same as the electric field due to a point charge with the same net charge as the sphere located at the center of the sphere.

10.3.B.2: While in electrostatic equilibrium, the excess charge of an insulator is distributed throughout the interior of the insulator as well as at the surface, and the electric field within the insulator may have a nonzero value.

AP® only

10.4.A: Describe the electric potential energy of a system.

10.4.A.1: The electric potential energy of a system of two point charges equals the amount of work required for an external force to bring the point charges to their current positions from infinitely far away.

10.4.A.2: The general form for the electric potential energy of two charged objects is given by the equation $U_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}$.

10.4.A.3: The total electric potential energy of a system can be determined by finding the sum of the electric potential energies of the individual interactions between each pair of charged objects in the system.

10.5.A: Describe the electric potential due to a configuration of charged objects.

10.5.A.1: Electric potential describes the electric potential energy per unit charge at a point in space.

10.5.A.2: The electric potential due to multiple point charges can be determined by the principle of scalar superposition of the electric potential due to each of the point charges.

10.5.A.3: The electric potential difference between two points is the change in electric potential energy per unit charge when a test charge is moved between the two points.

10.5.A.3.i: Electric potential difference may also result from chemical processes that cause positive and negative charges to separate, such as in a battery.

10.5.A.4: When conductors are in electrical contact, electrons will be redistributed such that the surfaces of the conductors are at the same electric potential.

10.5.B: Describe the relationship between electric potential and electric field.

10.5.B.1: The average electric field between two points in space is equal to the electric potential difference between the two points divided by the distance between the two points.

10.5.B.2: Electric field vector maps and equipotential lines are tools to describe the field produced by a charge or configuration of charges and can be used to predict the motion of charged objects in the field.

10.5.B.2.i: Equipotential lines represent lines of equal electric potential in space. These lines are also referred to as isolines of electric potential.

10.5.B.2.ii: Isolines are perpendicular to electric field vectors. An isoline map of electric potential can be constructed from an electric field vector map, and an electric field map may be constructed from an isoline map.

10.5.B.2.iii: An electric field vector points in the direction of decreasing potential.

AP® only

10.5.B.2.iv: There is no component of an electric field along an isoline.

10.7.A: Describe changes in energy in a system due to a difference in electric potential between two locations.

10.7.A.1: When a charged object moves between two locations with different electric potentials, the resulting change in the electric potential energy of the object-field system is given by the following equation.

10.7.A.2: The movement of a charged object between two points with different electric potentials results in a change in kinetic energy of the object consistent with the conservation of energy.

Skills learned & applied in this chapter:

- Working with isolines.

Electric Charge

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.1.A, 10.1.A.1, 10.1.A.1.i, 10.1.A.1.ii, 10.1.A.1.iii, 10.1.A.iv, 10.2.A, 10.2.A.1, 10.2.A.1.i, 10.2.A.1.ii, 10.2.A.1.iii, 10.2.A.2, 10.2.A.2.i, 10.2.A.2.ii, 10.2.A.3

Mastery Objective(s): (Students will be able to…)

- Describe properties of positive and negative electric charges.
- Describe properties of conductors and insulators.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the mass of the pendulum does not affect its period.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- charged balloon making hairs repel, attracting water molecules.
- charged balloon sticking to wall (draw on one side of balloon to show that charges do not move)
- charged balloon pulling meter stick
- build & demonstrate electroscope
- Wimshurst machine
- Van de Graaff generator

Notes:

<u>charge:</u>

1. A physical property of matter which causes it to experience a force when near other electrically charged matter. (Sometimes called "electric charge".) Measured in coulombs (C).
2. A single microscopic object (such as a proton or electron) that carries an electric charge. (Sometimes called a "point charge.) Denoted by the variable *q*.
3. The total amount of electric charge on a macroscopic object (caused by an accumulation of microscopic charged objects). Denoted by the variable *Q*.

4. (verb) To cause an object to acquire an electric charge.

positive charge: the charge of a proton. Originally defined as the charge left on a piece of glass when rubbed with silk. The glass becomes positively charged because the silk pulls electrons off the glass.

negative charge: the charge of an electron. Originally defined as the charge left on a piece of amber (or rubber) when rubbed with fur (or wool). The amber becomes negatively charged because the amber pulls the electrons off the fur.

static electricity: stationary electric charge, such as the charge left on silk or amber in the above definitions.

elementary charge: the magnitude (amount) of charge on one proton or one electron. One elementary charge equals 1.60×10^{-19} C. Because ordinary matter is made of protons and electrons, the amount of charge carried by any object must be an integer multiple of the elementary charge.

Note however that quarks, which protons and neutrons are made of, carry fractional charges; up-type quarks carry a charge of +⅔ of an elementary charge, and down-type quarks carry a charge of −⅓ of an elementary charge. A proton is made of two up quarks and one down quark and carries a charge of +1 elementary charge. A neutron is made of one up quark and two down quarks and carries no charge.

<u>electric current</u> (sometimes called electricity): the movement of electrons through a medium (substance) from one location to another. Note, however, that electric current is defined as the direction a *positively* charged particle would move. Thus electric current "flows" in the opposite direction from the actual electrons.

WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.

Some Devices that Produce, Use or Store Charge

<u>capacitor</u>: a device that stores electric charge.

<u>battery</u>: a device that uses chemical reactions to produce an electric current.

<u>generator</u>: a device that converts mechanical energy (motion) into an electric current.

<u>motor</u>: a device that converts an electric current into mechanical energy.

Conductors *vs.* Insulators

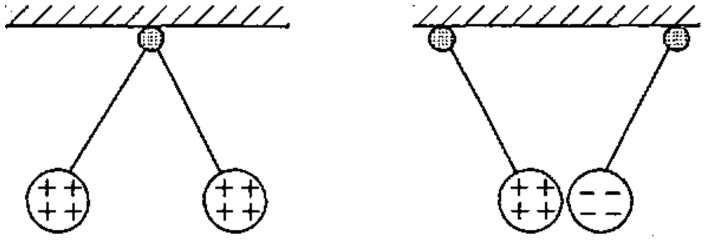
<u>conductor</u>: a material that allows charges to move freely through it. Examples of conductors include metals and liquids with positive and negative ions dissolved in them (such as salt water). When charges are transferred to a conductor, the charges distribute themselves evenly throughout the substance.

<u>insulator</u>: a material that does not allow charges to move freely through it. Examples of insulators include nonmetals and most pure chemical compounds (such as glass or plastic). When charges are transferred to an insulator, they cannot move, and remain where they are placed.

Behavior of Charged Particles

- **Like charges repel.** A pair of the same type of charge (two positive charges or two negative charges) exert a force that pushes the charges away from each other.

- **Opposite charges attract.** A pair of opposite types of charge (a positive charge and a negative charge) exert a force that pulls the charges toward each other.



Note that if you were to place a charge (either positive or negative) on a solid metal sphere, the charges would repel, and the result would be that the charges would be spread equally over the *outside* surface, but not inside the sphere.

Conservation of Charge

Because electric charges are an inherent property of subatomic particles (protons and electrons), and because mass is conserved (as you should have learned in chemistry), charge is therefore also a conserved quantity.

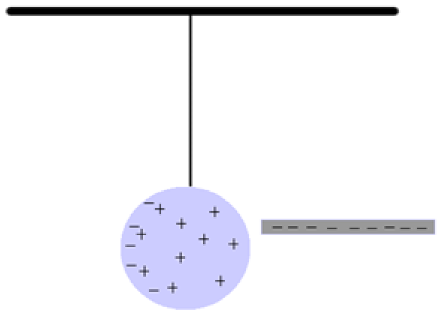
This means that electric charges can be moved from one object to another, but cannot be created or destroyed.

This means that the net charge on a system is constent unless electrons are transferred to or from the surroundings. For example, if you wear shoes with rubber soles and scuff your feet on a carpet, this action transfers electrons from the carpet to your shoes.

Charging by Induction

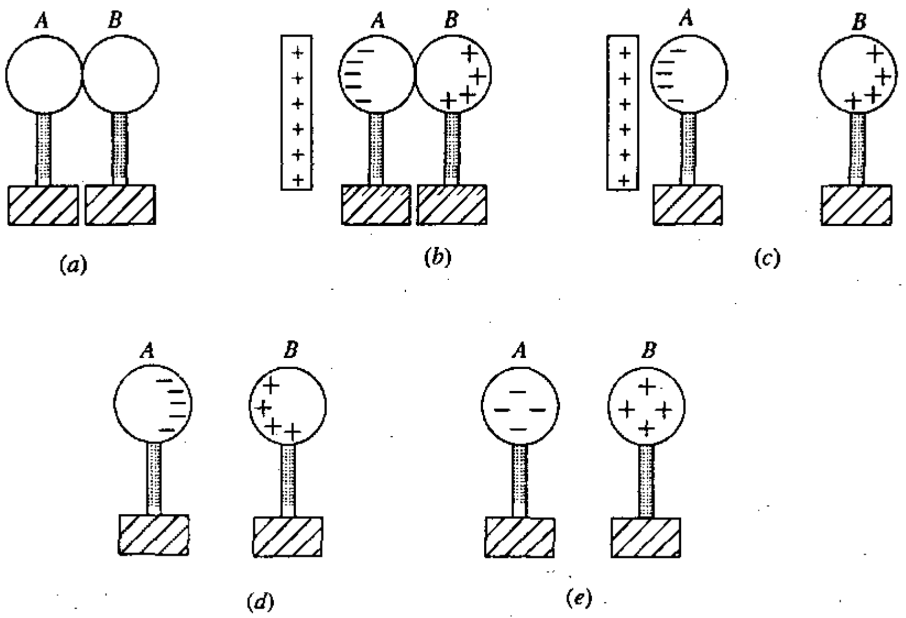
<u>induction</u>: when an electrical charge on one object causes a charge in a second object.

When a charged rod is brought near a neutral object, the charge on the rod attracts opposite charges and repels like charges that are near it. The diagram to the right shows a negatively-charged rod repelling negative charges.



If the negatively-charged rod were touched to the sphere, some of the charges from the rod would be transferred to the sphere at the point of contact. This would cause the sphere to have an overall negative charge.

A procedure for inducing charges in a pair of metal spheres is shown below:



(a) Metal spheres *A* and *B* are brought into contact.

(b) A positively charged object is placed near (but not in contact with) sphere *A*. This induces a negative charge in sphere *A*, which in turn induces a positive charge in sphere *B*.

(c) Sphere *B* (which is now positively charged) is moved away.

(d) The positively charged object is removed.

(e) The charges distribute themselves throughout the metal spheres.

honors (not AP®)

Charge Density

The amount of electric charge on a surface is called the charge density. As with density (in the mass/volume sense), the variable used is usually the Greek letter rho with a subscript *q* indicating charge (ρ_q). Charge density can be expressed in terms of length, area, or volume, which means, the units for charge density can be $\frac{C}{m}$, $\frac{C}{m^2}$, or $\frac{C}{m^3}$.

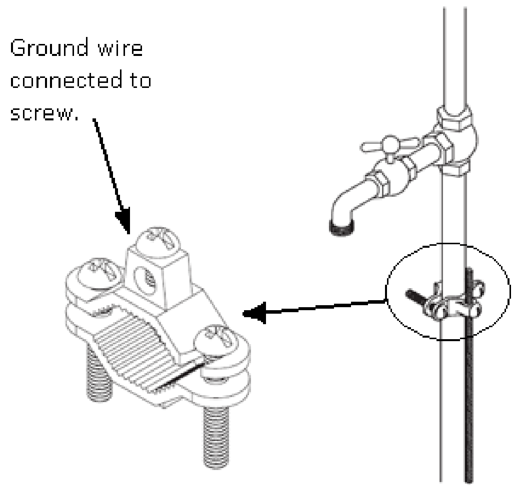
Grounding

For the purposes of our use of electric charges, the ground (Earth) is effectively an endless supply of both positive and negative charges. Under normal circumstances, if a charged object is touched to the ground, electrons will move to neutralize the charge, either by flowing from the object to the ground or from the ground to the object.

Grounding a charged object or circuit means neutralizing the electrical charge on an object or portion of the circuit by connecting it to a much larger and approximately neutral system, such as the Earth.

The charge of any object that is connected to ground is zero, by definition.

The term "grounding" comes from the fact that this is often accomplished by connecting the system via a wire to a metal pipe or stake that is partially or fully buried in the ground.



In buildings, the metal pipes that bring water into the building are often used to ground the electrical circuits. The metal pipe is a good conductor of electricity, and carries the unwanted charge out of the building and into the ground outside.

Electric Permittivity

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.1.A, 10.1.A.2, , 10.1.A.3, 10.1.A.3.i, 10.1.A.3.ii, 10.1.A.4, 10.1.B, 10.1.B.1, 10.1.B.2, 10.1.B.3

Mastery Objective(s): (Students will be able to…)

- Solve problems using Coulomb's Law
- Quantitatively predict the effects on the electrostatic force when one of the variables (amount of electric charge or distance) in Coulomb's Law is changed.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how force and distance both affect the amount of force between two charged objects.

Tier 2 Vocabulary: charge

Notes:

force field: a region in which objects experience a force that is proportional to the amount that the object has of the property that the field acts upon.

electric field: a force field that acts upon objects with a nonzero electrical charge.

For example, in physics 1, you learned about the gravitational force field, which acts upon objects with mass. In the *Electric Fields* topic, starting on page 171, you will learn more about electric fields, which act upon objects with charge.

polarized: an object or region that has opposite values of some property at each end. An object is electrically polarized if one end is charged more positively or less negatively than the other.

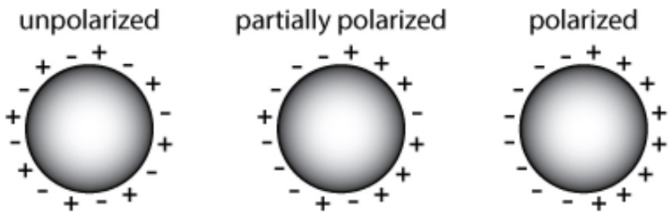


Image © 2012 by Jeff Cruzan, xaktly.com. Used with permission.

electric permittivity: the degree to which a material or medium is polarized in the presence of an electric field.

permittivity of free space (ε_o): the degree to which empty space (a vacuum) can support an electric field. The permittivity of free space is defined in terms of two constants: the magnetic permeability of a vacuum (μ_o; see *Magnetism*, starting on page 282) and the speed of light (c).)

$$\varepsilon_o = \frac{1}{\mu_o c^2} = 8.85\times10^{-12}\ \tfrac{\text{F}}{\text{m}}$$

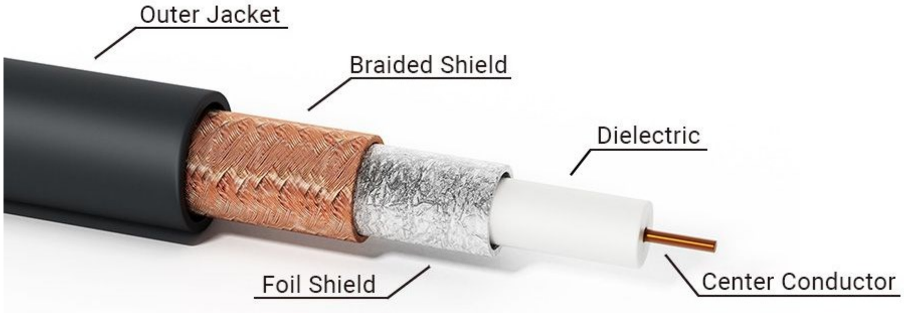
relative permittivity (dielectric constant) (κ* or ε_r): the ratio of the electric permittivity of a substance to the electric permittivity of empty space.

A vacuum (empty space) has a relative permittivity of 1.

Relative permittivity is most often used to determine the ability of an insulating material to prevent electric charges from moving through itself. The lower the relative permittivity, the better the insulator.

Substance	κ	Substance	κ
vacuum	1	rubber	~7
air	1.0006	ethanol @ 25 °C	24.3
paper	1.4	methanol @ 25 °C	32.7
polyethylene	2.25	water @ 25 °C	78.4
silicon dioxide	3.9	calcium copper titanate	>250 000

Polyethylene is often used as a dielectric (insulator) between the center conductor and the outside shield in coaxial cables.



* Note that κ is the Greek letter "kappa," not the Roman letter "k".

Coulomb's Law

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.1.A, 10.1.A.2, , 10.1.A.3, 10.1.A.3.i, 10.1.A.3.ii, 10.1.A.4, 10.1.B, 10.1.B.1, 10.1.B.2, 10.1.B.3

Mastery Objective(s): (Students will be able to…)

- Solve problems using Coulomb's Law
- Quantitatively predict the effects on the electrostatic force when one of the variables (amount of electric charge or distance) in Coulomb's Law is changed.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how force and distance both affect the amount of force between two charged objects.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- Charged balloon or Styrofoam sticking to wall.
- Charged balloon pushing meter stick.
- Van de Graaff generator with negative electrode attached to inertia balance pan.

Notes:

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

+1 C is the charge of 6.2415×10^{18} protons.
−1 C is the charge of 6.2415×10^{18} electrons.

A single proton or electron therefore has a charge of $\pm 1.6022 \times 10^{-19}$ C. This amount of charge is called the elementary charge, because it is the charge of one elementary particle.

An object can only have an integer multiple of this amount of charge, because it is impossible* to have a charge that is a fraction of a proton or electron.

Because charged particles attract or repel each other, that attraction or repulsion must be a force, which can be measured and quantified. The force is directly proportional to the strengths of the charges, and inversely proportional to the square of the distance. The formula is:

$$F_e = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{r^2} = k\frac{q_1 q_2}{r^2}$$

where:

F_e = electrostatic force of repulsion between electric charges. A positive value of F_e denotes that the charges are repelling (pushing away from) each other; a negative value of F_e denotes that the charges are attracting (pulling towards) each other.†

ε_o = electric permittivity of free space $\approx 8.85 \times 10^{-12}\ \frac{\text{F}}{\text{m}}$.

k = electrostatic constant = $= \frac{1}{4\pi\varepsilon_o} \approx 9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$.

q_1 , q_2 = the charges on objects #1 and #2 respectively

r = distance (radius, because it goes outward in every direction) between the centers of the two charges

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Coulomb's Law has several parallels with Newton's Law of Universal Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

gravitational force

$$F_e = k\frac{q_1 q_2}{r^2}$$

electrostatic force

The situations are similar in that there are two objects, each exerting a force on the other separated by some distance, *r*. With the gravitational force, masses can only attract, which means F_g is always attractive. However, F_e can be either attractive or repulsive, because charges attract or repel, depending on whether they have opposite or like signs. If the charges have the same sign (repulsive), the value of F_e will be positive; if the charges have opposite signs (attractive), the value of F_e will be negative.

---

* This is true for macroscopic objects. Certain quarks, which are the particles that protons and neutrons are made of, have charges of $\frac{1}{3}$ or $\frac{2}{3}$ of an elementary charge.

† It is unfortunate that a positive value for force denotes attraction in the gravitational force equation, but repulsion in the electrostatic force equation.

Sample problems:

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is 1.60×10^{-19} C, and the charge of a single electron is -1.60×10^{-19} C.

$$37.1\,\text{pm} = 3.71 \times 10^{-11}\,\text{m}$$

$$F_e = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9)(1.60 \times 10^{-19})(-1.60 \times 10^{-19})}{(3.71 \times 10^{-11})^2} = -1.67 \times 10^{-7}\,\text{N}$$

The value of the force is negative, which signifies that the force is attractive. However, rather than memorize whether a positive or negative indicates attraction or repulsion, it's easier to reason that the charges are opposite, so the objects attract. *Never memorize what you can understand!*

Q: Two charged particles, each with charge $+q$ (which means $q_1 = q_2 = q$) are separated by distance *d*. If the amount of charge on one of the particles is halved and the distance is doubled, what will be the effect on the force between them?

A: To solve this problem, we first set up Coulomb's Law:

$$F_e = \frac{kq_1q_2}{r^2}$$

Now, we replace one of the charges with half of itself—let's say q_1 will become $(0.5\,q_1)$. Similarly, we replace the distance *r* with $(2r)$. This gives:

$$F_e = \frac{k(0.5q_1)q_2}{(2r)^2}$$

Simplifying and rearranging this expression gives:

$$F_e = \frac{0.5kq_1q_2}{4r^2} = \frac{0.5}{4} \cdot \frac{kq_1q_2}{r^2} = \frac{1}{8} \cdot \frac{kq_1q_2}{r^2}$$

Therefore, the new F_e will be $\frac{1}{8}$ of the old F_e.

An easier way to solve this problem is to do a "before and after" calculation. Set the value of every quantity in the "before" equation to 1:

$$F_e = \frac{kq_1q_2}{r^2} \quad \rightarrow \quad \frac{1\cdot 1\cdot 1}{1^2} = 1$$

For the "after" equation, replace quantities that change with their multipliers:

$$F_e = \frac{kq_1q_2}{r^2} \quad \rightarrow \quad \frac{1\cdot 1\cdot 0.5}{2^2} = \frac{0.5}{4} = \frac{1}{8}$$

The "before" value for F_e was 1, and the "after" value was $\frac{1}{8}$, which means the new force will be $\frac{1}{8}$ of the original force.

This method is sometimes called "Bertha's Rule of Ones," or the "factor of change" method.

Homework Problems

1. **(M)** What is the magnitude of the electric force between two objects, each with a charge of $+2.00\times10^{-6}$ C, which are separated by a distance of 1.50 m? Is the force attractive or repulsive?

   Answer: 0.016 N, repulsive

2. **(M)** An object with a charge of $+q_1$ is separated from a second object with an unknown charge by a distance *r*. If the objects attract each other with a force *F*, what is the charge on the second object?
   (*If you are not sure how to do this problem, do #3 below and use the steps to guide your algebra.*)

   Answer: $q_2 = -\frac{Fr^2}{kq_1}$

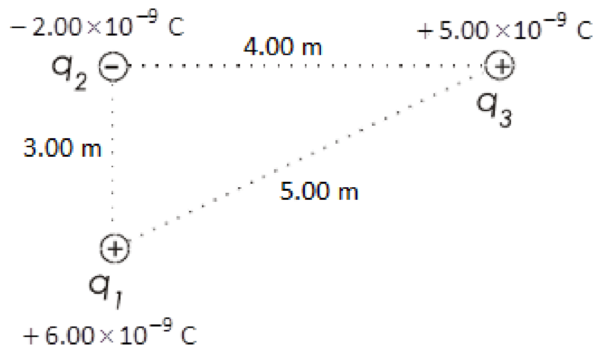
3. **(S)** An object with a charge of $+1.50\times10^{-2}$ C is separated from a second object with an unknown charge by a distance of 0.500 m. If the objects attract each other with a force of 1.35×10^{6} N, what is the charge on the second object?
   (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #2 above as a starting point if you have already solved that problem.*)

   Answer: -2.50×10^{-3} C

4. **(M)** The distance between an alpha particle (+2 elementary charges) and an electron (−1 elementary charge) is 2.00×10^{-25} m. If that distance is tripled, what will be the effect on the force between the charges?

Answer: The new F_e will be $\frac{1}{9}$ of the old F_e.

5. **(A)** Three elementary charges, particle q_1 with a charge of $+6.00\times10^{-9}$ C, particle q_2 with a charge of -2.00×10^{-9} C, and particle q_3 with a charge of $+5.00\times10^{-9}$ C, are arranged as shown in the diagram below.



What is the net force (magnitude and direction) on particle q_3?
(*Hint: this is a forces-at-an-angle problem like you saw in Physics 1.*)

Answer: 7.16×10^{-9} N at an angle of 65.2° above the *x*-axis.

Electric Fields and Electric Potential

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-1, HS-PS3-2, HS-PS3-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.3.A, 10.3.A.1, 10.3.A.2, 10.3.A.2.i, 10.3.A.2.ii, 10.3.A.2.iii, 10.3.A.3, 10.3.A.3.i, 10.3.A.3.ii, 10.3.A.3.iii, 10.3.B, 10.3.B.1, 10.3.B.1.i, 10.3.B.1.ii, 10.3.B.2, 10.4.A, 10.4.A.1, 10.4.A.2, 10.4.A.3, 10.5.A, 10.5.A.1, 10.5.A.2, 10.5.A.3, 10.5.A.3.i, 10.5.A.4

Mastery Objective(s): (Students will be able to…)

- Sketch electric field lines and vectors around charged particles or objects.
- Solve problems involving the forces on a charge due to an electric field.

Success Criteria:

- Sketches show arrows pointing from positive charges to negative charges.
- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how the electric force on a charged particle changes as you get closer to or farther away from another charged object.

Tier 2 Vocabulary: charge, field

Labs, Activities & Demonstrations:

- students holding copper pipe in one hand and zinc-coated steel pipe in other—measure with voltmeter. (Can chain students together.)

Notes:

<u>force field</u>: a region in which an object experiences a force because of some intrinsic property of the object that enables the force to act on it. Force fields are vectors, which means they have both a magnitude and a direction.

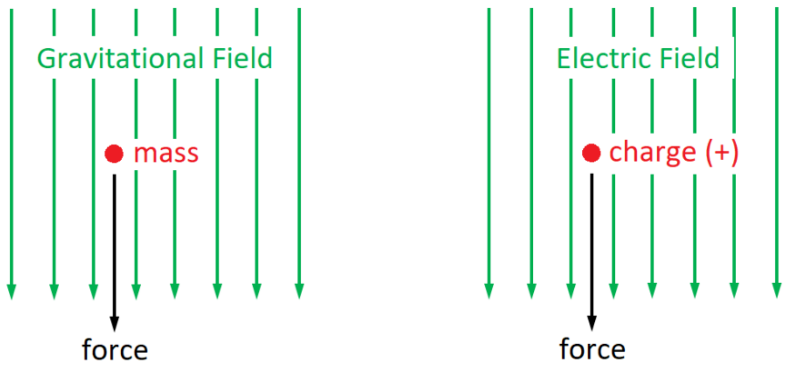
<u>electric field</u> ($\vec{\boldsymbol{E}}$) : an electrically charged region (force field) that exerts a force on any charged particle within the region.

An electric field applies a force to an object based on its electrical charge. $\vec{\boldsymbol{F}}_e = q\vec{\boldsymbol{E}}$, where $\vec{\boldsymbol{E}}$ represents the magnitude and direction of the electric field.

Because gravity is a familiar concept, it is useful to use gravitational fields as a way to explain force fields, and thus electric fields.

Recall that a gravitational field applies a force to an object based on its mass. $\vec{F}_g = m\vec{g}$, where $\vec{g}$ represents the magnitude and direction of the gravitational field.

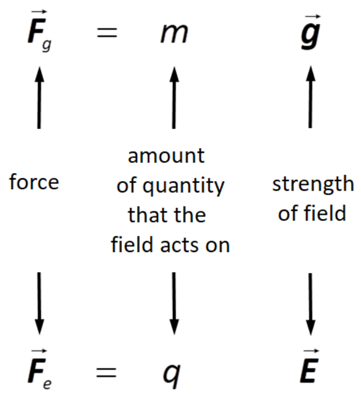
Just as a gravitational field applies a force to an object that has mass, an electric field applies a force to an object that has charge:



A key difference between the two situations is that there are two kinds of charges—positive and negative—whereas there is only one kind of mass.

The force on an object with mass is always in the direction of the gravitational field. However, the direction of the force on an object with charge depends on whether the charge is positive or negative. *The force on an object with **positive charge** is in the **same direction** as the electric field; the force on an object with **negative charge** is always in the **opposite direction** from the electric field.*

For any force field, the amount of force is the amount of the quantity that the field acts on times the strength of the field:

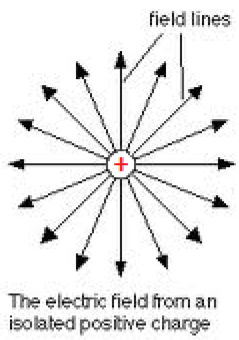


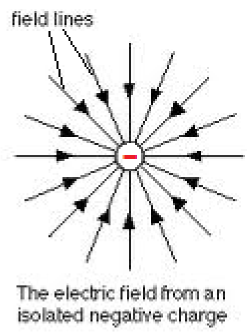
field lines: lines with arrows that show the direction of an electric field. In the above diagrams, the arrows are the field lines.

Field lines are lines that show the directions of force on an object. As described above, for an electric field, the object is assumed to be a positively-charged particle. This means that *the direction of the electric field is from positive to negative.* This means that field lines go outward in all directions from a positively-charged particle, and inward from all directions toward a negatively-charged particle.

This means that a positively-charged particle (such as a proton) would move in the direction of the arrows, and a negatively charged particle (such as an electron) would move in the opposite direction.

The simplest electric field is the region around a single charged particle:



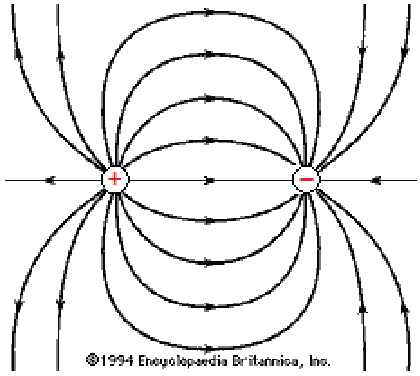


As stated in the *Electric Charge* topic (starting on page 157), the charges in a solid conductor repel one another, resulting in the charges moving to the outside of the conductor. This means that *the electric field inside of a conductor is zero.*

However, the same is not true for insulators. If you have an insulator (such as a dielectric) in an electric field, the excess electric charge is spread throughout the insulator, and the electric field can have a nonzero value.

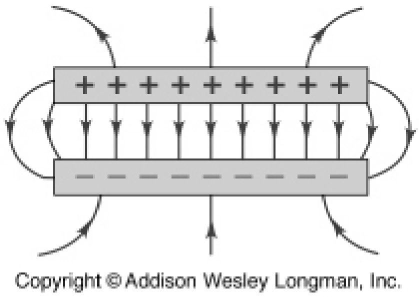
If you have a hollow conducting sphere (such as a hollow metal ball), the sphere will conduct the charges to the outside, and the electric field inside of the sphere will be zero. In this situation, the sphere may be considered as a point charge, as if all of the charge were placed at its center.

If a positive and a negative charge are near each other, the field lines go from the positive charge toward the negative charge:

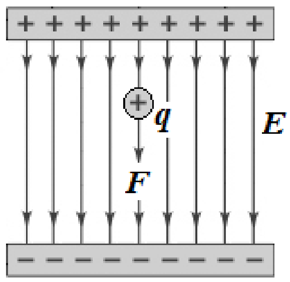


(Note that even though this is a two-dimensional drawing, the field itself is three-dimensional. Some field lines come out of the paper from the positive charge and go into the paper toward the negative charge, and some go behind the paper from the positive charge and come back into the paper from behind toward the negative charge.)

In the case of two charged plates (flat surfaces), the field lines would look like the following:



Electric Field Strength



We can measure the strength of an electric field by placing a particle with a positive charge (q) in the field and measuring the force ($\vec{F}$) on the particle.

Coulomb's Law tells us that the force on the charge is due to the charges from the electric field:

$$F_e = \frac{kq_1q_2}{r^2}$$

If the plates have equal charge densities, the repulsive force from the like-charged plate decreases as the particle moves away from it, but the attractive force from the oppositely-charged plate increases by the same amount as the particle moves toward it.

This means that if the positive and negative charges on the two surfaces that make the electric field have equal charge densities, *the force is the same everywhere in between the two surfaces*. The force on the particle is related only to the strength of the electric field and the charge of the particle.

This results in the equation that defines the electric field ($\vec{\boldsymbol{E}}$) as the force between the electric field and our particle, divided by the charge of our particle:

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}}{q} \quad \text{or} \quad \vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$$

Work Done on a Charge by an Electric Field

Recall that work is the dot product of force and displacement:

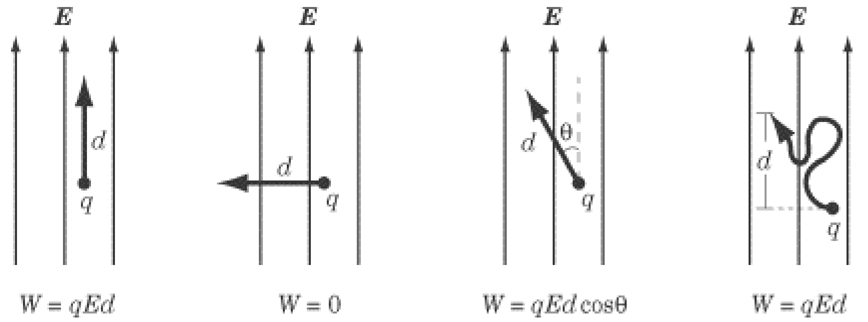
$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

Because $W = \Delta U$, the potential energy of an electric field is the work that it is able to do. This means:

$$U_e = \vec{F}_e \cdot \vec{d} = \frac{kq_1q_2}{r^2} \cdot r = \frac{kq_1q_2}{r}$$

Because $\vec{F} = q\vec{E}$, we can substitute:

$$W = \vec{F} \bullet \vec{d} = q\,\vec{E} \bullet \vec{d} = qEd\cos\theta$$



Electric Potential

Recall from the work-energy theorem that work equals a change in energy. Because an electric field can do work on a charged particle, an electric field must therefore apply energy to the particle.

electric potential (V): the electric potential energy of a charged particle in an electric field. Because the electric potential is caused by electric charges acting at a distance, the electric potential is given by the equation:

$$V = k\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\varepsilon_o} \cdot \sum_i \frac{q_i}{r_i}$$

electric potential difference (ΔV): the difference in electric potential between two points in space. This difference is caused by the action of an electric field at a distance:

$$\Delta V = \frac{\Delta U_e}{q} = \frac{W}{q} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}} = Ed\cos\theta$$

We can rearrange this equation to solve for work done by an electric field:

$$W = q\Delta V$$

Electric potential is measured in volts (V).

$$1\text{ V} \equiv 1\,\tfrac{\text{N}\cdot\text{m}}{\text{C}} \equiv 1\,\tfrac{\text{J}}{\text{C}}$$

Electric potential is analogous to gravitational potential energy. In a gravitational field, a particle has gravitational potential energy because gravity can make it move. In an electric field, a particle has electric potential (energy) because the electric field can make it move.

Redistribution of Charges

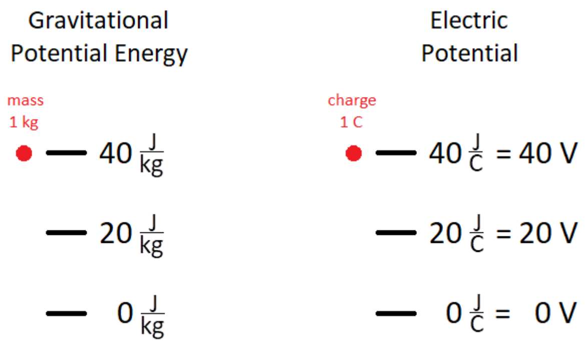
When conductors are placed in electrical contact, electrons will redistribute themselves, so that the surfaces of the conductors have the same electric potential.

This is why birds can sit on power lines, even if those power lines are made of bare coper wire. Because the wire conducts the charges freely, the potential difference between any one point on the wire and another is the same. This means there is no potential difference between one of each bird's legs and the other, so there is no energy to force the electric charges to go through the bird.

Image © 2011 by Paul Anderson. Used with permission.

For example, if we had a 1 kg mass and we placed it at a height of 4 m above the ground, its gravitational potential would be $U_g = mgh = (1)(10)(4) = 40\text{ J}$.

Similarly, if we put an object with a charge of 1 C at a location that has an electric potential of 40 V, that object would have 40 J of potential energy due to the electric field.



$$\frac{U_g}{m} = \vec{\boldsymbol{g}} \bullet \vec{\boldsymbol{h}}$$

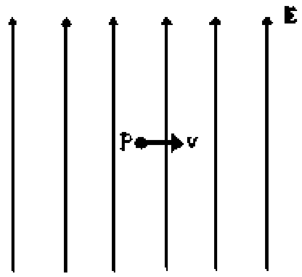
gravitational potential energy per unit of mass

$$V = \frac{W}{q} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$$

electric potential (already per unit of charge)

Sample Problem:

Q: A proton has a velocity of $1\times10^5\ \frac{\text{m}}{\text{s}}$ when it is at point *P* in a uniform electric field that has an intensity of $1\times10^4\ \frac{\text{N}}{\text{C}}$. Calculate the force (magnitude and direction) on the proton and sketch its path.

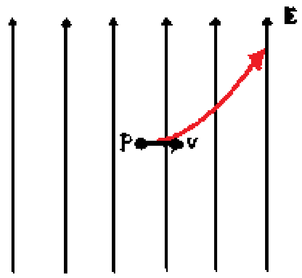


A: The force on the proton is given by:

$$\vec{F}_e = q\vec{E} = (1.6\times10^{-19})(1\times10^4) = 1.6\times10^{-15}\text{ N}$$

The direction of the force is the same direction as the electric field, which in this problem is upwards.

An upward force causes acceleration upwards. Because the proton starts with a velocity only to the right, upward acceleration means that its velocity will have a continuously increasing vertical component.



Homework Problems

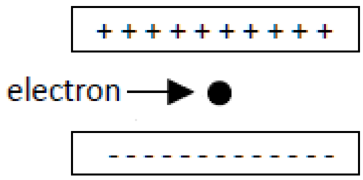
1. **(M)** Sketch the electric field in all directions around each of the following charged particles. (Assume that each particle has the same amount of charge.)

   a. 

   b. 

   c. 

2. **(M)** An electron is placed exactly halfway between two charged parallel plates, as shown in the diagram at the right. The electric field strength between the plates is $4.8\times10^{-11}\,\frac{\text{N}}{\text{C}}$.



   a. Sketch field lines to represent the electric field between the plates.

   b. Which direction does the electron move?

   c. As the electron moves, does the force acting on it increase, decrease, or remain the same?

   d. What is the net force on the electron?

   Answer: -7.68×10^{-30} N
   (Negative means the opposite direction of the electric field.)

Electric Field Vectors

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.5.B, 10.5.B. 1, 10.5.B.2, 10.5.B.2.i, 10.5.B.2.ii, 10.5.B.2.iii, 10.5.B.2.iv

Mastery Objective(s): (Students will be able to…)

- Sketch & interpret electric field vector diagrams.

Success Criteria:

- Sketches show arrows pointing from positive charges toward negative charges.
- Electric field vectors show longer arrows where charges are larger and shorter arrows where charges are smaller.

Language Objectives:

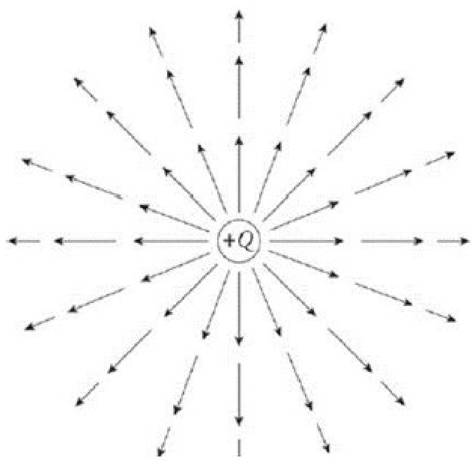
- Explain how the electric force on a charged particle changes as you get closer to or farther away from another charged object.

Tier 2 Vocabulary: charge, field

Notes:

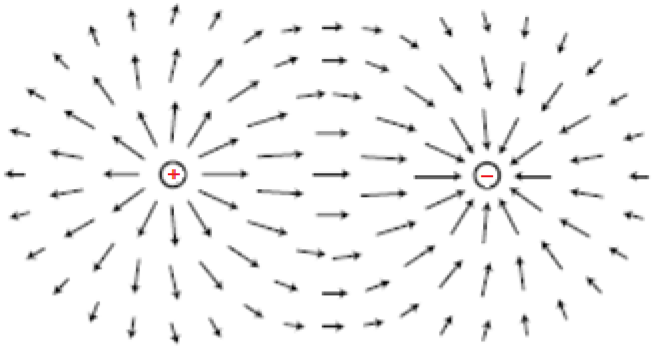
electric field vector: an arrow representing the strength and direction of an electric field at a point represented on a map.

A map of an electric field can be drawn using field vectors instead of field lines. Electric field vectors are preferred, because in addition to showing the direction of the electric field at a given location, they also show the relative strength. For example, this diagram shows the electric field around a positive charge. Notice that:

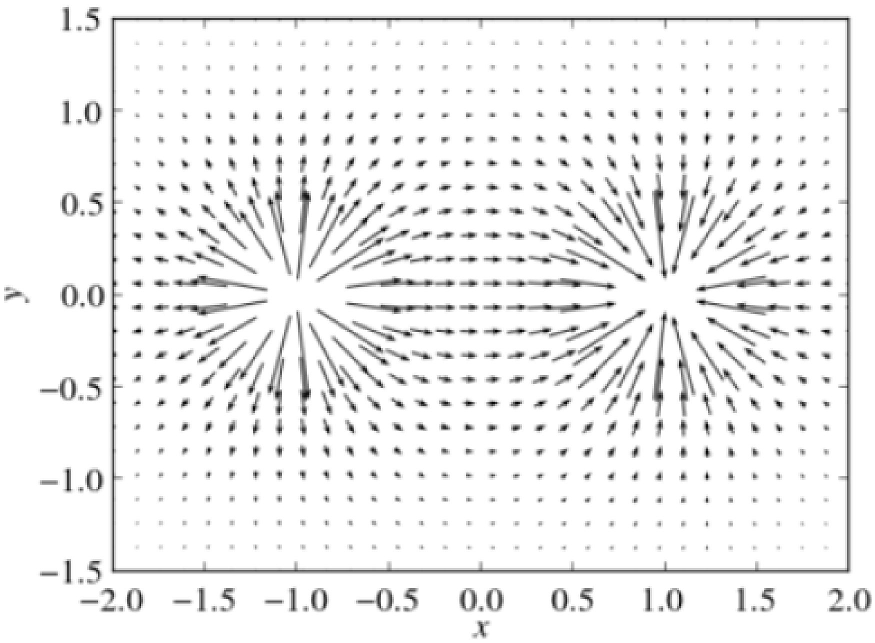


- The vectors point in the direction of the electric field (from positive to negative).
- The vectors are longer where the electric field is stronger and shorter where the electric field is weaker.

The electric field vectors around a pair of point charges, one positive and one negative, would look like the following:



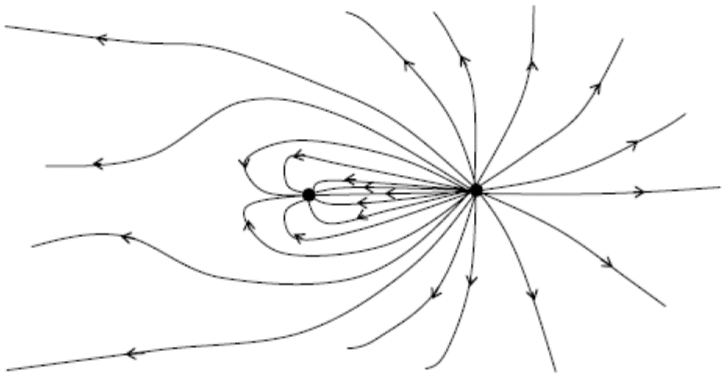
If the point charges were not shown, you could use a field vector diagram to determine their locations:



In the above example, there must be a positive point charge at coördinates (−1.0, 0) and a negative point charge at coördinates (+1.0, 0)

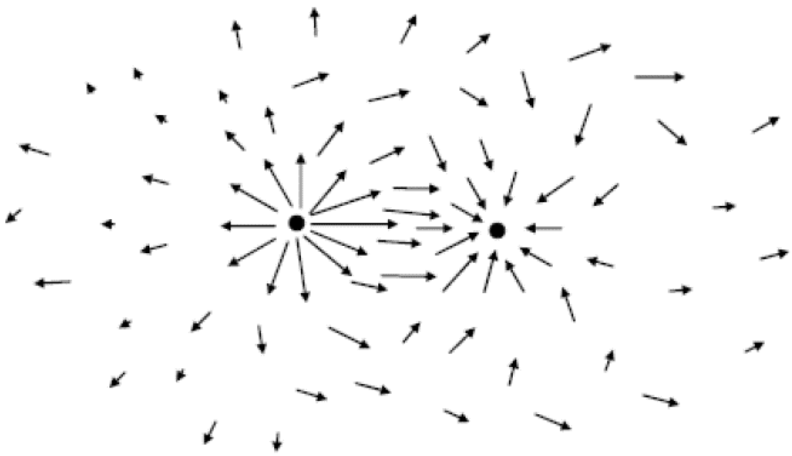
Homework Problems

1. **(M)** In the following electric field diagram: (Note that this is not an electric field vector diagram.)



   a. Label the point charges (the black dots) with the sign of their respective charges (positive or negative).
   b. Which of the two charges is stronger? Explain how you can tell.

2. **(M)** Consider the following electric field vector diagram:



   a. Label the point charges (the black dots) with the sign of their respective charges (positive or negative).
   b. Which of the two charges is stronger? Explain how you can tell.

Equipotential Lines & Maps

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.5.B, 10.5.B. 1, 10.5.B.2, 10.5.B.2.i, 10.5.B.2.ii, 10.5.B.2.iii, 10.5.B.2.iv

Mastery Objective(s): (Students will be able to…)

- Sketch & interpret equipotential ("isoline") maps.

Success Criteria:

- Isolines are perpendicular to electric field lines.
- Isolines connect regions of equal electric potential.

Language Objectives:

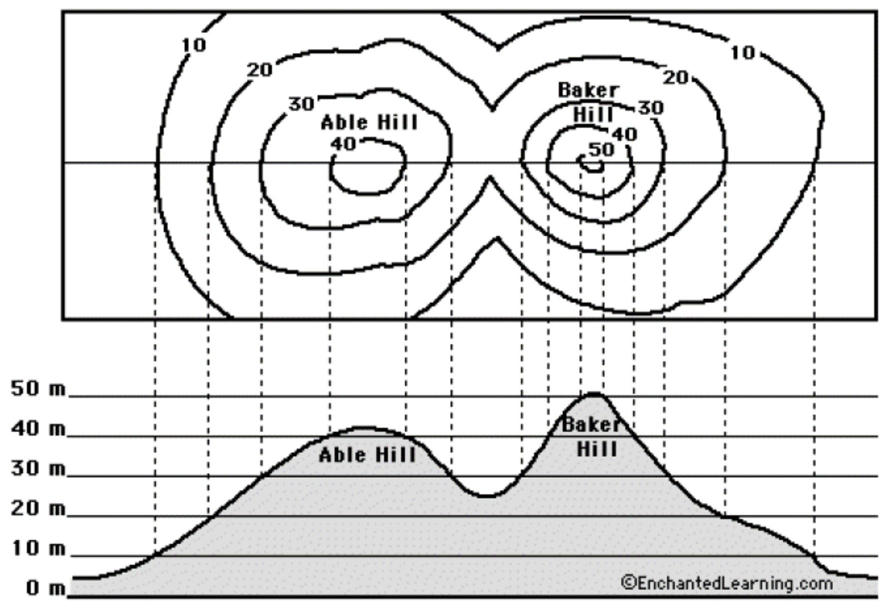
- Explain how isolines are like geographical contour maps.

Tier 2 Vocabulary: isoline, field, map

Notes:

isoline or equipotential line: a line on a map that connects regions of equal electric potential.

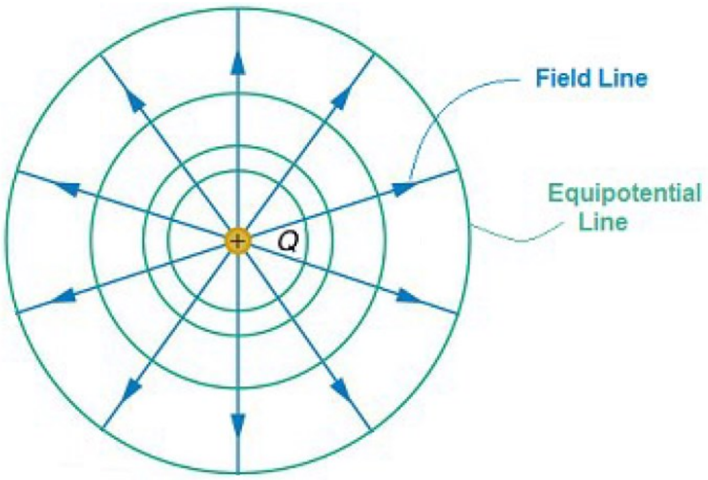
Isolines are the equivalent of elevation lines on a contour map. Below is a contour map (top) and side view of the same landscape (bottom):



The contour map (top) is a view from above. Each contour line connects points that have the same elevation.

Similar to contour lines, equipotential lines connect regions of the same electric potential.

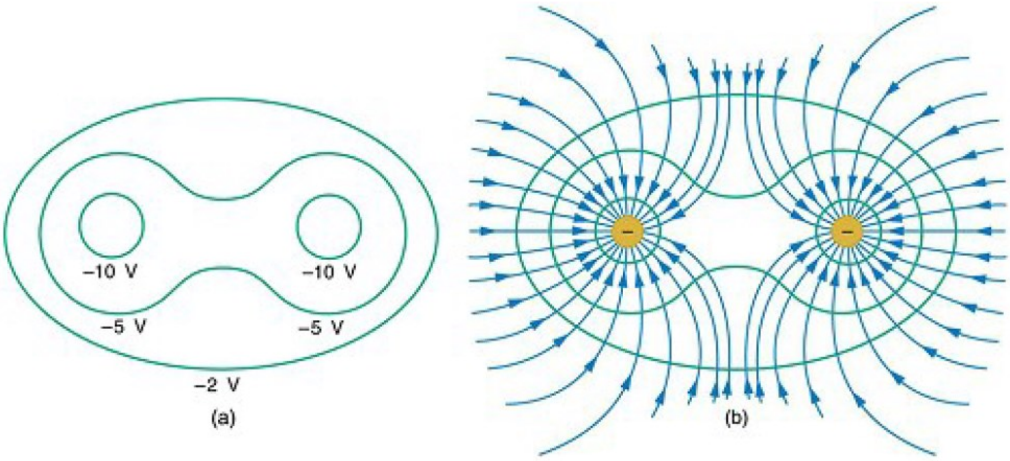
For example, the electric field direction is away from a positive point charge. The electric field strength decreases as you get farther away from the point charge. The equipotential lines (isolines) are therefore circles around the point charge; circles closer to the charge have higher electric potential, and circles farther away have lower electric potential.



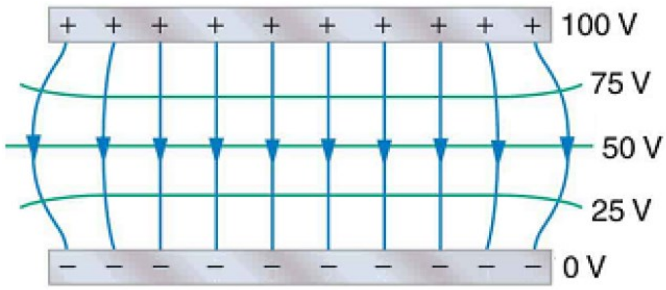
Notice that the equipotential lines are perpendicular to the field lines. As you travel along a field line, the electric potential becomes continuously less positive or more negative. The equipotential lines are the mileposts that show what the electric potential is at that point.

If you were given only the equipotential lines, you could determine what the resulting field lines looked like. For example, in the illustration below, if you were given the isolines in diagram (a), you would infer that the regions inside each of the smaller isolines must be negative point charges, and that the electric field becomes more and more negative as you approach those points.

Because the electric field lines go from positive to negative, the field lines must therefore point into the points, resulting in diagram (b). Notice again that the field lines are always perpendicular to the isolines.



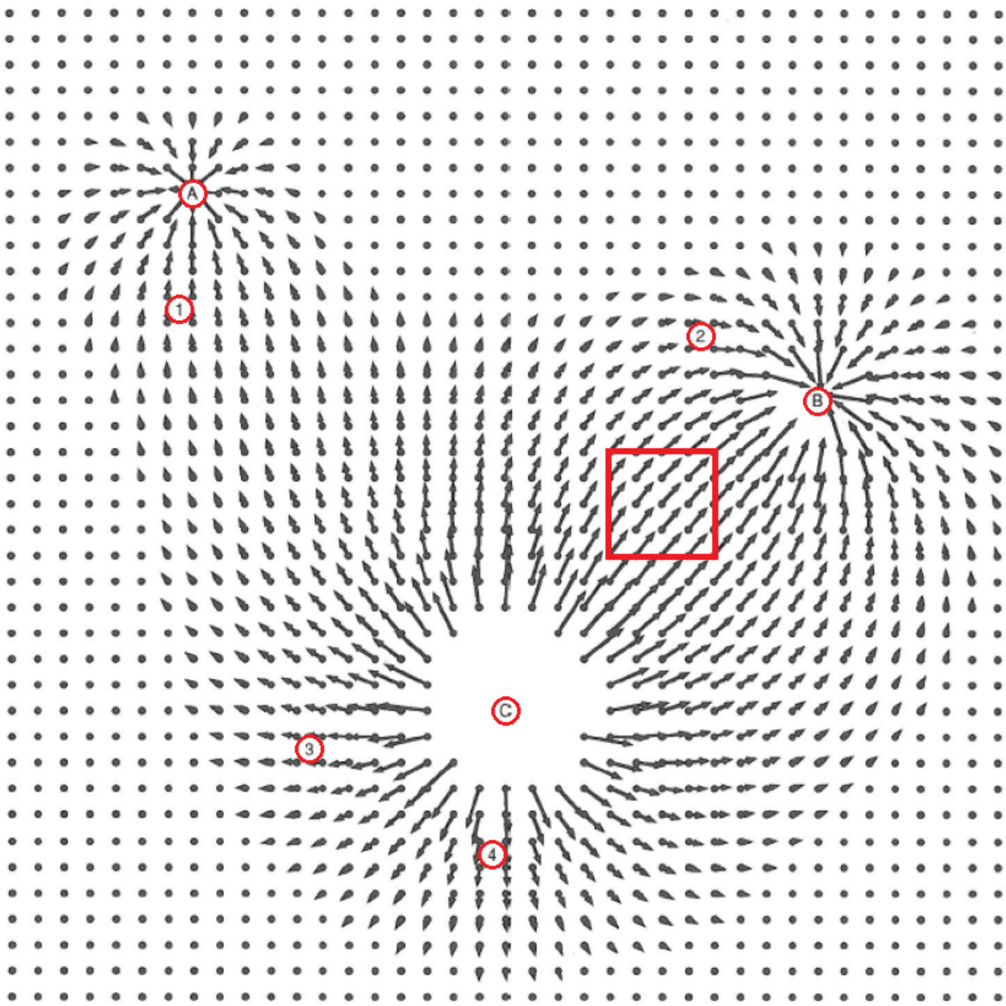
The electric field lines and isolines for the region between two parallel plates would look like the following:



Homework Problems

Consider the following electric field vector map. The length and direction of each vector arrow represents the magnitude and direction of the electric field in that location. Empty space (such as the space around point C) represents regions where the electric field is extremely strong.

There are three point charges, labeled A, B, and C. There are four numbered locations, 1, 2, 3, and 4, and a region of interest surrounded by a square box between points B and C.



1. **(M)** On the diagram on the previous page, indicate the sign (positive or negative) of charges A, B, and C.

2. **(M)** Rank the charges from strongest to weakest (regardless of sign).

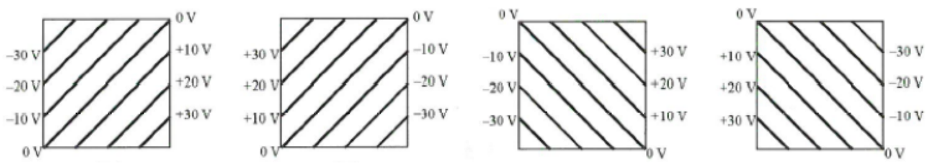
3. **(M)** Based on the lengths and directions of the field vectors, draw equipotential lines (isolines) connecting regions of the same electric potential on the diagram on the previous page.

4. **(M)** Indicate the direction of the force that would act on a proton placed at point 1 and point 2.

5. **(M)** Indicate the direction of the force that would act on an electron placed at point 3 and point 4.

6. **(M)** At which numbered point is the electric field strongest in magnitude?

7. **(M)** Circle the diagram that could represent isolines in the boxed region between charges B and C.



Explain your rationale.

Introduction: DC Circuits

Unit: DC Circuits

Topics covered in this chapter:

This chapter discusses DC Circuits, particularly those containing batteries, resistors and/or capacitors., how they behave, and how they relate to each other.

- *Electric Current & Ohm's Law* describes equations and calculations involving the flow of charged particles (electric current).
- *Electrical Components* shows pictures of and circuit diagram symbols for common electrical components.
- *EMF & Internal Resistance of a Battery* explains the difference between the voltage supplied by the chemical cells in a battery and the voltage that the battery is actually able to supply in a circuit.
- *Circuits*, *Series Circuits*, *Parallel Circuits*, and *Mixed Series & Parallel Circuits* describe arrangements of circuits that contain batteries and resistors (or other components that have resistance) and the equations that relate to them.
- *Measuring Voltage, Current & Resistance* describes how to correctly measure those quantities for components in a circuit.
- *Capacitance* and *Capacitors in Series & Parallel Circuits* describes capacitors and how they behave in circuits.

- *DC Resistor-Capacitor (RC) Circuits* describes calculations for time-varying circuits that contain a resistor and a capacitor.

One of the new challenges encountered in this chapter is interpreting and simplifying circuit diagrams, in which different equations may apply to different parts of the circuit.

Standards addressed in this chapter:

NGSS Standards/MA Curriculum Frameworks (2016):

HS-PS2-4. Use mathematical representations of Newton's Law of Gravitation and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

HS-PS3-1. Use algebraic expressions and the principle of energy conservation to calculate the change in energy of one component of a system when the change in energy of the other component(s) of the system, as well as the total energy of the system including any energy entering or leaving the system, is known. Identify any transformations from one form of energy to another, including thermal, kinetic, gravitational, magnetic, or electrical energy, in the system.

HS-PS3-2. Develop and use a model to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles and objects or energy stored in fields.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

10.6.A: Describe the physical properties of a parallel-plate capacitor.

10.6.A.1: A parallel-plate capacitor consists of two separated parallel conducting surfaces that can hold equal amounts of charge with opposite signs.

10.6.A.2: Capacitance relates the magnitude of the charge stored on each plate to the electric potential difference created by the separation of those charges.

10.6.A.2.i: The capacitance of a capacitor depends only on the physical properties of the capacitor, such as the capacitor's shape and the material used to separate the plates.

AP® only

10.6.A.2.ii: The capacitance of a parallel-plate capacitor is proportional to the area of one of its plates and inversely proportional to the distance between its plates. The constant of proportionality is the product of the dielectric constant, κ (ε_r), of the material between the plates and the electric permittivity of free space, ε_o.

10.6.A.3: The electric field between two charged parallel plates with uniformly distributed electric charge, such as in a parallel-plate capacitor, is constant in both magnitude and direction, except near the edges of the plates.

10.6.A.3.i: The magnitude of the electric field between two charged parallel plates, where the plate separation is much smaller than the dimensions of the plates, can be described with the equation $E_c = \frac{Q}{\kappa \varepsilon_o A}$.

10.6.A.3.ii: A charged particle between two oppositely charged parallel plates undergoes constant acceleration and therefore its motion shares characteristics with the projectile motion of an object with mass in the gravitational field near Earth's surface.

10.6.A.4: The electric potential energy stored in a capacitor is equal to the work done by an external force to separate that amount of charge on the capacitor.

10.6.A.5: The electric potential energy stored in a capacitor is described by the equation $U_c = \frac{1}{2} Q \Delta V$

10.6.A.6: Adding a dielectric between two plates of a capacitor changes the capacitance of the capacitor and induces an electric field in the dielectric in the opposite direction to the field between the plates.

11.1.A: Describe the movement of electric charges through a medium.

11.1.A.1: Current is the rate at which charge passes through a cross-sectional area of a wire.

11.1.A.1.i: Electric charge moves in a circuit in response to an electric potential difference, sometimes referred to as electromotive force, or emf (ε).

11.1.A.1.ii: If the current is zero in a section of wire, the net motion of charge carriers in the wire is also zero, although individual charge carriers will not have zero speed.

11.1.A.2: Although current is not a vector quantity, it does have a direction. The direction of current is associated with what the motion of positive charge would be but not with any coordinate system in space.

11.1.A.2.i: The direction of conventional current is chosen to be the direction in which positive charge would move.

AP® only

11.1.A.2.ii: In common circuits, current is actually due to the movement of electrons (negative charge carriers).

11.2.A: Describe the behavior of a circuit.

11.2.A.1: A circuit is composed of electrical loops, which may include circuit elements such as wires, batteries, resistors, lightbulbs, capacitors, switches, ammeters, and voltmeters.

11.2.A.2: A closed electrical loop is a closed path through which charges may flow.

11.2.A.2.i: A closed circuit is one in which charges would be able to flow.

11.2.A.2.ii: An open circuit is one in which charges would not be able to flow.

11.2.A.2.iii: A short circuit is one in which charges would be able to flow with no change in potential difference.

11.2.A.3: A single circuit element may be part of multiple electrical loops.

11.2.A.4: Circuit schematics are representations used to describe and analyze electric circuits.

11.2.A.4.i: The properties of an electric circuit are dependent on the physical arrangement of its constituent elements.

11.2.A.4.ii: Circuit elements have common symbols that are used to create schematic diagrams. Variable elements are indicate by a diagonal strikethrough arrow across the standard symbol for that element.

11.3.A: Describe the resistance of an object using physical properties of that object.

11.3.A.1: Resistance is a measure of the degree to which an object opposes the movement of electric charge.

11.3.A.2: The resistance of a resistor with uniform geometry is proportional to its resistivity and length and is inversely proportional to its cross-sectional area.

11.3.A.2.i: Resistivity is a fundamental property of a material that depends on its atomic and molecular structure and quantifies how strongly the material opposes the motion of electric charge.

11.3.A.2.ii: The resistivity of a conductor typically increases with temperature.

11.3.B: Describe the electrical characteristics of elements of a circuit.

11.3.B.1: Ohm's law relates current, resistance, and potential difference across a conductive element of a circuit.

11.3.B.1.i: Materials that obey Ohm's law have constant resistance for all currents and are called ohmic materials.

AP® only

11.3.B.1.ii: The resistivity of an ohmic material is constant regardless of temperature.

11.3.B.1.iii: Resistors can also convert electrical energy to thermal energy, which may change the temperature of both the resistor and the resistor's environment.

11.3.B.1.iv: The resistance of an ohmic circuit element can be determined from the slope of a graph of the current in the element as a function of the potential difference across the element.

11.4.A: Describe the transfer of energy into, out of, or within an electric circuit, in terms of power.

11.4.A.1: The rate at which energy is transferred, converted, or dissipated by a circuit element depends on the current in the element and the electric potential difference across it.

11.4.A.2: The brightness of a bulb increases with power, so power can be used to qualitatively predict the brightness of bulbs in a circuit.

11.5.A: Describe the equivalent resistance of multiple resistors connected in a circuit.

11.5.A.1: Circuit elements may be connected in series and/or in parallel.

11.5.A.1.i: A series connection is one in which any charge passing through one circuit element must proceed through all elements in that connection and has no other path available. The current in each element in series must be the same.

11.5.A.1.ii: A parallel connection is one in which charges may flow through one of two or more paths. Across each path, the potential difference is the same.

11.5.A.2: A collection of resistors in a circuit may be analyzed as though it were a single resistor with an equivalent resistance R_{eq}.

11.5.A.2.i: The equivalent resistance of a set of resistors in series is the sum of the individual resistances.

11.5.A.2.ii: The inverse of the equivalent resistance of a set of resistors connected in parallel is equal to the sum of the inverses of the individual resistances.

11.5.A.2.iii: When resistors are connected in parallel, the number of paths available to charges increases, and the equivalent resistance of the group of resistors decreases.

AP® only

11.5.B: Describe a circuit with resistive wires and a battery with internal resistance.

11.5.B.1: Ideal batteries have negligible internal resistance. Ideal wires have negligible resistance.

11.5.B.1.i: The resistance of wires that are good conductors may normally be neglected, because their resistance is much smaller than that of other elements of a circuit.

11.5.B.1.ii: The resistance of wires may only be neglected if the circuit contains other elements that do have resistance.

11.5.B.1.iii: The potential difference a battery would supply if it were ideal is the potential difference measured across the terminals when there is no current in the battery and is sometimes referred to as its emf (ε).

11.5.B.2: The internal resistance of a nonideal battery may be treated as the resistance of a resistor in series with an ideal battery and the remainder of the circuit.

11.5.B.3: When there is current in a nonideal battery with internal resistance r, the potential difference across the terminals of the battery is reduced relative to the potential difference when there is no current in the battery.

11.5.C: Describe the measurement of current and potential difference in a circuit.

11.5.C.1: Ammeters are used to measure current at a specific point in a circuit.

11.5.C.1.i: Ammeters must be connected in series with the element in which current is being measured.

11.5.C.1.ii: Ideal ammeters have zero resistance so that they do not affect the current in the element that they are in series with.

11.5.C.2: Voltmeters are used to measure electric potential difference between two points in a circuit.

11.5.C.2.i: Voltmeters must be connected in parallel with the element across which potential difference is being measured.

11.5.C.2.ii: Ideal voltmeters have an infinite resistance so that no charge flows through them.

11.5.C.3: Nonideal ammeters and voltmeters will change the properties of the circuit being measured.

11.6.A: Describe a circuit or elements of a circuit by applying Kirchhoff's loop rule.

11.6.A.1: Energy changes in simple electrical circuits may be represented in terms of charges moving through electric potential differences within circuit elements.

AP® only

11.6.A.2: Kirchhoff's loop rule is a consequence of the conservation of energy.

11.6.A.3: Kirchhoff's loop rule states that the sum of potential differences across all circuit elements in a single closed loop must equal zero.

11.6.A.4: The values of electric potential at points in a circuit can be represented by a graph of electric potential as a function of position within a loop.

11.7.A: Describe a circuit or elements of a circuit by applying Kirchhoff's junction rule.

11.7.A.1: Kirchhoff's junction rule is a consequence of the conservation of electric charge.

11.7.A.2: Kirchhoff's junction rule states that the total amount of charge entering a junction per unit time must equal the total amount of charge exiting that junction per unit time.

11.8.A: Describe the equivalent capacitance of multiple capacitors.

11.8.A.1: A collection of capacitors in a circuit may be analyzed as though it were a single capacitor with an equivalent capacitance C_{eq}.

11.8.A.1.i: The inverse of the equivalent capacitance of a set of capacitors connected in series is equal to the sum of the inverses of the individual capacitances.

11.8.A.1.ii: The equivalent capacitance of a set of capacitors in series is less than the capacitance of the smallest capacitor.

11.8.A.1.iii: The equivalent capacitance of a set of capacitors in parallel is the sum of the individual capacitances.

11.8.A.2: As a result of conservation of charge, each of the capacitors in series must have the same magnitude of charge on each plate.

11.8.B: Describe the behavior of a circuit containing combinations of resistors and capacitors.

11.8.B.1: The time constant is a significant feature of an RC circuit.

11.8.B.1.i: The time constant of an RC circuit is a measure of how quickly the capacitor will charge or discharge and is defined as $\tau = R_{eq}C_{eq}$.

11.8.B.1.ii: For a charging capacitor, the time constant represents the time required for the capacitor's charge to increase from zero to approximately 63 percent of its final asymptotic value.

11.8.B.1.iii: For a discharging capacitor, the time constant represents the time required for the capacitor's charge to decrease from fully charged to approximately 37 percent of its initial value.

AP® only

11.8.B.2: The potential difference across a capacitor and the current in the branch of the circuit containing the capacitor each change over time as the capacitor charges and discharges, but both will reach a steady state after a long time interval.

11.8.B.2.i: Immediately after being placed in a circuit, an uncharged capacitor acts like a wire, and charge can easily flow to or from the plates of the capacitor.

11.8.B.2.ii: As a capacitor charges, changes to the potential difference across the capacitor affect the charge on the plates of the capacitor, the current circuit branch in which the capacitor is located, and the electric potential energy stored in the capacitor.

11.8.B.2.iii: The potential difference across a capacitor, the current in the circuit branch in which the capacitor is located, and the electric potential energy stored in the capacitor all change with respect to time and asymptotically approach steady state conditions.

11.8.B.2.iv: After a long time, a charging capacitor approaches a state of being fully charged, reaching a maximum potential difference at which there is zero current in the circuit branch in which the capacitor is located.

11.8.B.2.v: Immediately after a charged capacitor begins discharging, the amount of charge on the capacitor plates and the energy stored in the capacitor begin to decrease.

11.8.B.2.vi: As a capacitor discharges, the amount of charge on the capacitor, the potential difference across the capacitor, and the current in the circuit branch in which the capacitor is located all decrease until a steady state is reached.

11.8.B.2.vii: After either charging or discharging for times much greater than the time constant, the capacitor and the relevant circuit branch may be modeled using steady-state conditions.

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Identifying electric circuit components.
- Simplifying circuit diagrams.

Electric Current & Ohm's Law

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.3.A, 11.3.A.1, 11.3.A.2, 11.3.A.2.i, 11.3.A.2.ii, 11.3.B, 11.3.B.1, 11.3.B.1.i, 11.3.B.1.ii, 11.3.B.1.iii, 11.3.B.1.iv, 11.4.A, 11.4.A.1

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between voltage, current, resistance and power.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

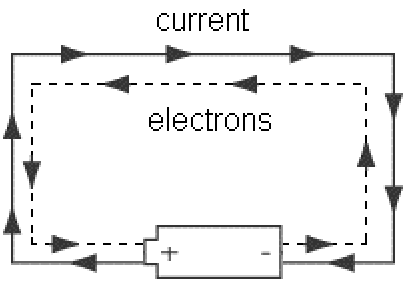
- Describe the relationships between voltage, current, resistance, and power.

Tier 2 Vocabulary: current, resistance, power

Labs, Activities & Demonstrations:

- modeling resistivity with straws
- make a light bulb out of a pencil "lead" (graphite)

Notes:



<u>electric current</u> (*I*): the flow of charged particles through a conductor, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged.

This means that electric current "travels" in the ***<u>opposite</u>*** direction from the electrons. We will use conventional current (pretending that positive particles are flowing through the circuit) throughout this course.

Electric current ($\vec{I}$) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta Q}{t}$$

Note that when electric current is flowing, charged particles move from where they are along the circuit. For example, when a light bulb is illuminated, the electrons that do the work for the first few minutes are already in the filament.

voltage (potential difference) (ΔV)*: the difference in electric potential energy between two locations, per unit of charge.

$$\Delta V = \frac{W}{q}$$

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (ΔV) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

resistance (R): the amount of electromotive force (electric potential) needed to force a given amount of current through an object in a DC circuit.

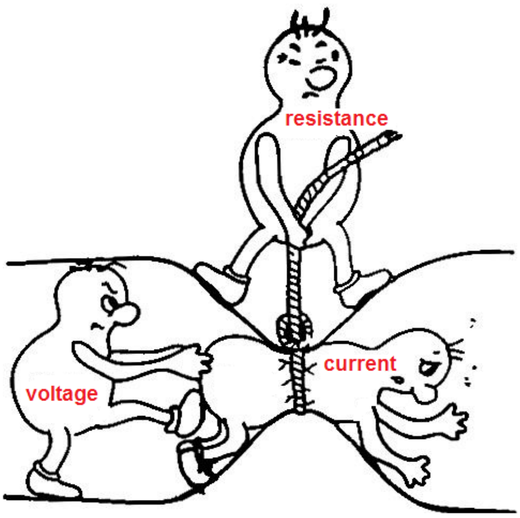
$$R = \frac{\Delta V}{I}$$

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{\Delta V}{R} \quad \text{or} \quad \Delta V = IR$$

Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.



Resistance is an intrinsic property of a substance. In this course, we will limit problems that involve calculations to ohmic resistors, which means their resistance does not change with temperature.

* Note that most physics texts (and most physicists and electricians) use V for both electric potential and voltage, and students have to rely on context to tell the difference. In these notes, to make the distinction clear (and to be consistent with the AP® Physics 2 exam), we will use V for electric potential, and ΔV for voltage (potential difference).

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use resistors in circuits to reduce the amount of current that flows through the components.

Every physical object has resistance.

- Substances that are good conductors have minimal resistance. *The resistance of wires is small enough that it can be ignored, unless the wire is the only element of the circuit.*
- Substances that are good insulators have very large resistance. Air, for example, has a resistance of 10^{12} to $10^{16}\ \frac{\Omega}{\text{cm}}$; it takes about 21 100 V to create a spark that can bridge a gap of 1 cm of air. This means that *an air gap is considered to be an open circuit, in which no current flows*.

honors (not AP®)

impedance (Z): the opposition that a circuit presents to a current when a voltage is applied. In a DC circuit, impedance and resistance are equivalent. In an AC circuit, the oscillating voltage creates changing electric and magnetic fields, which themselves resist the changes caused by the alternating current. This means the opposition to current is constantly changing at the same frequency as the oscillation of the current.

Mathematically, impedance is represented as a complex number, in which the real part is resistance and the imaginary part is reactance, a quantity that takes into account the effects of the oscillating electric and magnetic fields.

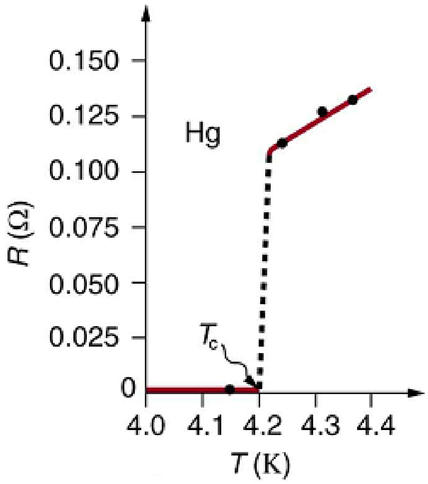
resistivity (ρ): the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters (Ω·m).

$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100°C), resistivity is given by:

$$\rho = \rho_o(1 + \alpha \Delta T)$$

where ρ_o is the resistivity at some reference temperature and α is the temperature coëfficient of resistivity for that substance. For conductors, α is positive (which means their resistivity increases with temperature). For metals at room temperature, resistivity typically varies from +0.003 to +0.006 K^{-1}.



Some materials become superconductors (essentially zero resistance) at very low temperatures. The temperature below which a material becomes a superconductor is called the critical temperature (T_c). For example, the critical temperature for mercury is 4.2 K, as shown in the graph to the right.

honors (not AP®)

<u>conductivity</u> (σ): the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). (Note that "mho" is "ohm" spelled backwards.)

$$\sigma = \frac{1}{\rho}$$

<u>ohmic resistor</u>: a resistor whose resistance is the same regardless of voltage and current. The filament of an incandescent light bulb is an example of a non-ohmic resistor, because the current heats up the filament, which increases its resistance. (This is necessary in order for the filament to also produce light.)

<u>capacitance</u> (C): the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt.

$$C = \frac{Q}{\Delta V}$$

<u>power</u> (P): as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

<u>work</u> (W): recall from mechanics that work (W) equals power times time, and is measured in either newton-meters (N·m) or joules (J):

$$W = Pt = I\Delta V t = I^2 R t = \frac{(\Delta V)^2 t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours (kW·h).

$$1\,\text{kW}\cdot\text{h} \equiv 3.6\times 10^6\ \text{J} \equiv 3.6\,\text{MJ}$$

Summary of Terms, Units and Variables

Term	Variable	Unit	Term	Variable	Unit
point charge	q	coulomb (C)	resistance	R	ohm (Ω)
charge	Q	coulomb (C)	capacitance	C	farad (F)
current	I	ampere (A)	power	P	watt (W)
voltage	ΔV	volt (V)	work	W	joule (J)

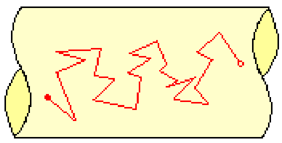
honors (not AP®)

Alternating Current *vs.* Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

However, the net flow of charged particles through a wire is very slow. Electrons continually collide with one another in all directions as they drift slowly through the circuit. Individual electrons in a DC circuit have a net velocity of about one meter per hour.



alternating current: electric current flows back and forth in one direction and then the other, like a wave. The current alternates at a particular frequency. In the U.S., household current is 110 – 120 volts AC with a frequency of 60 Hz. In most of the rest of the world, household current is 230 volts AC with a frequency of 50 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Sample Problems:

Q: A simple electrical device uses 1.5 A of current when plugged into a 110 V household electrical outlet. How much current would the same device draw if it were plugged into a 12 V outlet in a car?

A: Resistance is a property of a specific object. Because we are not told otherwise, we assume the device is ohmic and the resistance is the same regardless of the current.

Therefore, our strategy is to use the information about the device plugged into a household outlet to determine the device's resistance, then use the resistance to determine how much current it draws in the car.

In the household outlet:

$$R = \frac{\Delta V}{I} = \frac{110}{1.5} = 73.\bar{3}\,\Omega$$

In the car:

$$I = \frac{\Delta V}{R} = \frac{12}{73.\bar{3}} = 0.163\text{ A}$$

Q: A laptop computer uses 10 W of power. The laptop's power supply adjusts the current so that the power is the same regardless of the voltage supplied. How much current would the computer draw from a 110 V household outlet? How much current would the same laptop computer need to draw from a 12 V car outlet?

A: The strategy for this problem is the same as the previous one.

Household outlet:

$$P = I\Delta V$$

$$I = \frac{P}{\Delta V} = \frac{10}{110} = 0.091\text{ A}$$

Car outlet:

$$I = \frac{P}{\Delta V} = \frac{10}{12} = 0.8\bar{3}\text{ A}$$

Q: A 100 Ω resistor is 0.70 mm in diameter and 6.0 mm long. If you wanted to make a 470 Ω resistor out of the same material (with the same diameter), what would the length need to be? If, instead, you wanted to make a resistor the same length, what would the new diameter need to be?

A: In both cases, $R = \frac{\rho L}{A}$.

For a resistor of the same diameter (same cross-sectional area), ρ and A are the same, which means:

$$\frac{R'}{R} = \frac{L'}{L}$$
$$L' = \frac{R'L}{R} = \frac{(470)(6.0)}{100} = 28.2\,\text{mm}$$

For a resistor of the same length, ρ and L are the same, which means:

$$\frac{R'}{R} = \frac{A}{A'} = \frac{\pi r^2}{\pi (r')^2} = \frac{\pi (d/2)^2}{\pi (d'/2)^2} = \frac{d^2}{(d')^2}$$
$$d' = \sqrt{\frac{Rd^2}{R'}} = d\sqrt{\frac{R}{R'}} = 0.70\sqrt{\frac{100}{470}} = 0.70\sqrt{0.213} = 0.323\,\text{mm}$$

Homework Problems

1. **(S)** An MP3 player uses a standard 1.5 V battery. How much resistance is in the circuit if it uses a current of 0.010 A?

   Answer: 150 Ω

2. **(M)** How much current flows through a hair dryer plugged into a 110 V circuit if it has a resistance of 25 Ω?

   Answer: 4.4 A

3. **(S)** A battery pushes 1.2 A of charge through the headlights in a car, which has a resistance of 10 Ω. What is the potential difference across the headlights?

   Answer: 12 V

4. **(M)** A circuit used for electroplating copper applies a current of 3.0 A for 16 hours. How much charge is transferred?

   Answer: 172 800 C

5. **(S)** What is the power when a voltage of 120 V drives a 2.0 A current through a device?

   Answer: 240W

6. **(S)** What is the resistance of a 40. W light bulb connected to a 120 V circuit?

   Answer: 360 Ω

7. **(M)** If a component in an electric circuit dissipates 6.0 W of power when it draws a current of 3.0 A, what is the resistance of the component?

   Answer: 0.67 Ω

8. **(S)** A 0.7 mm diameter by 60 mm long pencil "lead" is made of graphite, which has a resistivity of approximately $1.0\times10^{-4}\ \Omega\cdot\text{m}$. What is its resistance?

   Hints:
   - *You will need to convert mm to m.*
   - *You will need to convert the diameter to a radius before using* $A=\pi r^2$ *to find the area.*

   Answer: 15.6 Ω

9. **(M)** A cylindrical object has radius *r* and length *L* and is made from a substance with resistivity *ρ*. A potential difference of Δ*V* is applied to the object. Derive an expression for the current that flows through it.

   Hint: this is a two-step problem.

   Answer: $I=\dfrac{(\Delta V)A}{\rho L}$

10. **(S)** Some children are afraid of the dark and ask their parents to leave the hall light on all night. Suppose the hall light in a child's house has two 75. W incandescent light bulbs (150 W total), the voltage is 120 V, and the light is left on for 8.0 hours.

    a. How much current flows through the light fixture?

    Answer: 1.25 A

    b. How many kilowatt-hours of energy would be used in one night?

    Answer: 1.2 kW·h

    c. If the power company charges 22 ¢ per kilowatt-hour, how much does it cost to leave the light on overnight?

    Answer: 26.4 ¢

    d. If the two incandescent bulbs are replaced by LED bulbs that use 12.2 W each (24.4 W total) how much would it cost to leave the light on overnight?

    Answer: 4.3 ¢

Electrical Components

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.2.A, 11.2.A.4.ii

Mastery Objective(s): (Students will be able to…)

- Identify electrical components using the components themselves and/or the symbols used in circuit diagrams.
- Describe the purpose of various electrical components and how they are used in circuits.

Success Criteria:

- Descriptions correctly identify the component.
- Purpose and use of component is correct.

Language Objectives:

- Explain the components in an actual circuit or a circuit diagram, and describe what each one does.

Tier 2 Vocabulary: component, resistor, fuse

Labs, Activities & Demonstrations:

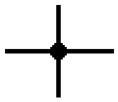
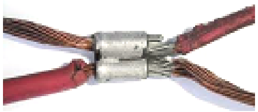
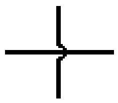
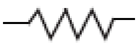
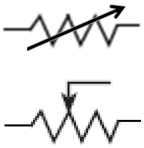
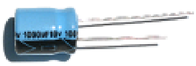
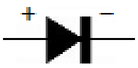
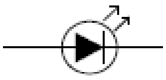
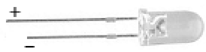
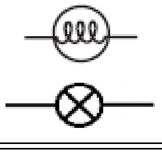
- Show & tell with actual components.
- How a fuse works.

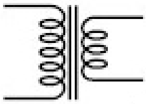
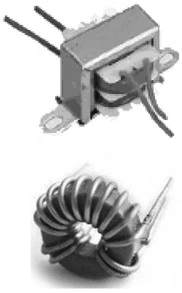
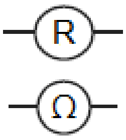
Notes:

electrical component: an object that performs a specific task in an electric circuit. A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

The following table describes some of the common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

Component	Symbol	Picture	Description
wire			Carries current in a circuit.
junction			Connection between two or more wires.
unconnected wires			Wires pass by each other but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor			Stores charge.
diode			Allows current to flow in only one direction (from + to −).
light-emitting diode (LED)			Diode that gives off light when current flows through it.
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).

Component	Symbol	Picture	Description
inductor (transformer)			Increases or decreases voltage in an AC circuit.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground		 (clamps to water pipe)	Neutralizes charge.

For the potentiometer, notice the use of the diagonal arrow across the resistor symbol. The diagonal arrow indicates that the resistance can be adjusted (variable resistance). The diagonal arrow can be used with other symbols (*e.g.,* capacitors) in the same manner.

AP® only Note that the AP® Physics 2 exam will use just a diagonal slash (without an arrow head) to indicate variability.

honors (not AP®)

Voltaic Cells (Batteries)*

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain how a battery works.
- Identify the components of a battery and their function.

Success Criteria:

- Descriptions are accurate and components are identified correctly

Language Objectives:

- Explain how a battery works. (Domains: speaking, writing)
- Identify the components of a battery and their function (Domains: speaking, writing)

Tier 2 Vocabulary: battery, current

Labs, Activities & Demonstrations:

- building a voltaic cell

Notes:

<u>voltaic cell</u>: (also called a galvanic cell) a chemical apparatus that uses an electrochemical reaction to produce electricity. (A battery is a type of galvanic cell.)

<u>electrochemistry</u>: using chemical oxidation & reduction (redox) reactions to produce electricity or vice-versa. In an electrochemical reaction, oxidation and reduction reactions occur in separate containers, and electrons travel from one container to the other. In physics, the chemical energy from the combination of the two reactions is the potential difference (voltage) that moves those electrons through an electric circuit.

<u>electrolytic cell</u>: a cell similar to a galvanic cell, except that the reaction is nonspontaneous, and electricity is used to add the energy needed to make the reaction occur. (Electrolysis of water is an example.)

<u>electrode</u>: a solid metal strip where either oxidation or reduction occurs. The metal strips also conduct the electrons into or out of the electric circuit.

* Voltaic cells are taught in AP® Chemistry as part of the topic of electrochemistry. The topic is presented here in order to explain where the electric potential in a battery comes from.

honors (not AP®)

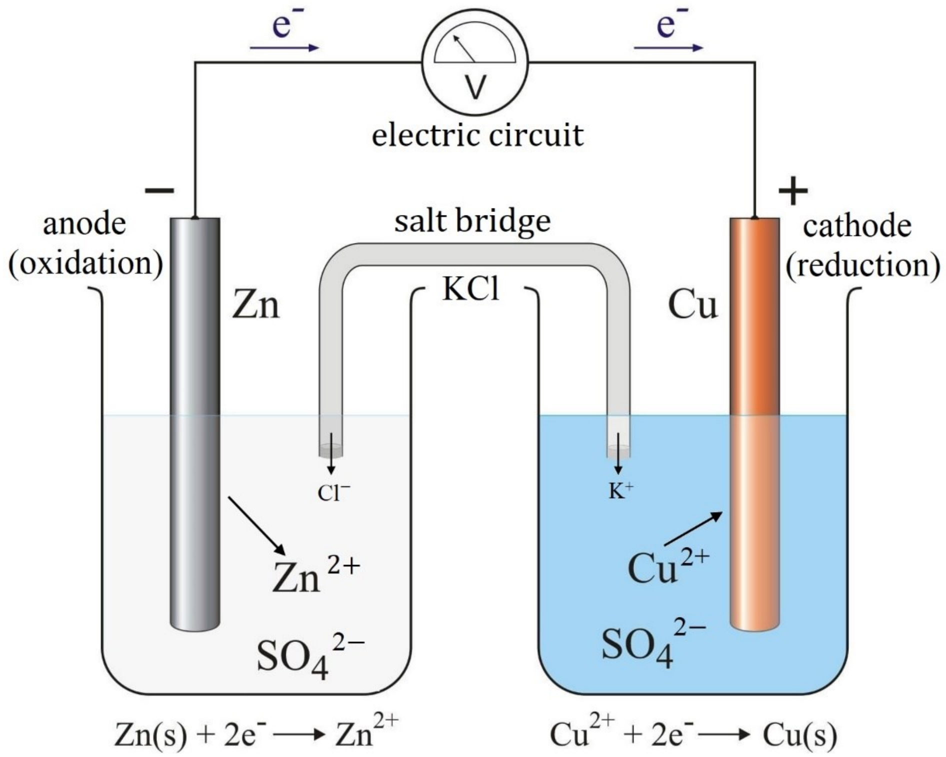
anode: the negatively (−) charged electrode. At the anode:

- Oxidation occurs. (Atoms from the anode are oxidized into positive ions.)
- These positive ions are released into the solution. (*I.e.,* the anode loses mass.)
- The electrons produced by oxidation are forced through the wire toward the cathode via the electric circuit.

cathode: the positively (+) charged electrode. At the cathode:

- Reduction occurs. (Ions from the solution are reduced to neutral metal atoms.)
- These metal atoms are deposited onto the cathode. (*I.e.,* the cathode gains mass.)
- The electrons needed for reduction are brought in through the wire from the anode via the electric circuit.

salt bridge: a salt solution that is connected to both half-cells. The salt bridge provides ions for the two half-cells in order to keep the charges balanced. (If the charges are not allowed to balance, opposite charges would build up in both cells and the reaction would stop.) The salt solution must be made of ions that do not take part in the reactions at the cathode or anode. (KNO_3 is commonly used.)



honors (not AP®)

standard voltage ($E°$): the voltage (electric potential) of an electrochemical reaction under "standard conditions".

- "Standard conditions" means temperature is 25 °C, all ion concentrations are $1\frac{\text{mol}}{\text{L}}$, and all gas pressures are 1 atm.*
- The actual voltage of the cell, V, depends on the temperature, ion concentrations and gas pressures. At standard conditions, $V = E^o$.
- $E°$ values for reduction reactions are published in tables of Standard Reduction Potentials.
- $E°$ for an oxidation reaction is the negative of the $E°$ for the reverse (reduction) reaction. *(I.e.,* if you reverse the reaction, change the sign of $E°$.)
- The standard voltage of a cell is the sum of the standard voltages for the oxidation and reduction half-cells:
  - $E^o = E^o_{reduction} + E^o_{oxidation}$
- If $E^o > 0$, then the reaction happens spontaneously. This is what happens when a battery is used to power a circuit.
- If $E^o < 0$, the reaction does not occur spontaneously, and energy is required to force the reaction to occur. This is what happens while a battery is charging.

* You can tell that "standard conditions" were defined by chemists. If they were defined by physicists, standard pressure would be 1 bar rather than 1 atm.

EMF & Internal Resistance of a Battery

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.5.B, 11.5.B.1.iii, 11.5.B.2, 11.5.B.3

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between voltage, current, resistance and power.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe the relationships between voltage, current, resistance, and power.

Tier 2 Vocabulary: current, resistance, power

Labs, Activities & Demonstrations:

- batteries and resistors (< 10 Ω) to measure emf in a low-resistance circuit

Notes:

An ideal battery always supplies current at the voltage of the electrochemical cells inside of it. In a real battery, the voltage is a little less when the battery is "under load" (supplying current to a circuit) than when it is tested with no load. This difference is caused by real-world limitations of the chemical and physical processes that occur inside of the battery.

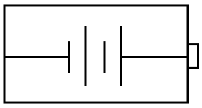
electromotive force (emf): the potential difference (voltage) supplied by a battery without load. This is the stated voltage of the battery. The term "electromotive force" literally means "electron-moving force". EMF is often represented by the variable ε.

voltage: the observed potential difference between two points in a circuit. The voltage of a battery usually means the voltage under load.

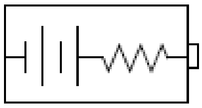
internal resistance (r): a model that explains the difference between the emf of a battery and the voltage (potential difference) it can supply as if the drop in voltage were caused by a resistor inside of the battery. In this course, we will use the variable R for the resistance of components in a circuit, and r for internal resistance.

ideal battery: a battery that has no internal resistance.

To account for internal resistance, we model a battery as if it were a power supply with the ideal voltage, plus a resistor that is physically inside of the battery.



ideal



model

Note that this is a model; the actual situation is more complex, because in addition to the resistivity of the battery's component materials, the difference between the internal voltage and the supplied voltage also depends on factors such as electrolyte conductivity, ion mobility, and electrode surface area.

The following table shows the nominal voltage and internal resistance of common Duracell (coppertop) dry cell batteries of different sizes. These numbers are given by the manufacturer for a new battery at room temperature (25°C):

Size	**AAA**	**AA**	**C**	**D**	**9V**
V_{NL} **(V)**	1.5	1.5	1.5	1.5	9
R_{int} **(mΩ)**	250	120	150	137	1 700

The internal resistance can be used to calculate the maximum current that a battery could theoretically supply. If you were to connect a wire from the positive terminal of a battery to the negative terminal, the only resistance in the circuit should be the battery's internal resistance.

The theoretical maximum current that the battery can supply is therefore the current that would be supplied when the only resistance is the battery's internal resistance, and can be calculated from Ohm's Law:

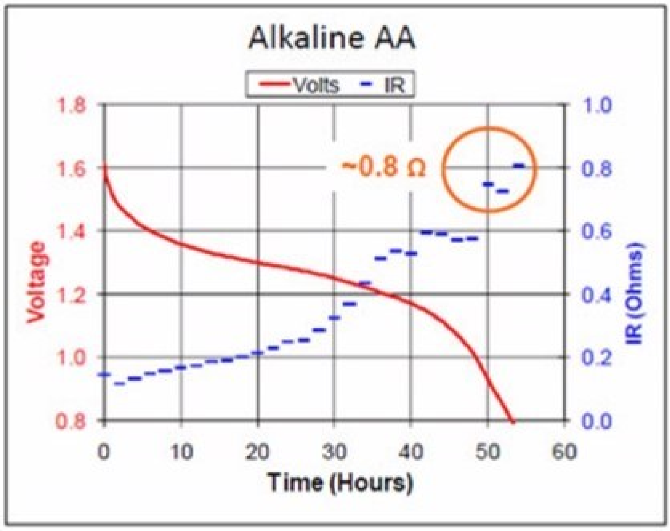
$$I_{\text{max}} = \frac{\Delta V}{r}$$

This concept can be used in circuit analysis by using the following equation:

$$\Delta V_{terminal} = \varepsilon - Ir$$

This equation states that the voltage (potential difference) across the terminals of the battery is the battery's emf minus the potential difference that is "used up" by the internal resistance.

Note also that the factors that affect a battery's internal resistance change as the battery ages. The following graph shows the changes in voltage and internal resistance of an alkaline AA battery as it supplied a current of 50 mA over a duration of about 52 hours



honors (not AP®)

Internal resistance can be calculated by measuring the voltage with no "load" on the battery (*i.e.,* the voltmeter is connected directly to the battery with nothing else in the circuit) and the voltage with "load" (*i.e.,* the battery is connected to a circuit with measurable resistance):

$$R_{\text{int}} = \left(\frac{\Delta V_{NL}}{\Delta V_{FL}} - 1 \right) R_L$$

where:

- R_{int} = internal resistance of battery
- ΔV_{FL} = voltage measured with full load (resistor with resistance R_L in circuit)
- ΔV_{NL} = voltage measured with no load (voltmeter connected directly to battery)
- R_L = resistance of the load (resistor) that is used to experimentally determine the internal resistance of the battery

Circuits

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.2.A, 11.2.A.1, 11.2.A.2, 11.2.A.2.i, 11.2.A.2.ii, 11.2.A.2.iii, 11.2.A.3, 11.2.A.4, 11.2.A.4.i, 11.2.A.4.ii

Mastery Objective(s): (Students will be able to…)

- Identify electrical circuits or sections of circuits as series or parallel.

Success Criteria:

- Descriptions correctly identify the component.
- Descriptions correctly describe which type of circuit (series or parallel) the component is in.

Language Objectives:

- Identify which components are in series *vs.* parallel in a mixed circuit.

Tier 2 Vocabulary: series, parallel

Labs, Activities & Demonstrations:

- Example circuit with light bulbs & switches.
- Fuse demo using a single strand from a multi-strand wire.

Notes:

circuit: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

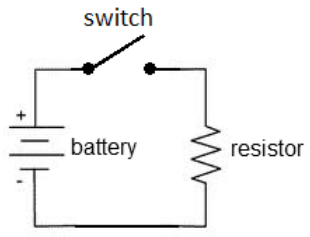
closed circuit: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

open circuit: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

short circuit: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

If we assume that wires have essentially no resistance, then a short circuit draws essentially infinite current. In a household with 110 V wiring, a short circuit can quickly produce enough heat to start a fire. This is why circuits need to be protected with fuses or circuit breakers.

A diagram of a simple electric circuit might look like the diagram to the right.



When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and back to the negative terminal of the battery.

An electric circuit needs a power supply (often a battery) that provides current at a specific voltage (electric potential difference), and one or more components that use the energy provided.

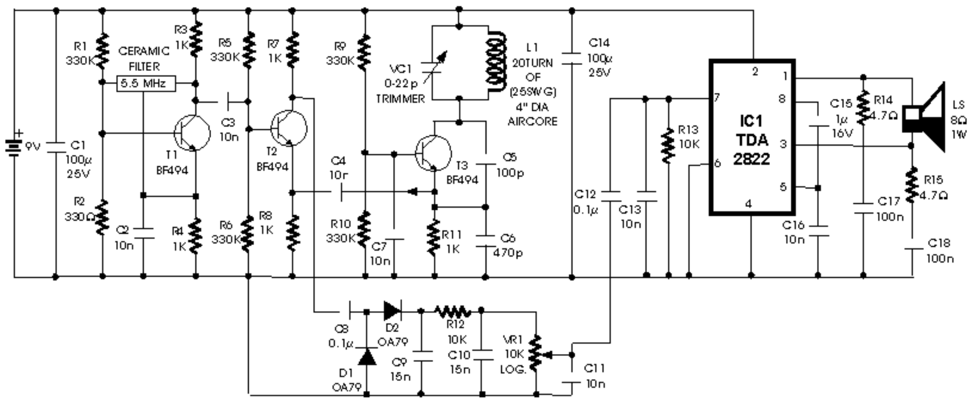
The battery or power supply continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and

2. The total resistance of the circuit is small enough to allow the current to flow.

If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

As circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:

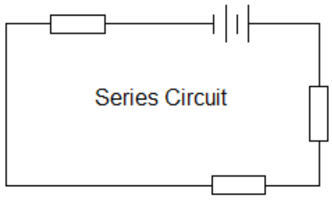


Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

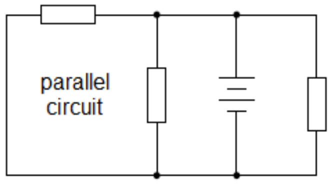
Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

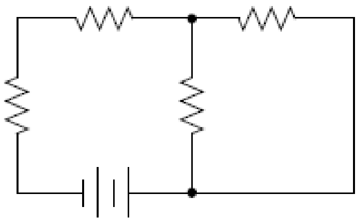
series: Components in series lie along the same path, one after the other.



parallel: Components in parallel lie in separate paths.



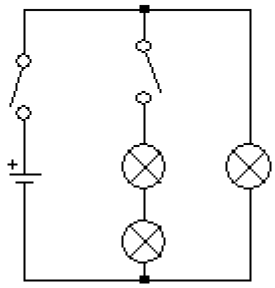
Note that complex circuits may have some components that are in series with each other and other components that are in parallel.



Sample Problem:

Q: A circuit consists of a battery, two switches, and three light bulbs. Two of the bulbs are in series with each other, and the third bulb is in parallel with the others. One of the switches turns off the two light bulbs that are in series with each other, and the other switch turns off the entire circuit. Draw a schematic diagram of the circuit, using the correct symbol for each component.

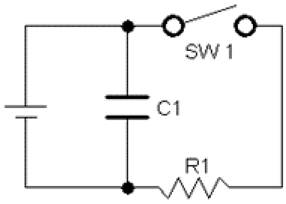
A:



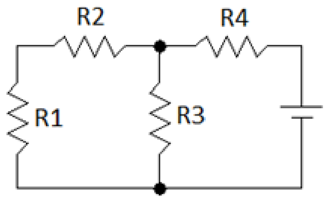
Note that no sensible person would intentionally wire a circuit this way. It would make much more sense to have the second switch on the branch with the one light bulb, so you could turn off either branch separately or both branches by opening both switches. This is an example of a strange circuit that a physics teacher would use to make sure you really can follow exactly what the question is asking!

Homework Problems

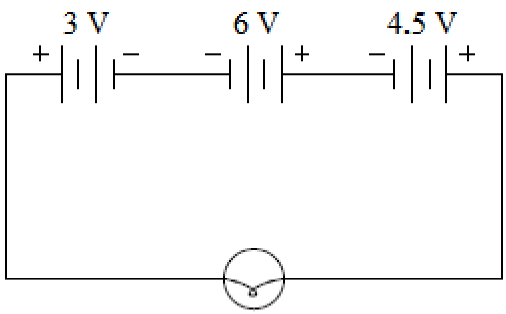
1. **(M)** The circuit shown to the right contains a battery, switch (SW1), capacitor (C1), and resistor (R1). Which of components C1 and SW1 are in series with R1? Which are in parallel with R1?



2. **(M)** The circuit shown to the right contains a battery and four resistors (R1, R2, R3, and R4). Which resistors are in series with R1? Which are in parallel with R1?



3. **(M)** The following bizarre circuit contains three batteries and a light bulb. What is the potential difference across the light bulb?
(*Hint: remember to check the +/− orientation of the batteries.*)



Answer: 7.5 V

Kirchhoff's Rules

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.6.A, 11.6.A.1, 11.6.A.2, 11.6.A.3, 11.6.A.4, 11.7.A, 11.7.A.1, 11.7.A.2

Mastery Objective(s): (Students will be able to…)

- Apply Kirchhoff's junction and loop rules to determine voltages and currents in circuits.

Success Criteria:

- Loop rule correctly applied (electric potential differences add to zero).
- Junction rule correctly applied (total current into a junction equals total current out).

Language Objectives:

- Explain why electric potential has to add to zero around a loop and why current into a junction has to add up to current out.

Tier 2 Vocabulary: loop, junction

Labs, Activities & Demonstrations:

- model a circuit by walking up & down stairs

Notes:

In 1845, the German physicist Gustav Kirchhoff came up with two simple rules that describe the behavior of current in complex circuits. Those rules are:

Kirchhoff's junction rule: the total current coming into a junction must equal the total current coming out of the junction.

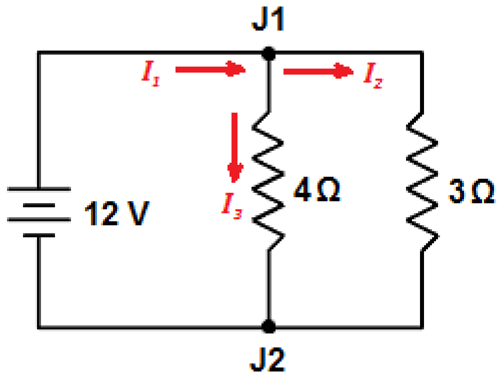
The junction rule is based on the concept of conservation of electric charge. Current is simply the flow of electric charge, so any charges that come into a junction must also come out of it.

Kirchhoff's loop rule: the sum of the voltages around a closed loop must add up to zero.

The loop rule is based on the concept that voltage is the difference in electric potential between one location in the circuit and another. If you come back to the same point in the circuit, the difference in electric potential between where you started and where you ended (the same place) must be zero. Therefore, any increases and decreases in voltage around the loop must cancel.

Junction Rule Example:

As an example of the junction rule, consider the following circuit:



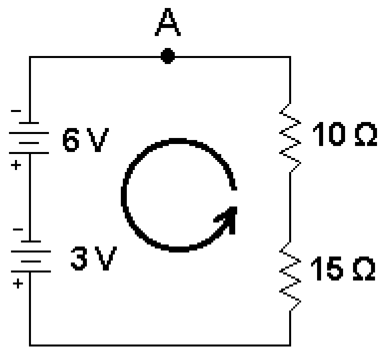
The junction rule tells us that the current flowing into junction J1 must equal the current flowing out. If we assume current I_1 flows into the junction, and currents I_2 and I_3 flow out of it, then $I_1 = I_2 + I_3$.

We know that the voltage across both resistors is 12 V. From Ohm's Law we can determine that the current through the 3 Ω resistor is $I_2 = 4\text{ A}$, and the current through the 4 Ω resistor is $I_3 = 3\text{ A}$. The junction rule tells us that the total current must therefore be:

$$I_1 = I_2 + I_3 = 4\text{A} + 3\text{A} = 7\text{A}.$$

Loop Rule Example:

For the loop rule, consider the circuit to the right:



If we start at point A and move counterclockwise around the loop (in the direction of the arrow), the voltage should be zero when we get back to point A.

For this example, we are moving around the circuit in the same direction that the current flows, because that makes the most intuitive sense. However, it wouldn't matter if we moved clockwise instead—just as with vector quantities, we choose a positive direction and assign each quantity to a positive or negative number accordingly, and the math tells us what is actually happening.

Starting from point A, we first move through the 6 V battery. We are moving from the negative pole to the positive pole of the battery, so the voltage increases by +6 V. When we move through the second battery, the voltage increases by +3 V.

Next, we move through the 15 Ω resistor. When we move through a resistor in the positive direction (of current flow), the voltage drops, so we assign the resistor a voltage of $-15I$ (based on $V = IR$, where I is the current through the resistor). Similarly, the voltage across the 10 Ω resistor is $-10I$. Applying the loop rule gives:

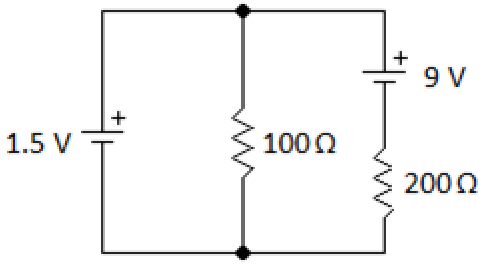
$$\begin{aligned} 6+3+(-15I)+(-10I) &= 0 \\ 9-25I &= 0 \\ 9 &= 25I \\ I = \frac{9}{25} &= 0.36\text{ A} \end{aligned}$$

Now that we know the total current, we can use it to find the voltage drop (potential difference) across the two resistors.

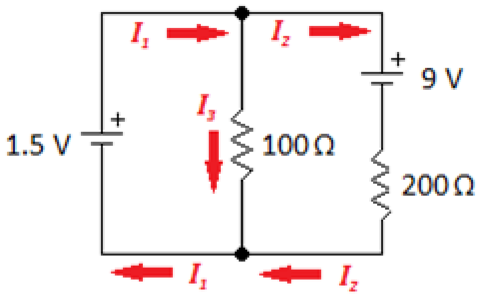
$$\Delta V_{10\,\Omega} = IR = (0.36)(10) = 3.6\text{ V} \qquad \Delta V_{15\,\Omega} = IR = (0.36)(15) = 5.4\text{ V}$$

Sample Problem using Kirchhoff's Rules:

Find the voltage and current across each resistor in the following circuit:

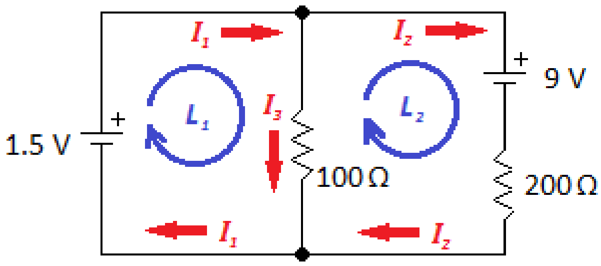


Applying the junction rule, we choose arbitrary directions for current:



$$I_1 = I_2 + I_3$$

By closer inspection, we can see that the direction for I_2 is probably going to be wrong. This means we expect that I_2 will come out to a negative number.



Now that we have defined the directions for current using the junction rule, we apply the loop rule. Again, we choose the direction of each loop arbitrarily, without worrying about which direction we have chosen for current. The loop direction is simply the order in which we inspect each element. The numbers will be determined by the direction we chose for current.

As we inspect our way around the loop, there are two rules for determining the voltage across each component:

1. Voltage across a battery is *positive* if the loop direction is *from negative to positive* (the "forward" direction).
   Voltage across a battery is *negative* if the loop direction is f*rom positive to negative* (the "backward" direction).

2. Voltage across a resistor is *negative* if the loop direction is *with the current* (the resistor is "using up" voltage).
   Voltage across a resistor is *positive* if the loop direction is *against the current* (we are traveling from a place where the electric potential is lower to a place where it is higher).

Now we inspect our way around each loop, writing the equations for the voltages:

L1: $+1.5-100I_3=0 \rightarrow I_3=0.015\text{ A}$

L2: $-9-200I_2+100I_3=0$

$-9-200I_2+1.5=0$

$-7.5=200I_2$

$I_2=-0.0375\text{ A}$

$I_1=I_2+I_3=-0.0375+0.015=-0.0225\text{ A}$

I_3 came out to a positive number, meaning that the current is flowing in the direction that we chose initially. However, I_1 and I_2 both came out negative, meaning that the current in those two segments of the circuit is actually flowing in the opposite direction from the arbitrary direction that we chose at the beginning of the problem.

Now that we know the current and resistance, we can find the voltage drop across each resistor using Ohm's Law.

100 Ω: $V=I_3R=(0.015)(100)=1.5\text{ V}$

200 Ω: $V=I_2R=(-0.0375)(200)=-7.5\text{ V}$

Again, the negative sign shows that the voltage drop (from positive to negative) is in the opposite direction from what we originally chose.

Series Circuits (Resistance Only)

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.5.A, 11.5.A.1, 11.5.A.1.i, 11.5.A.2, 11.5.A.2.i, 11.5.B, 11.5.B.1, 11.5.B.1.i, 11.5.B.1.ii

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in series circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in series circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in series circuits.

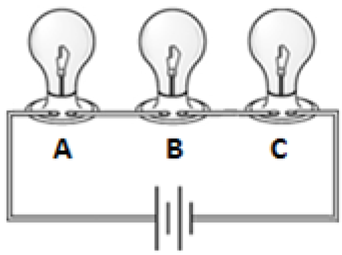
Tier 2 Vocabulary: series, circuit

Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in series.

Notes:

<u>series</u>: Components in <u>series</u> lie along the same path, one after the other.

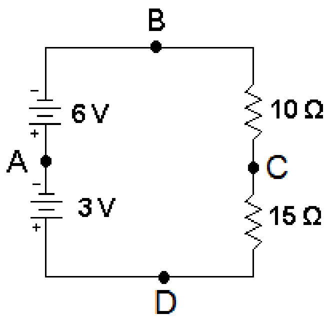


In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of the light bulbs A, B, or C is removed, no current can flow and none of the light bulbs will be illuminated.

Because some of the electric potential energy (voltage) is "used up" by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

The diagram to the right shows two batteries and two resistors in series.



Current

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \ldots$$

Voltage

In a series circuit, if there are multiple voltage sources (*e.g.,* batteries), the voltages add:

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots$$

In the above circuit, there are two batteries, one that supplies 6 V and one that supplies 3 V. The voltage from A to B is +6 V, the voltage from A to D is −3 V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3 V) + (+6 V) = +9 V.

Resistance

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

In the above circuit, the resistance between points B and D is 10Ω + 15Ω = 25Ω.

Power

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

Calculations

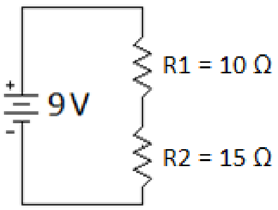
You can calculate the voltage, current, resistance, and power of each component separately, any subset of the circuit, or entire circuit, using the equations:

$$\Delta V = IR \qquad P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

"Solving" the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Sample Problem:

Suppose we are given the following series circuit:



and we are asked to fill in the following table:

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	
Power (P)			

First, we recognize that resistances in series add, which gives us:

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	***25 Ω***
Power (P)			

Now, we know two variables in the "Total" column, so we use $\Delta V = IR$ to find the current.

$$\Delta V = IR$$

$$9 = (I)(25)$$

$$I = \frac{9}{25} = 0.36\text{ A}$$

Because this is a series circuit, the total current is also the current through R_1 and R_2.

	R_1	R_2	Total
Voltage (ΔV)			9 V
Current (I)	***0.36 A***	***0.36 A***	***0.36 A***
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

As soon as we know the current, we can find the voltage across R_1 and R_2, again using $\Delta V = IR$.

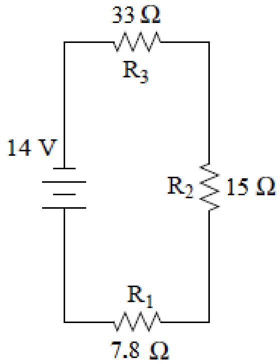
	R_1	R_2	Total
Voltage (ΔV)	***3.6 V***	***5.4 V***	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

Finally, we can fill in the power, using $P = I\Delta V$:

	R_1	R_2	Total
Voltage (ΔV)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)	***1.30 W***	***1.94 W***	***3.24 W***

Homework Problems

1. **(M)** Fill in the table for the following circuit:

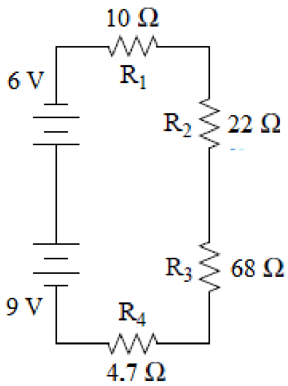


	R_1	R_2	R_3	Total
Voltage (ΔV)				14 V
Current (I)				
Resist. (R)	7.8 Ω	15 Ω	33 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

2. **(M)** Fill in the table for the following circuit.

(*Hint: Notice that the batteries are oriented in opposite directions.*)



	R_1	R_2	R_3	R_4	Total
Voltage (ΔV)					
Current (I)					
Resist. (R)	10 Ω	22 Ω	68 Ω	4.7 Ω	
Power (P)					

(The space below is intentionally left blank for calculations.)

Parallel Circuits (Resistance Only)

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.5.A, 11.5.A.1, 11.5.A.1.ii, 11.5.A.2, 11.5.A.2.ii, 11.5.A.2.iii

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

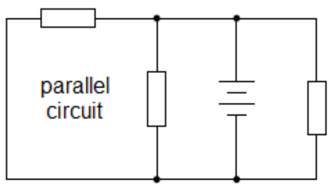
Language Objectives:

- Explain the relationships for voltages, current, resistance and power in parallel circuits.

Tier 2 Vocabulary: parallel

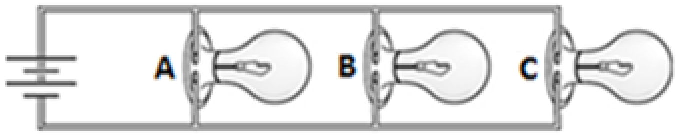
Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in parallel.parallel: Components in parallel lie in separate paths.



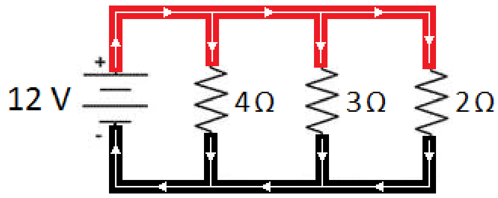
In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths.

For example, in the parallel circuit below, if any of light bulbs A, B, or C is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, each separate light bulb draws the same amount of current as if it were the only thing in the circuit, so ***the total current in the circuit increases with each new branch***. This is why you trip a circuit breaker or blow a fuse if you have too many high-power components plugged into the same circuit.)

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \ldots$$

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \ldots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \ldots$$

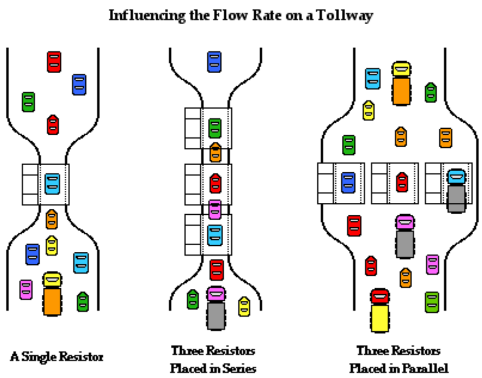
Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.

The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

The water will flow through the *set* of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

Another common analogy is to compare resistors with toll booths on a highway.



One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in a row would slows traffic down more. This is analogous to resistors in series.

Multiple toll booths next to each other (in parallel) make traffic flow faster because there are more paths for the cars to follow. Each toll booth further reduces the resistance to the flow of traffic.

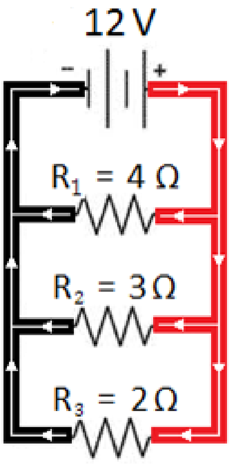
Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$\Delta V = IR \qquad P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$$

Sample Problem

Suppose we are given the following parallel circuit:



and we are asked to fill in the following table:

	R_1	R_2	R_3	Total
Voltage (ΔV)				12 V
Current (I)				
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches, so we can fill in 12 V for each resistor.

The next thing we can do is use $\Delta V = IR$ to find the current through each resistor:

	R_1	R_2	R_3	Total
Voltage (ΔV)	***12 V***	***12 V***	***12 V***	***12 V***
Current (I)	***3 A***	***4 A***	***6 A***	***13 A***
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

In a parallel circuit, the current adds, so the total current is 3 + 4 + 6 = 13 A.

Now, we have two ways of finding the total resistance. We can use $\Delta V = IR$ with the total voltage and current, or we can use the formula for resistances in parallel:

$$\Delta V = IR$$
$$12 = 13R$$
$$R = \frac{12}{13} = 0.923\,\Omega$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$
$$R_{total} = \frac{12}{13} = 0.923\,\Omega$$

Now we have:

	R_1	R_2	R_3	Total
Voltage (ΔV)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	***0.923 Ω***
Power (P)				

As we did with series circuits, we can calculate the power, using $P = I\,\Delta V$:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	***36 W***	***48 W***	***72 W***	***156 W***

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts and the chemicals leak out.

Light Bulbs

Electric light bulbs, which were invented by Thomas Edison in 1880, use electrical energy to produce light. For about 100 years, most light bulbs were incandescent bulbs, pass electricity through a tungsten filament until it glows white-hot.

Newer light bulbs, such as fluorescent bulbs or light-emitting diode (LED) bulbs, produce similar amounts of light but much less heat, making them much more energy efficient.

The S.I. unit for light intensity is the lumen (lm).

Intensity (lm)	450	800	1100	1600
Power (W)*	40	60	75	100
Resistance (Ω)†	360	240	192	144

The amount of light that a light bulb produces is proportional to the amount of power it consumes. For over 100 years, incandescent bulbs were sold according to their power rating (in watts), and people developed an understanding of how much light a typical incandescent bulb would produce in a 120 V household circuit. Note that the power consumed by a light bulb is a function of both the current and the voltage: $P = I\Delta V$.

- In a parallel circuit (such as you would find in your house), the voltage is constant. The resistance of a component determines how much current it draws. A component with lower resistance (*e.g.,* a light bulb with a higher "wattage" rating) draws more current, and therefore uses more power. This means a bulb with a higher "wattage" rating will be ***brighter*** in a parallel circuit.

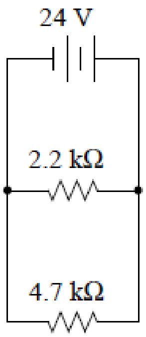
* For an incandescent bulb, assuming a 120 V household circuit.

† For an incandescent bulb. Note that light bulbs are not ohmic resistors, meaning their resistance changes as the current changes. These values are for bulbs in a 120 V household circuit.

- In a series circuit, the current through each bulb is constant (because there is only one path). The voltage across the entire circuit is fixed, but the voltage across each component splits according to the component's resistance. A component with less resistance (*e.g.,* a light bulb with a higher "wattage" rating) "uses up" less of the total voltage and therefore uses less power. This means a bulb with a higher "wattage" rating will be ***dimmer*** in a series circuit.

Homework Problems

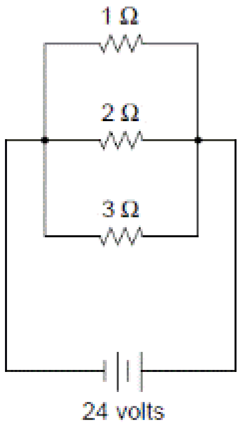
1. **(M)** Fill in the table for the following circuit:



	R_1	R_2	Total
Voltage (ΔV)			24 V
Current (I)			
Resist. (R)	2 200 Ω	4 700 Ω	
Power (P)			

(The space below is intentionally left blank for calculations.)

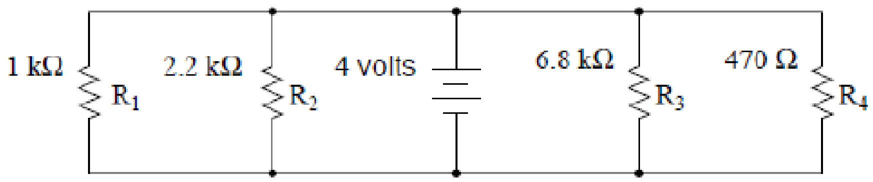
2. **(S)** Fill in the table for the following circuit:



	R_1	R_2	R_3	Total
Voltage (ΔV)				24 V
Current (I)				
Resist. (R)	1 Ω	2 Ω	3 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

3. **(M)** Fill in the table for the following circuit:



	R_1	R_2	R_3	R_4	Total
Voltage (ΔV)					4 V
Current (I)					
Resistance (R)	1 000 Ω	2 200 Ω	6 800 Ω	470 Ω	
Power (P)					

(The space below is intentionally left blank for calculations.)

Mixed Series & Parallel Circuits (Resistance Only)

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-9(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.5.A, 11.5.A.1, 11.5.A.1.i, 11.5.A.1.ii, 11.5.A.2, 11.5.A.2.i, 11.5.A.2.ii, 11.5.A.2.iii

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, current, resistance and power in mixed series & parallel circuits.

Success Criteria:

- Correct relationships are applied for voltage, current, resistance and power in mixed series & parallel circuits.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the relationships for voltages, current, resistance and power in mixed series & parallel circuits.

Tier 2 Vocabulary: series, parallel

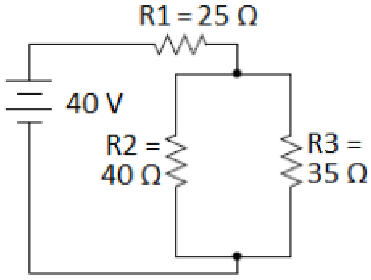
Labs, Activities & Demonstrations:

- light bulb mystery circuits

Notes:

If a circuit has mixed series and parallel sections, you can determine the various voltages, currents and resistances by applying Kirchhoff's Rules and/or by "simplifying the circuit." Simplifying the circuit, in this case, means replacing groups of resistors that are in series or parallel with a single resistor of equivalent resistance.

For example, suppose we need to solve the following mixed series & parallel circuit for voltage, current, resistance and power for each resistor:



Because the circuit has series and parallel sections, we cannot simply use the series and parallel rules across the entire table.

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	
Power (P)				

We can use Ohm's Law $(\Delta V = IR)$ and the power equation $(P = I\Delta V)$ on each individual resistor and the totals for the circuit (columns), but we need two pieces of information for each resistor in order to do this.

Our strategy will be:

1. Simplify the resistor network until all resistances are combined into one equivalent resistor to find the total resistance.
2. Use $\Delta V = IR$ to find the total current.
3. Work backwards through your simplification, using the equations for series and parallel circuits in the appropriate sections of the circuit until you have all of the information.

Step 1: If we follow the current through the circuit, we see that it goes through resistor R1 first. Then it splits into two parallel pathways. One path goes through R2 and R3, and the other goes through R4 and R5.

There is no universal shorthand for representing series and parallel components, so let's define the symbols "—" to show resistors in series, and "||" to show resistors in parallel. The above network of resistors could be represented as:

R1 — (R2 || R3)

Now, we simplify the network just like a math problem—start with the innermost parentheses and work your way out.

Step 2: Combine the parallel 40 Ω and 35 Ω resistors into a single equivalent resistance:

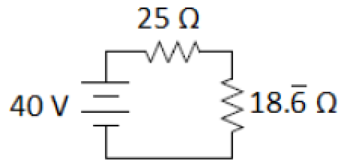
$$25\,\Omega - (40\,\Omega \parallel 35\,\Omega) \longrightarrow 25\,\Omega - (R_{eq.,||})$$

$$\frac{1}{R_{total}} = \frac{1}{40} + \frac{1}{35}$$

$$\frac{1}{R_{total}} = 0.0250 + 0.0286 = 0.0536$$

$$R_{total} = \frac{1}{0.0536} = 18.\overline{6}\,\Omega$$

Now our circuit is equivalent to:



Step 3: Add the two resistances in series to get the total combined resistance of the circuit:

$$25\,\Omega - 18.\overline{6}\,\Omega \longrightarrow R_{total}$$

$$18.\overline{6} + 25 = 43.\overline{6}\,\Omega$$

This gives:

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				
Resistance (R)	25 Ω	40 Ω	35 Ω	***$43.\overline{6}\,\Omega$***
Power (P)				

Step 4: Now that we know the total voltage and resistance, we can use Ohm's Law to find the total current:

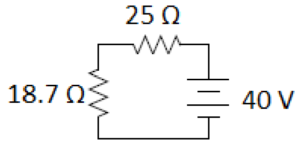
$$\Delta V = IR$$
$$40 = I(43.\overline{6})$$
$$I = \frac{40}{43.\overline{6}} = 0.916\text{ A}$$

While we're at it, let's use $P = I\Delta V = (0.916)(40) = 36.6\text{ W}$ to find the total power.

	R_1	R_2	R_3	Total
Voltage (ΔV)				40 V
Current (I)				***0.916 A***
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)				***36.6 W***

Now we work backwards.

The next-to-last simplification step was:



The 25 Ω resistor is R1. All of the current goes through it, so the current through R1 must be 0.916 A. Using Ohm's Law, this means the voltage drop across R1 must be:

$$\Delta V = IR = (0.916)(25) = 22.9\text{ V}$$

and the power must be:

$$P = I\Delta V = (0.916)(22.9) = 21.0\text{ W}$$

This means that the voltage across the parallel portion of the circuit (R2 ‖ R3) must be 40 – 22.9 = 17.1 V. Therefore, the voltage is 17.1 V across *both* parallel branches (because voltage is the same across parallel branches).

	R_1	R_2	R_3	Total
Voltage (ΔV)	***22.9 V***	***17.1 V***	***17.1 V***	40 V
Current (I)	***0.916 A***			0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	***21.0 W***			36.6 W

We can use this and Ohm's Law to find the current through one branch:

$$\Delta V_{40\,\Omega} = \Delta V_{35\,\Omega} = 40 - \Delta V_1 = 40 - 22.9 = 17.1\,V$$

$$\Delta V_{40\,\Omega} = I_{40\,\Omega} R_{40\,\Omega}$$

$$I_{40\,\Omega} = \frac{\Delta V_{40\,\Omega}}{R_{40\,\Omega}} = \frac{17.1}{40} = 0.428\,A$$

We can use Kirchhoff's Junction Rule to find the current through the other branch:

$$I_{total} = I_{40\,\Omega} + I_{35\,\Omega}$$

$$0.916 = 0.428 + I_{35\,\Omega}$$

$$I_{35\,\Omega} = 0.488\text{ A}$$

This gives us:

	R_1	R_2	R_3	Total
Voltage (ΔV)	22.9 V	17.1 V	17.1 V	40 V
Current (I)	0.916 A	***0.428 A***	***0.488 A***	0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	21.0 W			36.6 W

Finally, because we now have current and resistance for each of the resistors R_2 and R_3, we can use $P = I\Delta V$ to find the power:

$$P_2 = I_2 \Delta V_2 = (0.428)(17.1) = 7.32\text{ W}$$

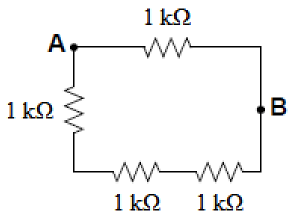
$$P_3 = I_3 \Delta V_3 = (0.488)(17.1) = 8.34\text{ W}$$

	R_1	R_2	R_3	Total
Voltage (ΔV)	22.9 V	17.1 V	17.1 V	40 V
Current (I)	0.916 A	0.428 A	0.488 A	0.916 A
Resistance (R)	25 Ω	40 Ω	35 Ω	$43.\overline{6}\,\Omega$
Power (P)	21.0 W	***7.32 W***	***8.34 W***	36.6 W

Alternately, because the total power is the sum of the power of each component, once we had the power in all but one resistor, we could have subtracted from the total to find the last one.

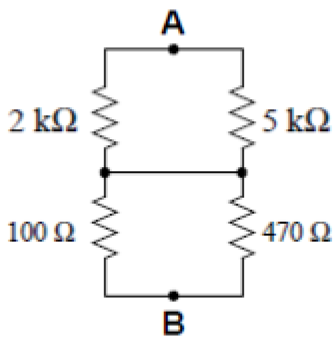
Homework Problems

1. **(M)** What is the equivalent resistance between points **A** and **B**?



Answer: 750 Ω

2. **(M)** What is the equivalent resistance between points **A** and **B**?

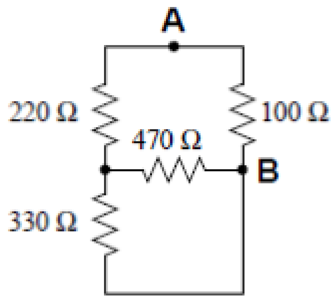


Hints:

- *Convert the resistances from* kΩ *to* Ω*.*
- *Redrawing the circuit to separate the top and bottom halves may make it easier to understand what is going on.*

Answer: 1511 Ω or 1.511 kΩ

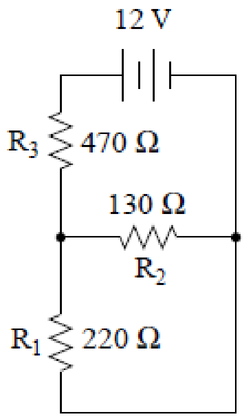
3. **(M)** What is the equivalent resistance between points **A** and **B**?



(The space below is intentionally left blank for calculations.)

Answer: 80.5 Ω

4. **(M)** Fill in the table for the circuit below:



	R_1	R_2	R_3	Total
Voltage (ΔV)				12 V
Current (I)				
Resistance (R)	220 Ω	130 Ω	470 Ω	
Power (P)				

(The space below is intentionally left blank for calculations.)

Measuring Voltage, Current & Resistance

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.5.C, 11.5.C.1, 11.5.C.1.i, 11.5.C.1.ii, 11.5.C.2, 11.5.C.2.i, 11.5.C.2.ii, 11.5.C.3

Mastery Objective(s): (Students will be able to…)

- Accurately measure voltage and current in a DC circuit.

Success Criteria:

- Multimeter wires are plugged in to the correct jacks and dial is set to the correct quantity.
- Measurements are taken at appropriate points in the circuit. (Voltage is measured in parallel and current is measured in series.)

Language Objectives:

- Explain how to set up the multimeter correctly.
- Explain where to take the measurements and why.

Tier 2 Vocabulary: meter

Labs, Activities & Demonstrations:

- Show & tell with digital multi-meter.
- Measurement of voltages and currents in a live DC circuit.

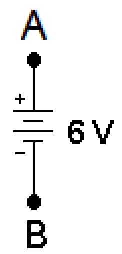
Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

Measuring Voltage

Voltage is measured with a voltmeter.

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:

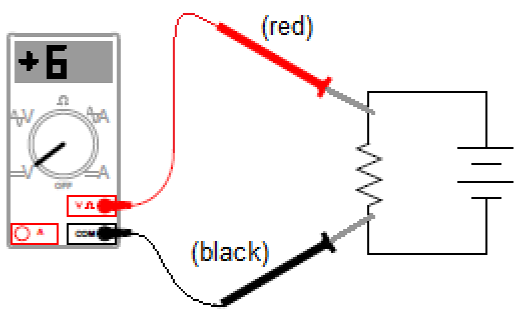


The voltage between points A and B is either +6 V or −6 V, depending on the direction. The voltage from A to B (positive to negative) is +6 V, and the voltage from B to A (negative to positive) is −6 V.

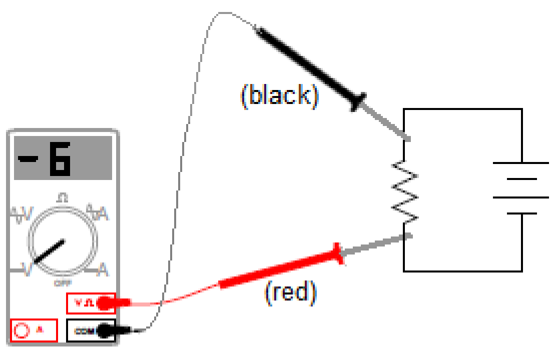
To measure voltage:

1. The circuit needs to be powered up with current flowing through it.
2. Make sure the red lead is plugged into the V Ω socket (for measuring volts or ohms).
3. Make sure that the voltmeter is set for volts (DC or AC, as appropriate).
4. Touch the two leads *in parallel* with the two points you want to measure the voltage across. (Remember that voltage is the same across all branches of a parallel circuit. You want the voltmeter to be one of the branches, and the circuit to be the other branch with the same voltage.)

On a voltmeter (a meter that measures volts or voltage), positive voltage means the current is going from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the end of a resistor that is closer to the positive terminal of the battery, and the black (−) lead on the end that is closer to the negative terminal, the voltage reading will be positive. In the circuit to the right, the voltmeter reads +6 V.



However, if you reverse the leads so that the black (−) lead is closer to the positive terminal of the battery and the red (+) lead is closer to the negative terminal, the voltage reading will be negative. In the circuit to the right, the voltmeter reads −6 V.



The reading of −6 V indicates that the current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (−) lead to the red (+) lead.

Voltmeters have a high resistance (typically 200 kΩ for older analog voltmeters, and 10 MΩ or more for modern digital voltmeters), so that the presence of the meter has a minimal effect on the circuit under test. An "ideal voltmeter" would have infinite resistance.

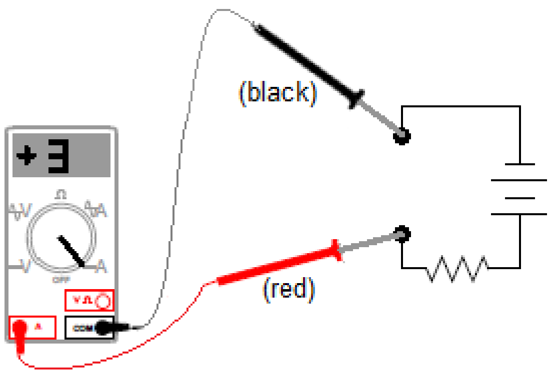
Measuring Current

Current (amperage) is measured with an ammeter.

To measure current:

1. The circuit needs to be open between the two points where you want to measure the current.
2. Make sure the red lead is plugged into appropriate socket (10 A if the current is expected to be 0.5 A or greater; 1 A or mA/μA if the current is expected to be less than 0.5 A).
3. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes (μA) AC or DC, depending on what you expect the current in the circuit to be.
4. Touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (Remember that current is the same through all components in a series circuit. You want to measure all of the current, so you want all of the current to flow through the meter.)

On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (−) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (−) lead on the end that is connected to the negative terminal, the current reading would be positive. In the circuit to the right, the current is +3 A.

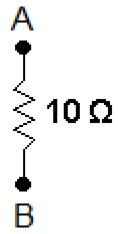


As with the voltage example above, if you switched the leads, the reading would be −3 A instead of +3 A.

Ammeters have a low resistance, so that the presence of the meter has a minimal effect on the circuit under test. Because of the way the meter is designed resistance varies depending on the amount of current being measured. (One popular ammeter has a resistance of 0.03 Ω when measuring currents in the 6 – 10 A range, 1.8 Ω when measuring current in the 60 – 400 mA range, and 100 Ω for currents in the 600 μA – 6 mA range.) An "ideal ammeter" would have zero resistance.

Measuring Resistance

Resistance is measured with an ohmmeter.



Resistance does not have a direction. If you placed an ohmmeter across points A and B, it would read 10 Ω regardless of which lead is on which point.

An ohmmeter supplies a voltage across the component and measures the current. Because the voltage supplied is constant, the Ohm's Law calculation is built into the meter and the readout displays the resistance.

To measure resistance:

1. The circuit needs to be open. Because the meter is applying a voltage and measuring current, you do not want other voltages or currents in the circuit.
2. Make sure the red lead is plugged into the V Ω socket (for measuring volts or ohms).
3. Make sure that the voltmeter is set for Ω.
4. Touch one lead to each end of the resistor.

If you need to measure the resistance of a component that is in a circuit under power, it can be more reliable to measure the voltage and current and calculate resistance using Ohm's Law ($\Delta V = IR$).

Capacitance

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.6.A, 10.6.A.1, 10.6.A.2, 10.6.A.2.i, 10.6.A.2.ii, 10.6.A.3, 10.6.A.3.i, 10.6.A.3.ii, 10.6.A.4, 10.6.A.5, 10.6.A.6

Mastery Objective(s): (Students will be able to…)

- Solve problems involving relationships between capacitance, charge and voltage.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Describe what a capacitor does.

Tier 2 Vocabulary: charge, capacitance

Labs, Activities & Demonstrations:

- build a capacitor

Notes:

capacitor: an electrical component that stores electrical charge but does not allow current to flow through.

When a voltage is applied to the circuit, one side of the capacitor will acquire a positive charge, and the other side will acquire an equal negative charge. This process is called *charging the capacitor*.

When a charged capacitor is placed in a circuit (perhaps it was charged previously, and then the voltage source is switched off), charge flows out of the capacitor into the circuit. This process is called *discharging the capacitor*.

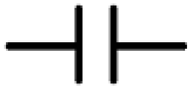
No current actually flows through the capacitor, but as it charges, the positive charges that accumulate on one side of the capacitor repel positive charges from the other side into the rest of the circuit. This means that ***an uncharged capacitor acts like a wire*** when it first begins to charge.

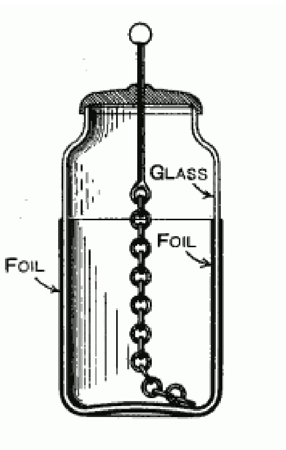
Once the capacitor is fully charged, the amount of potential difference in the circuit is unable to add any more charge, and no more charges flow. This means that ***a fully-charged capacitor*** *in a circuit that has a power supply (e.g., a battery)* ***acts like an open switch or a broken wire***.

If you disconnect the battery and reconnect the capacitor to a circuit that allows the capacitor to discharge, charges will flow out of the capacitor and through the circuit. This means that *a fully-charged capacitor in a circuit without a separate power supply acts like a battery* when it first begins to discharge.

Toys from joke shops that shock people use simple battery-and-capacitor circuits. The battery charges the capacitor gradually over time until a significant amount of charge has built up. When the person grabs the object, the person completes a circuit that discharges the capacitor, resulting in a sudden, unpleasant electric shock.

The simplest capacitor (conceptually) is a pair of parallel metal plates separated by a fixed distance. The symbol for a capacitor is a representation of the two parallel plates.





The first capacitors were made independently in 1745 by the German cleric Ewald Georg von Kleist and by the Dutch scientist Pieter van Musschenbroek. Both von Kleist and van Musschenbroek lined a glass jar with metal foil on the outside and filled the jar with water. (Recall that water with ions dissolved in it conducts electricity.) Both scientists charged the device with electricity and received a severe shock when they accidentally discharged the jars through themselves.

This type of capacitor is named after is called a Leyden jar, after the city of Leiden (Leyden) where van Musschenbroek lived.

Modern Leyden jars are lined on the inside and outside with conductive metal foil. As a potential difference is applied between the inside and outside of the jar, charge builds up between them. The glass, which acts as an insulator (a substance that does not conduct electricity), keeps the two pieces of foil separated and does not allow the charge to flow through.

Because the thickness of the jar is more or less constant, the Leyden jar behaves like a parallel plate capacitor.

Shortly after the invention of the Leyden jar, Daniel Gralath discovered that he could connect several jars in parallel to increase the total possible stored charge. Benjamin Franklin compared this idea with a "battery" of cannon. (The original meaning of the term "battery" was a collection of cannon for the purpose of battering the enemy.) The term is now used to describe a similar arrangement of electrochemical cells.

Franklin's most famous experiment was to capture the charge from a lightning strike in Leyden jars, proving that lightning is an electric discharge.

capacitance: a measure of the ability of a capacitor to store charge. Capacitance is measured in farads (F), named after the English physicist Michael Faraday.

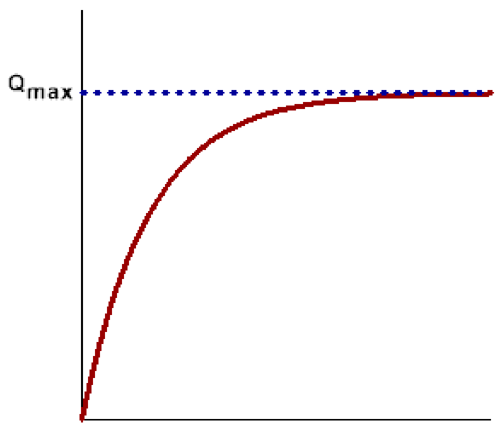
Capacitance is the ratio of the charge stored by a capacitor to the voltage applied:

$$C = \frac{Q}{\Delta V} \quad \text{which is often represented as} \quad Q = C\Delta V$$

Thus one farad is one coulomb per volt. Note, however, that one farad is a ridiculously large amount of capacitance. The capacitors in most electrical circuits are in the millifarad (mF) to picofarad (pF) range.

Capacitance is the theoretical limit of the charge that a capacitor could store at a given potential difference (voltage) if the charge were allowed to build up over an infinite amount of time.

As a capacitor is charged, the positive side increasingly repels additional positive charges coming from the voltage source, and the negative side increasingly repels additional negative charges. This means that the capacitor charges rapidly at first, but the amount of charge stored decreases exponentially as the charge builds up.



Note that Q_{max} is sometimes labeled Q_o. Be careful—in this case, the subscript 0 does ***not*** necessarily mean at time = 0.

Energy Stored in a Capacitor

Recall that energy is the ability to do work, and that $W = \Delta U$. Because $W = qV$, if we keep the voltage constant and add charge to the capacitor:

$$W = \Delta U = \Delta V \Delta q$$

Applying calculus* gives:

$$dU = \Delta V\, dq \qquad \text{and therefore} \qquad U = \int_0^Q \Delta V\, dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

Because $Q = C\Delta V$, we can substitute $C\Delta V$ for Q, giving the equation for the stored (potential) energy in a capacitor:

$$U_C = \frac{Q^2}{2C} = \tfrac{1}{2} QV = \tfrac{1}{2} CV^2$$

* Because this is not a calculus-based course, you are not responsible for understanding this derivation. However, you do need to be able to use the resulting equations.

Parallel-Plate Capacitors and Dielectrics

The capacitance of a parallel plate capacitor is given by the following equation:

$$C = \kappa \varepsilon_o \frac{A}{d}$$

where:

$C =$ capacitance

$\kappa = \varepsilon_r =$ relative permittivity (dielectric constant), vacuum $\equiv 1$

$\varepsilon_o =$ electrical permittivity of free space $= 8.85 \times 10^{-12} \, \frac{\text{F}}{\text{m}}$

$A =$ cross-sectional area

$d =$ distance between the plates of the capacitor

When a capacitor is fully charged, the distance between the plates can be so small that a spark could jump from one plate to the other, shorting out and discharging the capacitor. In order to prevent this from happening, the space between the plates is often filled with a chemical (often a solid material or an oil) called a dielectric.

A dielectric is an electrical insulator (charges do not move, which reduces the possibility of the capacitor shorting out), but has a relatively high value of electric permittivity (ability to support an electric field). (See *Electric Permittivity*, starting on page 163.)

Dielectrics in capacitors serve the following purposes:

- Keep the conducting plates from coming in contact, allowing for smaller plate separations and therefore higher capacitances.
- Increase the effective capacitance by spreading out the charge, reducing the electric field strength and allowing the capacitor to hold same charge at a lower voltage.
- Reduce the possibility of the capacitor shorting out by sparking (more formally known as dielectric breakdown) during operation at high voltage.

Note that a higher value of κ* and lower value of d both enable the capacitor to have a higher capacitance.

Commonly used solid dielectrics include porcelain, glass or plastic (such as polyethylene). Common liquid dielectrics include mineral oil or castor oil. Common gaseous dielectrics include air, nitrogen and sulfur hexafluoride.

* Note that κ is the Greek letter "kappa," not the Roman letter "k".

Electric Field in a Capacitor

From *Electric Fields and Electric Potential*, starting on page 171, $\Delta V = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$.

Because $C = \frac{Q}{\Delta V}$, which means $\Delta V = \frac{Q}{C}$, we can rearrange the above equation to give:

$$C = \kappa \varepsilon_o \frac{A}{d}$$

$$\Delta V = \frac{Q}{C} = \frac{Qd}{\kappa \varepsilon_o A} = \vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}} = Ed\cos\theta$$

Assuming the electric field and displacement are in the same direction, $\cos\theta = 1$, which means the electric field between the plates of a capacitor is:

$$E_c = \frac{Q}{\kappa \varepsilon_o A}$$

Capacitors in Series and Parallel Circuits

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.8.A, 11.8.A.1, 11.8.A.1.i, 11.8.A.1.ii, 11.8.A.1.iii, 11.8.A.2

Mastery Objective(s): (Students will be able to…)

- Calculate voltage, capacitance, charge and potential energy in series and parallel circuits.

Success Criteria:

- Correct relationships are applied for each quantity
- Variables are correctly identified and substituted correctly into the correct equations and algebra is correct.

Language Objectives:

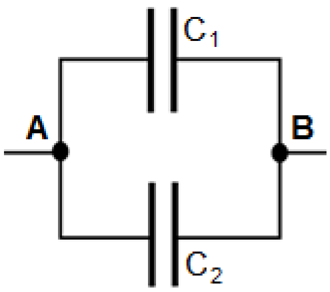
- Explain how capacitors behave similarly to and different from resistors in series and parallel circuits.

Tier 2 Vocabulary: charge, capacitance

Notes:

Capacitors in Parallel

When capacitors are connected in parallel:



The voltage between point **A** and point **B** must be $\Delta V = V_A - V_B$, regardless of the path.

The charge on capacitor C_1 must be $Q_1 = C_1 \Delta V$, and the charge on capacitor C_2 must be $Q_2 = C_2 \Delta V$.

The total charge must be:

$$Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V = (C_1 + C_2)\Delta V$$

Therefore:

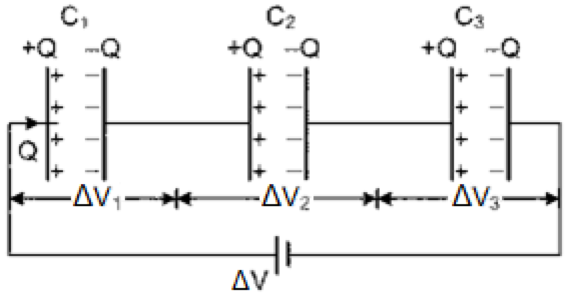
$$C_{eq} = \frac{Q_{total}}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} = C_1 + C_2$$

Generalizing this relationship, when capacitors are arranged in parallel, the total capacitance is the sum of the capacitances of the individual capacitors:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Capacitors in Series

In a series circuit, the voltage from one end to the other is divided among the components.



Note that the segment of the circuit that goes from the right side of C_1 to the left side of C_2 is isolated from the rest of the circuit. Current does not flow through a capacitor, which means charges cannot enter or leave this segment. Because charge is conserved (electrical charges cannot be created or destroyed), this means the negative charge on C_1 (which is $-Q_1$) must equal the positive charge on C_2 (which is $+Q_2$).

By applying this same argument across each of the capacitors, *all of the charges across capacitors in series must be equal to each other and equal to the total charge in that branch of the circuit*. (Note that this is true *regardless* of whether C_1, C_2 and C_3 have the same capacitance.)

Therefore: $Q = Q_1 = Q_2 = Q_3$

Because the components are in series, we also know that $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

Because $\Delta V = \frac{Q}{C}$:

$$\Delta V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

Because $Q = Q_1 = Q_2 = Q_3$:

$$\Delta V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{\Delta V}{Q} = \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Generalizing this relationship, when capacitors are arranged in series, the total capacitance is the sum of the capacitances of the individual capacitors:

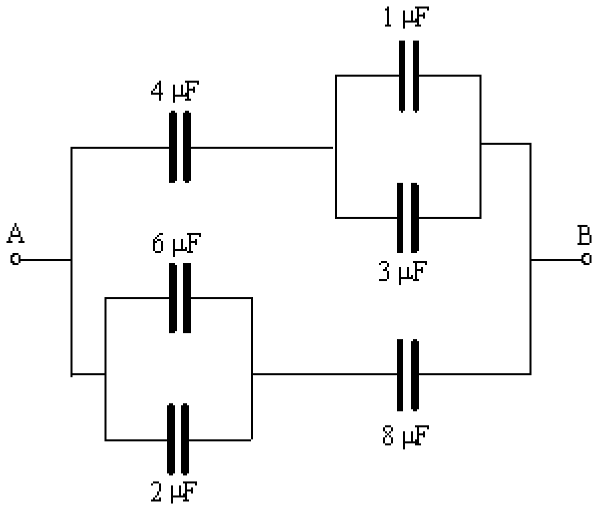
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Mixed Series and Parallel Circuits

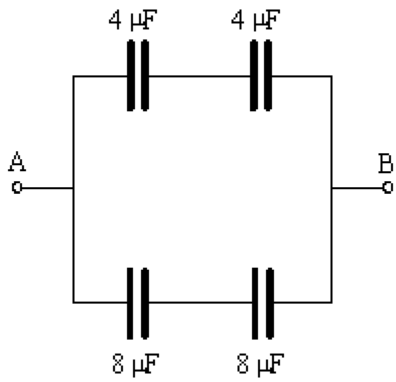
As with resistor networks, mixed circuits involving capacitors in series and in parallel can be simplified by replacing each set of capacitors with an equivalent capacitance, starting with the innermost set of capacitors and working outwards (much like simplifying an equation by starting with the innermost parentheses and working outwards).

Sample Problem:

Simplify the following circuit:



First we would add the capacitances in parallel. On top, $3\,\mu\text{F} + 1\,\mu\text{F} = 4\,\mu\text{F}$. On the bottom, $6\,\mu\text{F} + 2\,\mu\text{F} = 8\,\mu\text{F}$. This gives the following equivalent circuit:



Next, we combine the capacitors in series on top:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

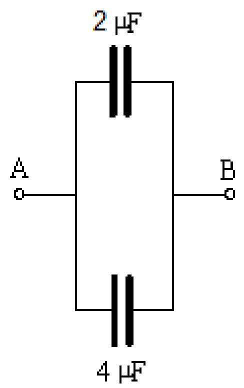
$$\frac{1}{C_{eq}} = \frac{1}{2}; \quad \therefore C_{eq} = 2\ \mu\text{F}$$

and the capacitors in series on the bottom:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\frac{1}{C_{eq}} = \frac{1}{4}; \quad \therefore C_{eq} = 4\ \mu\text{F}$$

This gives the following equivalent circuit:

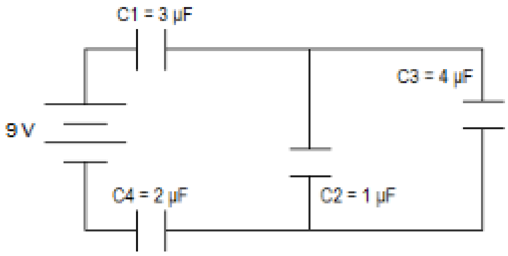


Finally, we combine the last two capacitances in parallel, which gives:

$$C_{eq} = C_1 + C_2 = 2 + 4 = 6\ \mu\text{F}$$

Sample Problem

Q: Fill in the table for the following circuit:



Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	3	1	4	2	
Charge (μC)						
Energy (μJ)						

A: Let's start by combining the two capacitors in parallel (C2 & C3) to make an equivalent capacitor.

$$C_* = C_2 + C_3 = 1\,\mu\text{F} + 4\,\mu\text{F} = 5\,\mu\text{F}$$

Now we have three capacitors in series: C_1, C_*, and C_4:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_*} + \frac{1}{C_4} = \frac{1}{3} + \frac{1}{5} + \frac{1}{2} = 0.\overline{3} + 0.2 + 0.5 = 1.0\overline{3}$$

$$C_{total} = \frac{1}{1.0\overline{3}} = 0.9677\,\mu\text{F}$$

Now we can calculate Q for the total circuit from $Q = C\Delta V$:

$$Q = C\Delta V = (9)(0.9677) = 8.710\,\mu\text{C}$$

Now we have:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	3	1	4	2	**0.9677**
Charge (μC)	Q					**8.710**
Energy (μJ)	U					

Next, because the charge is equal across all capacitors in series, we know that $Q_1 = Q_* = Q_4 = Q_{total}$, which gives:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	**8.710**			**8.710**	8.710
Energy (μJ)	U					

Now we can calculate V_1 and V_4 from $Q = C\Delta V$.

We can also calculate the energy from $U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV	**2.903**			**4.355**	9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	8.710			8.710	8.710
Energy (μJ)	U	**12.64**			**18.96**	**39.19**

We know that voltages in series add, so $\Delta V_{total} = \Delta V_1 + \Delta V_* + \Delta V_4$, which means $9 = 2.903 + \Delta V_* + 4.355$, which gives $\Delta V_* = 1.742\text{ V}$.

Because C_2 and C_3 (and therefore ΔV_2 and ΔV_3) are in parallel, $\Delta V_* = \Delta V_2 = \Delta V_3 = 1.742\text{ V}$:

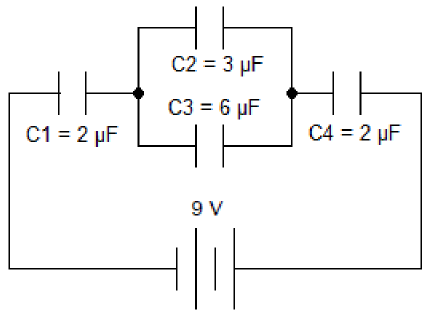
Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV	2.903	**1.742**	**1.742**	4.355	9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	8.710			8.710	8.710
Energy (μJ)	U	12.64			18.96	39.19

Finally, we can calculate Q from $Q = C\Delta V$ and U from $U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$:

Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV	2.903	1.742	1.742	4.355	9
Capacitance (μF)	C	3	1	4	2	0.9677
Charge (μC)	Q	8.710	**1.742**	**6.968**	8.710	8.710
Energy (μJ)	U	12.64	**1.52**	**6.07**	18.96	39.19

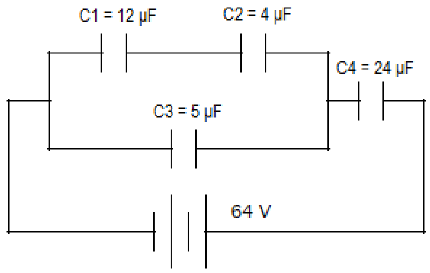
Homework Problems

1. **(S)** Fill in the table for the following circuit:



Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					9
Capacitance (μF)	C	2	3	6	2	
Charge (μC)	Q					
Energy (μJ)	U					

2. **(M)** Fill in the table for the following circuit:



Quantity (unit)	Var.	C1	C2	C3	C4	Total
Voltage (V)	ΔV					64
Capacitance (µF)	C	12	4	5	24	
Charge (µC)	Q					
Energy (µJ)	U					

DC Resistor-Capacitor (RC) Circuits

Unit: DC Circuits

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 11.8.B, 11.8.B.1, 11.8.B.1.i, 11.8.B.1.ii, 11.8.B.1.iii, 11.8.B.2, 11.8.B.2.i, 11.8.B.2.ii, 11.8.B.2.iii, 11.8.B.2.iv, 11.8.B.2.v, 11.8.B.2.vi, 11.8.B.2.vii

Mastery Objective(s): (Students will be able to…)

- Solve problems involving time-varying circuits with charging and discharging capacitors.

Success Criteria:

- Correct relationships are applied for each quantity
- Variables are correctly identified and substituted correctly into the correct equations and algebra is correct.

Language Objectives:

- Explain why a discharged capacitor behaves like a wire, and why a fully-charged capacitor behaves like an open switch.

Tier 2 Vocabulary: charge, capacitance, resistance

Labs, Activities & Demonstrations:

- RC circuit lab

Notes:

RC circuit: a circuit involving combinations of resistors and capacitors.

In an RC circuit, the amount and direction of current change with time as the capacitor charges or discharges. The amount of time it takes for the capacitor to charge or discharge is determined by the combination of the capacitance and resistance in the circuit. This makes RC circuits useful for intermittent (*i.e.,* with a built-in delay) back-and-forth switching. Some common uses of RC circuits include:

- clocks
- windshield wipers
- pacemakers
- synthesizers

When we studied resistor-only circuits, the circuits were steady-state, *i.e.,* voltage and current remained constant. RC circuits are time-variant, *i.e.,* the voltage, current, and charge stored in the capacitor(s) are all changing with time.

Charging a Capacitor

Recall that a capacitor is an electrical component that stores charge. No current actually flows through the capacitor. Recall also that capacitance (*C*) is a capacitor's ability to be charged by a given electric potential difference (voltage). Therefore, the maximum charge that a capacitor can hold is:

$$Q_{max} = C\Delta V$$

In the previous section, the charge that we calculated was actually this maximum charge Q_{max}, which is the amount of charge that the capacitor would hold if it had been charged for "a long time" such that it was fully charged.

However, recall also that:

- In a capacitor with zero charge, every charge placed on one side causes an equivalent charge on the opposite side. This means:

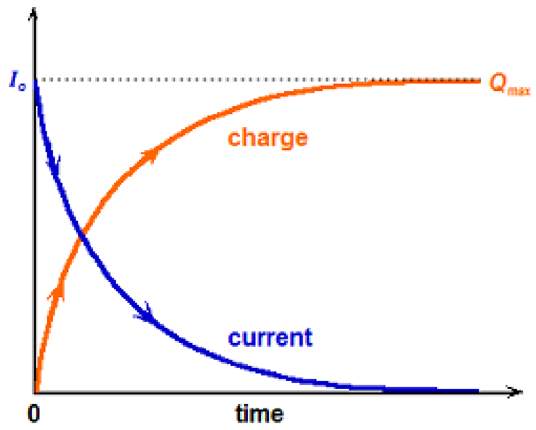
  With respect to current, a capacitor with zero charge initially behaves like a wire.

- When a capacitor is fully charged, no additional charge can be added (unless the voltage is increased). This means:

  With respect to current, a fully-charged capacitor behaves like an open circuit.

This means that the behavior of the capacitor changes as the charges build up inside of it.

When a capacitor that initially has zero charge is connected to a voltage source, the current that flows through the circuit decreases exponentially, and the charge stored in the capacitor asymptotically approaches Q_{max}, the maximum charge that can be stored in that capacitor for the voltage applied.



(Note that the graphs are not to scale; the y-axis scale and units are necessarily different for charge and current.)

honors (not AP®)

The equations for the charge in a capacitor and the current that flows "through" it as a function of time while a capacitor is charging are:

$$I = I_o e^{-t/RC} = \frac{\Delta V}{R} e^{-t/RC}$$

$$Q = Q_{\max}(1 - e^{-t/RC}) = C\Delta V(1 - e^{-t/RC})$$

where:

- I = current (A)
- I_o = initial current (just after switch was closed) (A)
- ΔV = voltage (V)
- Q = charge (C)
- $Q_{\max}$ = (theoretical) maximum charge stored by capacitor at the circuit's voltage (C)
- e = base of exponential function = 2.71828…
- t = time since switch was closed (s)
- R = resistance (Ω)
- C = capacitance (F)

We can rearrange the above equations to give:

$$\frac{I}{I_o} = 1 - \frac{Q}{Q_{\max}} = e^{-t/RC}$$

Note that the value *RC* is the time constant of the circuit, often denoted by the variable τ. Thus we can write the above equation as:

$$\frac{I}{I_o} = 1 - \frac{Q}{Q_{\max}} = e^{-t/\tau}$$

The *RC* term in the exponent is known as the time constant (τ) for the circuit. Larger values of *RC* mean the circuit takes longer to charge the capacitor. The following table shows the rate of decrease in current in the charging circuit and the rate of increase in charge on the capacitor as a function of time:

t	$\frac{I}{I_o} = \frac{\Delta V}{\Delta V_o} = e^{-t/RC}$	$\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$
0	1	0
¼ *RC*	0.78	0.22
½ *RC*	0.61	0.39
0.69 *RC*	0.5	0.5
RC	0.37	0.63
2 *RC*	0.14	0.86
4 *RC*	0.02	0.98
10 *RC*	4.5×10^{-5}	≈ 1

Note that the half-life of the charging (and discharging) process is approximately 0.69 *RC*.

Note also that while Q_{max} depends on the voltage applied, the rate of charging and discharging depend only on the resistance and capacitance in the circuit.

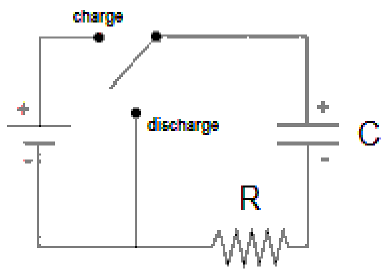
The AP® Physics 2 exam requires only a qualitative understanding of RC circuits. It is sufficient to understand that:

- The time constant $\tau = RC$ represents how quickly a capacitor will charge or discharge.
- After a "long time" (approximately 10τ), a capacitor that is charging can be assumed to be fully charged, and a capacitor that is discharging can be assumed to be fully discharged.
- After an amount of time equal to the time constant $(t = \tau = RC)$, a fully-discharged capacitor will charge to approximately 63 % of its full capacity.
- After an amount of time equal to the time constant $(t = \tau = RC)$, a fully-charged capacitor will discharge to approximately 37 % of its initial charge.

Discharging a Capacitor

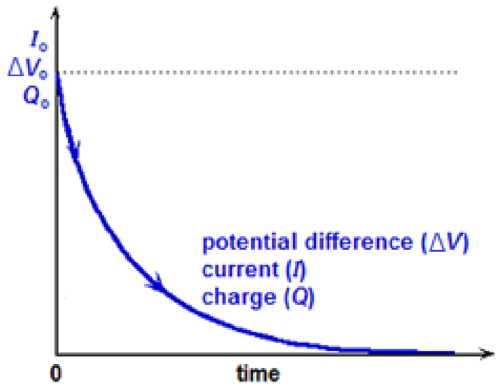
Just as a capacitor charges gradually, it also discharges gradually.

Imagine we have a circuit like the one at the right.



When the switch is in the "charge" position, the battery charges the capacitor. When the switch is flipped to the "discharge" position, the battery is switched out and the circuit contains only the capacitor and the resistor.

When this happens, the capacitor discharges (loses its charge). The capacitor acts as a temporary voltage source, and current temporarily flows out of the capacitor through the resistor.



(Note again that the graphs are not to scale; the y-axis scale and units are necessarily different for current, voltage and charge.)

The driving force for this temporary current is the repulsion from the stored charges in the capacitor. As charge leaves the capacitor there is less repulsion, which causes the voltage and current to decrease exponentially along with the charge.

honors (not AP®)

The equations for discharging a capacitor are therefore identical in form to the equations for charging it:

$$\Delta V = \Delta V_o e^{-t/RC} \qquad Q = Q_o e^{-t/RC} \qquad I = I_o e^{-t/RC}$$

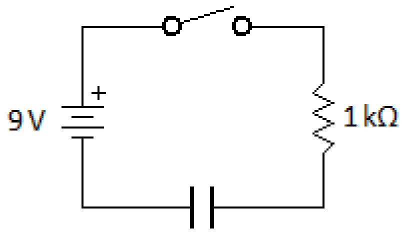
or, dividing each by its value at time zero:

$$\frac{\Delta V}{\Delta V_o} = \frac{Q}{Q_o} = \frac{I}{I_o} = e^{-t/RC}$$

Again the time constant, *RC*, is the relative amount of time it takes for the charge remaining in the capacitor and the voltage and current in the circuit to decay. (Refer to the table on page 271.)

Sample Problem

Q: A circuit has a 9 V battery, an open switch, a 1 kΩ resistor, and a capacitor in series. The capacitor has no residual charge.



When the switch is closed, the charge in the capacitor climbs to 86 % of its maximum value in 50 ms. What is the capacitance of the capacitor?

A: The charge increases at the rate of:

$$Q(t) = Q_{max}(1 - e^{-t/RC})$$

We are given that $\frac{Q}{Q_{max}} = 0.86$, $t = 0.05\,\text{s}$, and $R = 1000\,\Omega$.

$$\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$$

$$0.86 = 1 - e^{-0.05/1000\,C}$$

$$-0.14 = -e^{-0.05/1000\,C}$$

$$\ln(0.14) = \ln(e^{-0.05/1000\,C}) = \frac{-0.05}{1000\,C}$$

$$-1.97 = \frac{-0.05}{1000\,C}$$

$$1970\,C = 0.05$$

$$C = \frac{0.05}{1970} = 2.5 \times 10^{-5}\text{ F} = 25\,\mu\text{F}$$

Note that we could have solved this problem by using the table on page 271 to see that the capacitor is 86 % charged $\left(\frac{Q}{Q_{max}} = 0.86\right)$ when $t = 2RC$.

Homework Problems

1. **(S)** A series RC circuit consists of a 9 V battery, a 3 Ω resistor, a 6 μF capacitor and a switch. How long would it take after the switch is closed for the capacitor to reach 63 % of its maximum potential difference?

   Answer: 18 μs

2. **(M)** A circuit contains a 9 V battery, an open switch, a 1 kΩ resistor, and a capacitor, all in series. The capacitor initially has no charge. When the switch is closed, the charge on the capacitor climbs to 86 % of its maximum value in 50 ms. What is the capacitance of the capacitor?

   Answer: 25 μF

honors (not AP®)

3. **(S)** A heart defibrillator has a capacitance of 25 µF and is charged to a potential difference of 350 V. The electrodes of the defibrillator are attached to the chest of a patient who has suffered a heart attack. The initial current that flows out of the capacitor is 10 mA.

   a. How much time does it take for the current to fall to 0.5 mA?

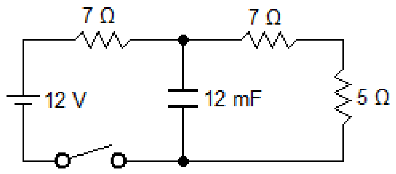
      Answer: 2.62 s

   b. How much charge is left on the defibrillator plates after 1.2 s?

      Answer: 2.22 mC

honors (not AP®)

4. **(M)** In the following RC circuit:



the switch (S) has been closed for a long time.

a. When the switch is opened, how much time does it takes for charge on the capacitor to drop to 13.5% of its original value?

Answer: 0.288 s

b. What is the maximum current through the 5 resistor the instant the switch is opened?

Answer: 0.63 A

Introduction: Magnetism & Electromagnetism

Unit: Magnetism & Electromagnetism

Topics covered in this chapter:

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Magnetism* describes properties of magnets and what causes objects to be magnetic.
- *Magnetic Fields & Magnetic Flux* explains magnetic fields and magnetic flux and how it is calculated.
- *Electromagnetism* describes the relationship between electric fields and magnetic fields, and how changes in one induce changes in the other.
- *Devices that Use Electromagnetism* lists devices that combine electricity and magnetism and explains how they work.

One of the challenges encountered in this chapter is understanding which set of equations applies to a given situation.

Standards addressed in this chapter:

NGSS Standards/MA Curriculum Frameworks (2016):

HS-PS2-5. Provide evidence that an electric current can produce a magnetic field and that a changing magnetic field can produce an electric current.

HS-PS3-5. Develop and use a model of magnetic or electric fields to illustrate the forces and changes in energy between two magnetically or electrically charged objects changing relative position in a magnetic or electric field, respectively.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

12.1.A: Describe the properties of a magnetic field.

12.1.A.1: A magnetic field is a vector field that can be used to determine the magnetic force exerted on moving electric charges, electric currents, or magnetic materials.

12.1.A.1.i: Magnetic fields can be produced by magnetic dipoles or combinations of dipoles, but never by monopoles.

12.1.A.1.ii: Magnetic dipoles have north and south polarity.

12.1.A.2: A magnetic field is a vector quantity and can be represented using vector field maps.

12.1.A.2.i: Magnetic field lines form closed loops.

12.1.A.2.ii: Magnetic fields in a bar magnet form closed loops, with the external magnetic field pointing away from one end (defined as the north pole) and returning to the other end (defined as the south pole).

12.1.B: Describe the magnetic behavior of a material as a result of the configuration of magnetic dipoles in the material.

12.1.B.1: Magnetic dipoles result from the circular or rotational motion of electric charges. In magnetic materials, this can be the motion of electrons.

12.1.B.1.i: Permanent magnetism and induced magnetism are system properties that both result from the alignment of magnetic dipoles within a system.

12.1.B.1.ii: No magnetic north pole is ever found in isolation from a south pole. For example, if a bar magnet is broken in half, both halves are magnetic dipoles.

12.1.B.1.iii: Magnetic poles of the same polarity will repel; magnetic poles of opposite polarity will attract.

12.1.B.1.iv: The magnitude of the magnetic field from a magnetic dipole decreases with increasing distance from the dipole.

12.1.B.2: A magnetic dipole, such as a magnetic compass, placed in a magnetic field will tend to align with the magnetic field.

12.1.B.3: A material's composition influences its magnetic behavior in the presence of an external magnetic field.

12.1.B.3.i: Ferromagnetic materials such as iron, nickel, and cobalt can be permanently magnetized by an external field that causes the alignment of magnetic domains or atomic magnetic dipoles.

AP® only

12.1.B.3.ii: Paramagnetic materials such as aluminum, titanium, and magnesium interact weakly with an external magnetic field, in that the magnetic dipoles of the material do not remain aligned after the external field is removed.

12.1.B.3.iii: All materials have the property of diamagnetism, in that their electronic structure creates a usually weak alignment of the dipole moments of the material opposite the external magnetic field.

12.1.B.4: Earth's magnetic field may be approximated as a magnetic dipole.

12.1.C: Describe the magnetic permeability of a material.

12.1.C.1: Magnetic permeability is a measurement of the amount of magnetization in a material in response to an external magnetic field.

12.1.C.2: Free space has a constant value of magnetic permeability, known as the vacuum permeability μ_o, that appears in equations epresenting physical relationships.

12.1.C.3: The permeability of matter has values different from that of free space and arises from the matter's composition and arrangement. It is not a constant for a material and varies based on many factors, including temperature, orientation, and strength of the external field.

12.2.A: Describe the magnetic field produced by moving charged objects.

12.2.A.1: A single moving charged object produces a magnetic field.

12.2.A.1.i: The magnetic field at a particular point produced by a moving charged object depends on the object's velocity and the distance between the point and the object.

12.2.A.1.ii: At a point in space, the direction of the magnetic field produced by a moving charged object is perpendicular to both the velocity of the object and the position vector from the object to that point in space and can be determined using the right-hand rule.

12.2.A.1.iii: The magnitude of the magnetic field is a maximum when the velocity vector and the position vector from the object to that point in space are perpendicular.

12.2.B: Describe the force exerted on moving charged objects by a magnetic field.

12.2.B.1: Magnetic forces describe interactions between moving charged objects.

AP® only

12.2.B.2: A magnetic field may exert a force on a charged object moving in that field.

12.2.B.2.i: The magnitude of the force exerted by a magnetic field on a moving charged object is proportional to the magnitude of the charge, the magnitude of the charged object's velocity, and the magnitude of the magnetic field and also depends on the angle between the velocity and magnetic field vectors.

12.2.B.2.ii: The direction of the force exerted by a magnetic field on a moving charged object is perpendicular to both the direction of the magnetic field and the velocity of the charge, as defined by the right-hand rule.

12.2.B.3: In a region containing both a magnetic field and an electric field, a moving charged object will experience independent forces from each field.

12.2.B.4: The Hall effect describes the potential difference created in a conductor by an external magnetic field that has a component perpendicular to the direction of charges moving in the conductor.

12.3.A: Describe the magnetic field produced by a current-carrying wire.

12.3.A.1: A current-carrying wire produces a magnetic field.

12.3.A.1.i: The magnetic field vectors around a long, straight, current-carrying wire are tangent to concentric circles centered on that wire. The field has no component toward, away from, or parallel to the long, straight, current-carrying wire.

12.3.A.1.ii: At a point in space, the magnitude of the magnetic field due to a long, straight, current-carrying wire is proportional to the magnitude of the current in the wire and inversely proportional to the perpendicular distance from the central axis of the wire to the point.

12.3.A.1.iii: The direction of the magnetic field created by a current-carrying wire is determined with the right-hand rule.

12.3.A.1.iv: The direction of the magnetic field at the center of a current-carrying loop is directed along the axis of the loop and can be found using the right-hand rule.

12.3.A.1.v: The magnetic field at a location near two or more current-carrying wires can be determined using vector addition principles.

AP® only

12.3.B: Describe the force exerted on a current-carrying wire by a magnetic field.

12.3.B.1: A magnetic field may exert a force on a currentcarrying wire.

12.3.B.1.i: The magnitude of the force exerted by a magnetic field on a current-carrying wire is proportional to the current, the length of the portion of the wire within the magnetic field, and the magnitude of the magnetic field, and also depends on the angle between the direction of the current in the wire and the direction of the magnetic field.

12.3.B.1.ii: The direction of the force exerted by the magnetic field on a current-carrying wire is determined by the right-hand rule.

12.4.A: Describe the induced electric potential difference resulting from a change in magnetic flux.

12.4.A.1: Magnetic flux is a description of the amount of the component of a magnetic field that is perpendicular to a cross-sectional area.

12.4.A.2: Magnetic flux through a surface is proportional to the magnitude of the component of the magnetic field perpendicular to the surface and to the cross-sectional area of the surface.

12.4.A.2.i: The area vector is defined to be perpendicular to the plane of the surface and directed outward from a closed surface.

12.4.A.2.ii: The sign of the magnetic flux indicates whether the magnetic field is parallel to or antiparallel to the area vector.

12.4.A.3: Faraday's law describes the relationship between changing magnetic flux and the resulting induced emf in a system.

12.4.A.4: Lenz's law is used to determine the direction of an induced emf resulting from a changing magnetic flux.

12.4.A.4.i: An induced emf generates a current that creates a magnetic field that opposes the change in magnetic flux.

12.4.A.4.ii: The right-hand rule is used to determine the relationships between current, emf, and magnetic flux.

12.4.A.5: A common example of electromagnetic induction is a conducting rod on conducting rails in a region with a uniform magnetic field.

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.

Magnetism & Magnetic Permeability

Unit: Magnetism & Electromagnetism

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 12.1.B, 12.1.B.1, 12.1.B.1.i, 12.1.B.1.ii, 12.1.B.1.iii, 12.1.B.1.iv, 12.1.B.2, 12.1.B.3, 12.1.B.3.i, 12.1.B.3.ii, 12.1.B.3.ii, 12.1.B.4, 12.1.C, 12.1.C.1, 12.1.C.2, 12.1.C.3

Mastery Objective(s): (Students will be able to…)

- List and explain properties of magnets.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Explain why we call the ends of a magnet "north" and "south".

Tier 2 Vocabulary: magnet

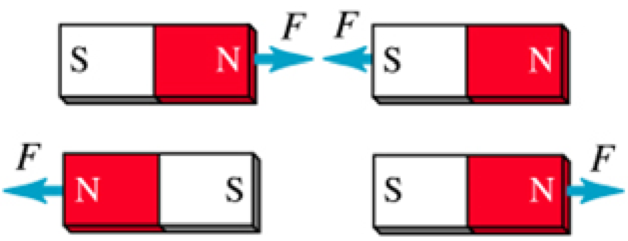
Labs, Activities & Demonstrations:

- neodymium magnets
- ring magnets repelling each other on a dowel
- magnets attracting each other across a gap

Notes:

magnet: a material with electrons that can align in a manner that attracts or repels other magnets.

A magnet is a dipole, meaning that it has has two opposite ends or "poles", called "north" and "south". If a magnet is allowed to move freely, the end that points toward the north on Earth is called the north end of the magnet. The end that points toward the south on Earth is called the south end of the magnet. (The Earth's magnetic poles are near, but not in exactly the same place as its geographic poles.) All magnets have a north and south pole. As with charges, opposite magnetic poles attract, and like poles repel.



There is no such thing as a magnetic monopole (*i.e.,* a north or south pole by itself). If you were to cut a magnet in half, each piece would be a magnet with its own north and south pole:

Electrons and Magnetism

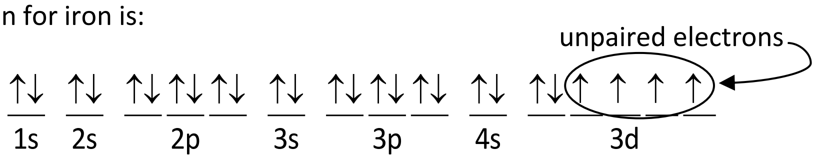
Magnetism is caused by the alignment of unpaired electrons in atoms.

As you (should) recall from chemistry, electrons within atoms reside in energy regions called "orbitals". Each orbital can hold up to two electrons.

Honors (not AP®)

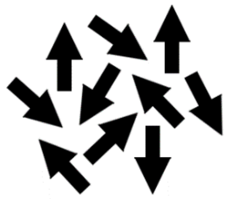
If two electrons share an orbital, they have opposite spins. (Note that the electrons are not actually spinning. "Spin" is the term for the intrinsic property of certain subatomic particles that is believed to be responsible for magnetism.) This means that if one electron aligns itself with a magnetic field, the other electron in the same orbital becomes aligned to oppose the magnetic field, and there is no net force.

However, if an orbital has only one electron, that electron is free to align with the magnetic field, which causes an attractive force between the magnet and the magnetic material. For example, as you learned in chemistry, the electron configuration for iron is:

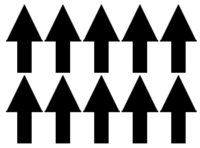


The inner electrons are paired up, but four of the electrons in the 3d sublevel are unpaired, and are free to align with an external magnetic field.

When the electrons in a substance are aligned randomly, the substance is not magnetic, and is not attracted to a magnet.



When the electrons in a substance are aligned, the substance is magnetic and will be attracted to a magnet.



diamagnetic: a material whose electrons are unable to align with a magnetic field. These substances will weakly repel a magnet.

paramagnetic: a material that has electrons that can move to align with a magnetic field. These materials are attracted to a magnet, but are not themselves magnets.

ferromagnetic: a material with crystals that have permanently-aligned electrons, resulting in a permanent magnet. Some naturally-occurring materials that exhibit ferromagnetism include iron, cobalt, nickel, gadolinium, dysprosium, and magnetite (Fe_3O_4).

Honors (not AP®)

Curie temperature (or Curie point): the temperature above which a ferromagnetic material becomes paramagnetic.

The Curie temperature is named after the French physicist Pierre Curie, who discovered this effect.

If a paramagnetic material is heated above its Curie temperature and then cooled in the presence of a magnetic field, the material becomes ferromagnetic. This is how permanent magnets are made.

Substance	**Curie temperature (°C)**	**Substance**	**Curie temperature (°C)**
iron	770	cobalt	1130
iron(III) oxide	675	nickel	354
magnetite (iron (II,III) oxide)	585	dysprosium	−185.2

Magnetic Permeability

Magnetic measurements and calculations involve fields that act over 3-dimensional space and change continuously with position. This means that most calculations relating to magnetic fields need to be represented using multivariable calculus, which is beyond the scope of this course.

magnetic permeability (magnetic permittivity): the ability of a material to support the formation of a magnetic field. Magnetic permeability is represented by the variable μ. The magnetic permeability of empty space is defined to be $\mu_0 = 4\pi \times 10^{-7}\,\frac{\text{N}}{\text{A}^2}$.

Honors (not AP®)

magnetic susceptibility (χ): a measure of the degree to which a material will become magnetized when it is placed in a magnetic field.

Diamagnetic materials have negative magnetic susceptibilities.

Paramagnetic materials have positive magnetic susceptibilities.

Ferromagnetic materials do not have well-defined magnetic susceptibilities, because these substances create their own magnetic fields, which interact with the external magnetic field.

Magnetic Fields

Unit: Magnetism & Electromagnetism

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 12.1.A, 12.1.A.1, 12.1.A.1.i, 12.1.A.1.ii, 12.1.A.2, 12.1.A.2.i, 12.1.A.2.ii, 12.1.B.2, 12.1.B.4

Mastery Objective(s): (Students will be able to…)

- Describe and draw magnetic fields.
- Calculate magnetic flux.

Success Criteria:

- Magnetic field lines connect north and south poles of the magnet.
- Arrows on field lines point from north to south.

Language Objectives:

- Explain how a compass works.

Tier 2 Vocabulary: field, north pole, south pole

Labs, Activities & Demonstrations:

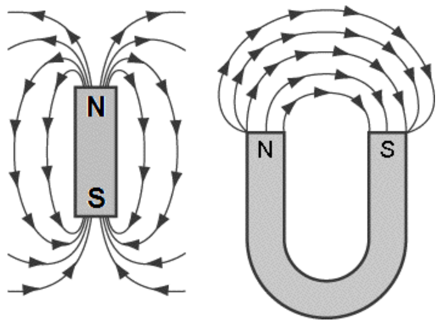
- magnetic field demonstrator plate
- placing various objects into the gap between two magnets
- ferrofluid
- representation of flux as dots on a balloon

Notes:

magnetic field ($\vec{B}$) : a force field (region in which a force acts on objects that have a certain property) in which magnetic attraction and repulsion are occurring.

Any object that is a magnetic dipole (*i.e.,* a magnet or something that behaves like one) creates its own magnetic field. If the object is allowed to move freely, the magnetic field will attract and repel the poles of the object so that it aligns with the magnetic field.

Similar to an electric field, we represent a magnetic field by drawing field lines. Magnetic field lines point from the north pole of a magnet toward the south pole, and they show the direction that the north end of a compass or magnet would be deflected if it was placed in the field:

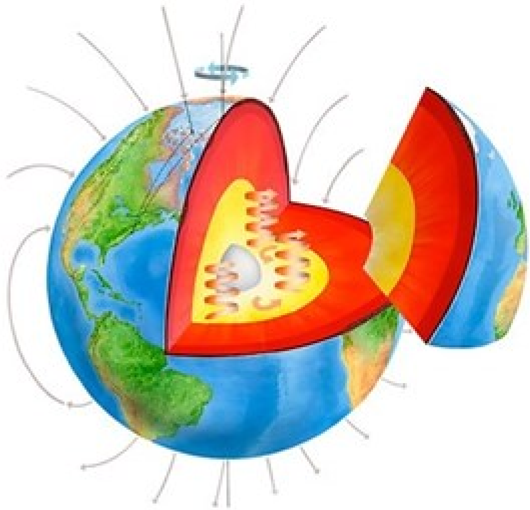


The strength of a magnetic field is measured in teslas (T), named after the physicist Nikola Tesla. One tesla is the magnetic field strength necessary to produce one newton of force when a particle that has a charge of one coulomb is moved through the magnetic field at a velocity of one meter per second. Because of the relationship between magnetism, forces, and electricity, one tesla can be expressed as many different combinations of units:

$$1\text{ T} \equiv 1\frac{\text{V}\cdot\text{s}}{\text{m}^2} \equiv 1\frac{\text{N}}{\text{A}\cdot\text{m}} \equiv 1\frac{\text{J}}{\text{A}\cdot\text{m}^2} \equiv 1\frac{\text{kg}}{\text{C}\cdot\text{s}} \equiv 1\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}} \equiv 1\frac{\text{kg}}{\text{A}\cdot\text{s}^2}$$

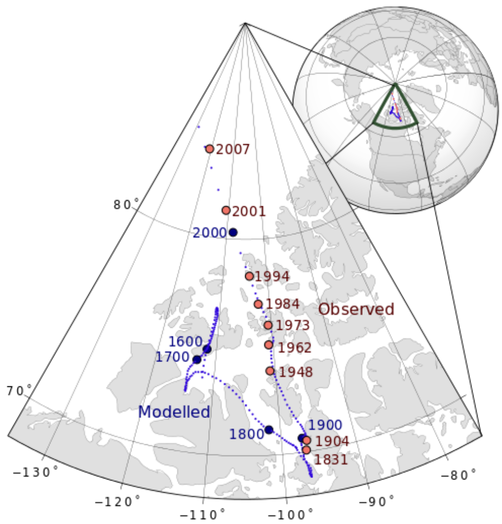
The Earth's Magnetic Field

The inner and outer core of the Earth are made of iron, which has a high magnetic susceptibility. The very high temperature of the inner core causes convection currents in the molten iron in the outer core.

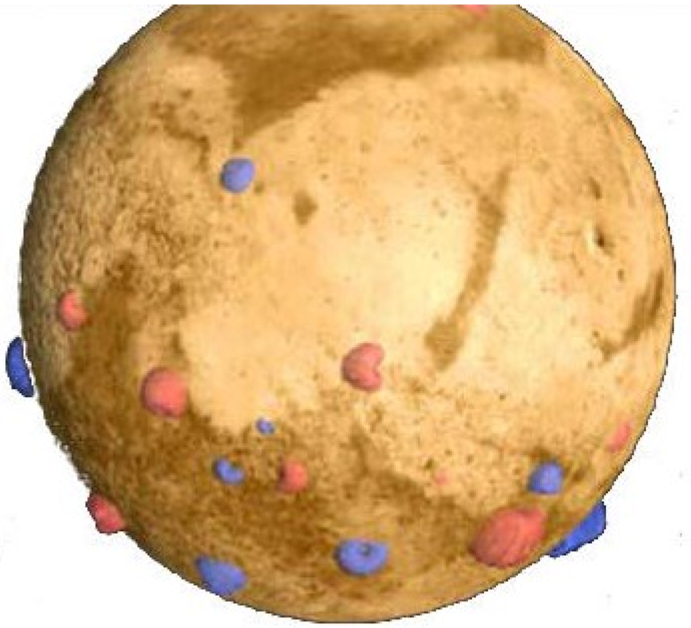


The rapid rotation of the Earth causes the molten iron in the outer core to swirl. The swirling iron causes a magnetic field over the entire planet.

Because the core of the Earth is in constant motion, the Earth's magnetic field is constantly changing. The exact location of the Earth's magnetic north and south poles varies by about 80 km over the course of each day because of the rotation of the Earth. Its average location (shown on the map of Northern Canada below) drifts by about 50 km each year:



Not all planets have a planetary magnetic field. Mars, for example, is believed to have once had a planetary magnetic field, but the planet cooled off enough to disrupt the processes that caused it. Instead, Mars has some very strong localized magnetic fields that were formed when minerals cooled down in the presence of the planetary magnetic field.



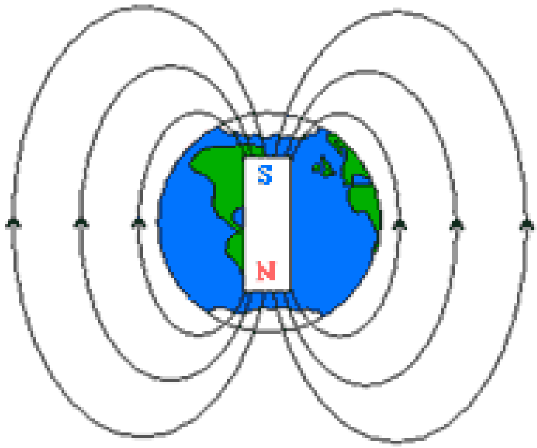
In this picture, the blue and red areas represent regions with strong localized magnetic fields. On Mars, a compass could not be used in the ways that we use a compass on Earth; if you took a compass to Mars, the needle would point either toward or away from each these regions.

Jupiter, on the other hand, has a planetary magnetic field twenty times as strong as that of Earth. This field may be caused by water with dissolved electrolytes or by liquid hydrogen.

Recall that the north pole of a magnet is the end that points toward the north on Earth. This must mean that if the Earth is a giant magnet, one of its magnetic poles must be near the geographic north pole, and the other magnetic pole must be near the geographic south pole.

For obvious reasons, the Earth's magnetic pole near the north pole is called the Earth's "north magnetic pole" or "magnetic north pole". Similarly, the Earth's magnetic pole near the south pole is called the Earth's "south magnetic pole" or "magnetic south pole".

However, because the north pole of a magnet points toward the north, the Earth's north magnetic pole (meaning its location) must therefore be the *south* pole of the giant magnet that is the Earth.



Similarly, because the south pole of a magnet points toward the south, the Earth's south magnetic pole (meaning its location) must therefore be the *north* pole of the giant Earth-magnet.

Unfortunately, the term "magnetic north pole," "north magnetic pole" or any other similar term almost always means the magnetic pole that is in the north part of the Earth. There is no universally-accepted way to name the poles of the Earth-magnet.

Magnetism & Moving Charges

Unit: Magnetism & Electromagnetism

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 12.2.A, 12.2.A.1, 12.2.A.1.i, 12.2.A.1.ii, 12.2.A.1.iii, 12.2.B, 12.2.B.1, 12.2.B.2, 12.2.B.2.i, 12.2.B.2.ii, 12.2.B.3, 12.2.B.4, 12.3.A, 12.3.A.1, 12.3.A.1.i, 12.3.A.1.ii, 12.3.A.1.iii, 12.3.A.1.iv, 12.3.A.1.v, 12.3.B.1, 12.3.B.1.i, 12.3.B.1.ii

Mastery Objective(s): (Students will be able to…)

- Explain how a moving charge (including current in a wire) creates a magnetic field.
- Explain how a magnetic field exerts a force on a moving charge.
- Solve problems involving moving charges in a magnetic field.
- Solve problems involving the magnetic field produced by a current-carrying wire.
- Solve problems involving the force produced by a current-carrying wire in an external magnetic field.

Success Criteria:

- Equations are set up correctly, with correct variable substitution.

Language Objectives:

- Identify the parts of a diagram or lab/demonstration set-up, and which quantity corresponds to which object.
- Explain how each object in the diagram or set-up contributes to the magnetic field and forces.

Tier 2 Vocabulary: field, wire, current

Labs, Activities & Demonstrations:

- current-carrying wire in a magnetic field
- electromagnet
- two coils wired together and two rare earth magnets, one on a spring
- electric motor
- magnet through copper pipe (Lenz's Law)
- wire & galvanometer jump rope
- neodymium magnet & CRT screen

Notes:

Recall that electrons have both charge and a property called "spin"*, which is responsible for magnetism, as discussed in *Magnetism & Magnetic Permeability*, starting on page 282.

The interaction of these properties has several consequences:

- When an electron moves, both its charge and its magnetic spin are moving. The movement of this magnetic spin creates a magnetic field.
- An electric field is produced by charged objects. (See *Electric Fields and Electric Potential*, starting on page 171.) When these charges move, the electric field changes, and the movement of the charges produces a magnetic field.

Therefore, ***a changing electric field produces a magnetic field***, and ***a changing magnetic field produces an electric field***.

Note that this and the remaining topics in this unit involve the cross-product of vectors and the right-hand rule. It would be useful to review *Vector Multiplication*, starting on page 28.

Forces and Moving Charges

The changing electric field given by the cross-product of the velocity of the charged particle and the magnetic field:

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \qquad \text{and} \qquad E = vB\sin\theta$$

Because the force is given by $\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}}$, the force $\vec{\boldsymbol{F}}$ on a point charge q that is moving through a magnetic field $\vec{\boldsymbol{B}}$ with velocity $\vec{\boldsymbol{v}}$ is therefore:

$$\vec{\boldsymbol{F}}_B = q(\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}) \qquad \text{and} \qquad F_B = qvB\sin\theta$$

The cross-product means that the direction of the force must be perpendicular to both the velocity of the charged object and the magnetic field.

* Remember that electrons are not actually spinning. The property was named based on a simpler understanding of the behavior of electrons than we have now.

Magnetism, Forces, and Current-Carrying Wires

Recall that current is a flow of charges, which means that an electric current moving through a magnetic field creates a force on the wire carrying the current.

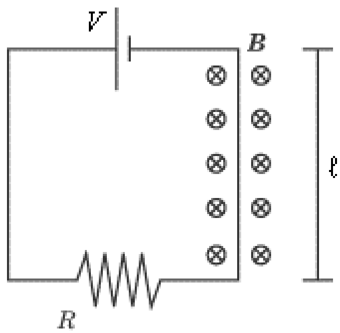
Recall from *Electric Current & Ohm's Law* starting on page 197, that $\vec{I} = \frac{\Delta q}{t}$. Recall from physics 1 that $\vec{v} = \frac{\vec{d}}{t} = \frac{\ell}{t}$. In this case, we can replace $\vec{d}$ with ℓ, where ℓ is the length (distance) of the section of the wire that passes through the magnetic field.

Combining these equations gives $q\vec{v} = \ell\vec{I}$, which we can use to create the equation:

$$\vec{F} = \ell(\vec{I} \times \vec{B}) \qquad \text{and} \qquad F = \ell I B \sin\theta$$

Note that the direction of the cross products $\vec{v} \times \vec{B}$ and $\vec{I} \times \vec{B}$ can be determined using the right-hand rule.

A current passing through a magnetic field would be represented like the diagram at the right.



In this diagram, the battery has voltage *V*, the resistor has resistance *R*, and the length of wire passing through the magnetic field is ℓ. The direction of the current, $\vec{I}$, as it passes through the magnetic field is from the bottom of the page to the top.

The magnetic field has strength ***B***, and is denoted by the symbols ⊗ ⊗ ⊗ ⊗ ⊗ which denote a magnetic field going <u>*into*</u> the page. (A field coming out of the page would be denoted by ⊙ ⊙ ⊙ ⊙ ⊙ instead. Think of the circle as an arrow inside a tube. The dot represents the tip of the arrow facing toward you, and the "X" represents the fletches (feathers) on the tail of the arrow facing away from you.)

By the right-hand rule, the direction of the force on the wire would be to the left.

For example, suppose we were given the following for the above diagram:

$$\vec{\boldsymbol{B}} = 4.0\times10^{-5}\text{ T}$$
$$V = 30\text{ V}$$
$$R = 5\,\Omega$$
$$\ell = 2\text{ m}$$

Current (from Ohm's Law):

$$V = IR$$
$$30 = I(5)$$
$$I = 6\text{A}$$

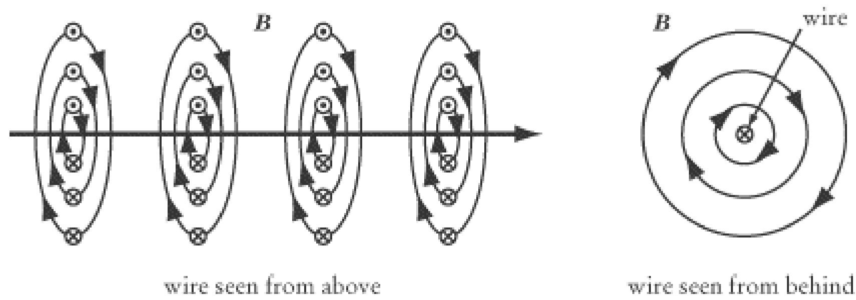
Force on the wire:

$$\vec{\boldsymbol{F}} = \ell(\vec{\boldsymbol{I}}\times\vec{\boldsymbol{B}})$$
$$F = \ell IB\sin\theta$$
$$F = (2)(6)(4.0\times10^{-5})\sin(90^\circ)$$
$$F = (2)(6)(4.0\times10^{-5})(1)$$
$$F = 4.8\times10^{-4}\text{N}$$

If the current is going upward through the magnetic field, and the magnetic field is pointing into the paper, then the right-hand rule tells us that the force would be directed to the left.

Magnetic Field Produced by Electric Current

An electric current moving through a wire also produces a magnetic field around the wire:



This time, we use the right-hand rule with our thumb pointing in the direction of the current, and our fingers curl in the direction of the magnetic field.

The strength of the magnetic field produced is given by the Ampere's Law, named for the French physicist, André-Marie Ampère, who discovered this relationship in the 1820s:

$$B = \frac{\mu_o}{2\pi}\cdot\frac{I}{r}$$

where *B* is the strength of the magnetic field, μ_o is the magnetic permeability of free space, I is the current, and *r* is the distance from the wire. (The variable *r* is used because the distance is in all directions, which means we would use polar or cylindrical coordinates.)

Combined Electric and Magnetic Fields

As stated above, the force on a charged particle due to an electric field is:

$$\vec{F}_e = q\vec{E}$$

and the force on a charged particle due to a magnetic field is:

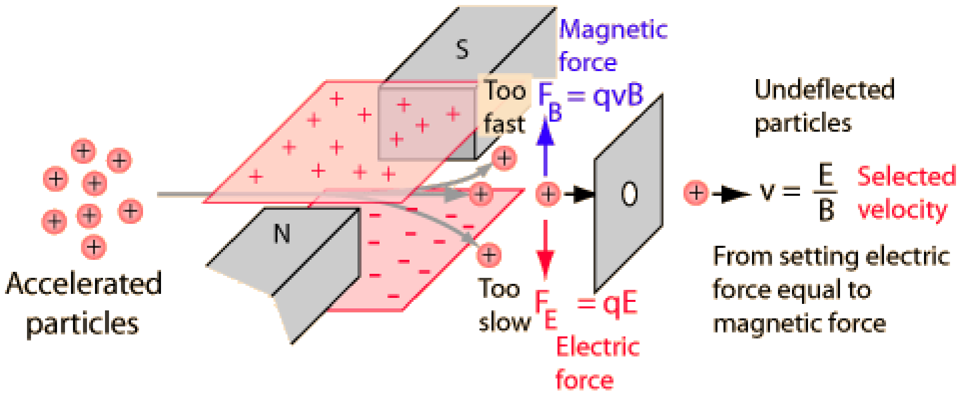
$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

Therefore, the force on a charged particle that interacts simultaneously with an electric field and a magnetic field must be the sum of the two:

$$\vec{F}_{EM} = q(\vec{E} + \vec{v} \times \vec{B})$$

An application of this combination is a particle sorter, which allows only particles with a given velocity to pass through.

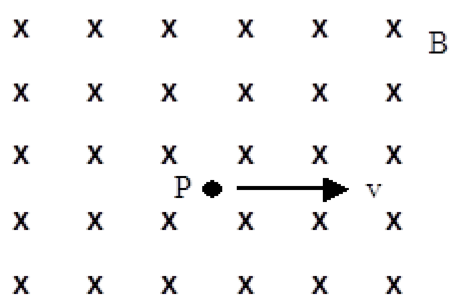
Particles that enter a mass spectrometer must have the correct velocity in order for the mass spectrometer to be able to separate the particles properly. Before the particles enter the mass spectrometer, they first pass through a particle sorter, which applies opposing electric and magnetic forces to the particle:



If the particles are moving too quickly, the magnetic force is stronger and the particles are deflected upwards. If the particles are moving too slowly, the electric force is stronger and the particles are deflected downwards. For particles with the desired velocity, the forces are equal, there is no net force, and the particles are not deflected.

Sample Problem:

Q: A proton has a velocity of $1\times 10^{5}\,\frac{\text{m}}{\text{s}}$ to the right when it is at point *P* in a uniform magnetic field of 0.1 T that is directed into the page. Calculate the force (magnitude and direction) on the proton, and sketch its path.



A: The force on the proton is given by:

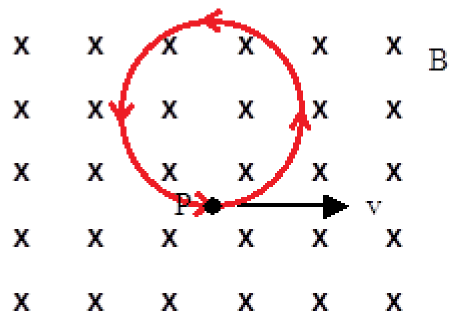
$$\vec{F}_B = q(\vec{v}\times\vec{B}) = qvB\sin\theta$$

Because the velocity of the particle and the direction of the magnetic field are perpendicular, $\sin\theta = \sin(90°) = 1$ and therefore the magnitude of $F_B = qvB$.

$$F_B = qvB$$
$$F_B = (1.6\times 10^{-19})(1\times 10^{5})(0.1)$$
$$F_B = 1.6\times 10^{-15}\,\text{N}$$

The direction is given by the right-hand rule. Start with the fingers of your right hand pointing straight in the direction of velocity (to the right) and rotate your right hand until you can bend your fingers toward the magnetic field (into the page). Your thumb will be pointing upwards, which means that the force on a positively charged particle moving to the right is upwards. (Note that if the particle had been negatively charged, it would have moved in the opposite direction.)

However, note that the action of the force causes a change in the direction of the proton's velocity, and the change in the direction of the proton's velocity changes the direction of the force. This feedback loop results in the proton moving in a continuous circle.



Homework Problems

1. **(M)** A wire 1 m long carries a current of 5 A. The wire is at right angles to a uniform magnetic field. The force on the wire is 0.2 N. What is the strength of the magnetic field?

   Answer: 0.04 T

2. **(S)** A wire 0.75 m long carries a current of 3 A. The wire is at an angle of 30° to a uniform magnetic field. The force on the wire is 0.5 N. What is the strength of the magnetic field?

   Answer: $0.\overline{4}$ T

3. **(M)** Two currents (one of 2 A and the other of 4 A) flow through wires that are parallel to each other, with the currents flowing in the same direction. The wires are 3 m long and are separated by 8 cm (0.08 m). What is the net magnetic field at the midpoint between the two wires?

   (*Hint: Find the magnetic field produced by each wire and add them, remembering that magnetic fields are vectors.*)

   Answer: 1×10^{-5} T

4. **(M)** A wire 2 m long moves perpendicularly through a 0.08 T field at a speed of $7\,\frac{\text{m}}{\text{s}}$.

   a. What emf is induced?

   Answer: 1.12 V

   b. If the wire has a resistance of 0.50 Ω, use Ohm's Law to find the current that flows through the wire.

   Answer: 2.24 A

Electromagnetic Induction & Faraday's Law

Unit: Magnetism & Electromagnetism

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 12.1.A, 12.1.A.1, 12.1.A.1.i, 12.1.A.1.ii, 12.1.A.2, 12.1.A.2.i, 12.1.A.2.ii, 12.1.B.2, 12.1.B.4

Mastery Objective(s): (Students will be able to…)

- Describe and draw magnetic fields.
- Calculate magnetic flux.

Success Criteria:

- Magnetic field lines connect north and south poles of the magnet.
- Arrows on field lines point from north to south.

Language Objectives:

- Explain how a compass works.

Tier 2 Vocabulary: field, north pole, south pole

Labs, Activities & Demonstrations:

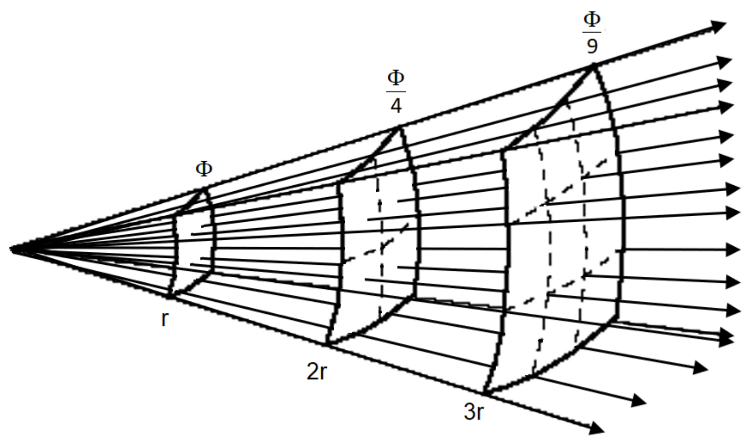
- ferrofluid
- representation of flux as dots on a balloon

Notes:

Magnetic Flux

<u>flux</u>: the flow of fluid, energy or particles across a given area.

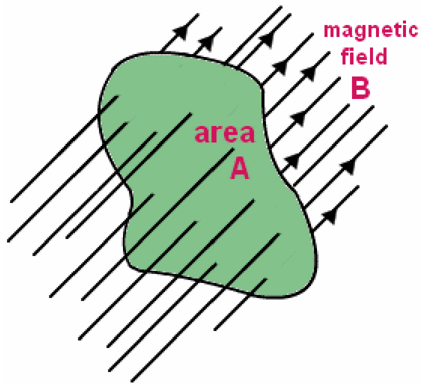
If a quantity (such as a magnetic field) originates from a point, the field spreads out and the amount of flux through a given area decreases as the square of the distance from that point.



magnetic flux (Φ): the total amount of a magnetic field that passes through a surface.

Stronger magnetic fields are generally shown with a higher density of field lines. Using this representation, you can think of the magnetic flux as the number of field lines that pass through an area.

The equation for magnetic flux is Faraday's Law, named for the English physicist Michael Faraday. The equation is usually presented as a surface integral, but in algebraic form it looks like the following:



$$\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}} = BA\cos\theta$$

where:

Φ = magnetic flux (Wb)

$\vec{\boldsymbol{B}}$ = strength of magnetic field (T)

$\vec{\boldsymbol{A}}$ = area of the region of interest that the magnetic field passes through (m^2)

The unit for magnetic flux is the weber (Wb). One tesla is one weber per square meter.

$$1\,\text{T} \equiv 1\,\frac{\text{Wb}}{\text{m}^2}$$

Magnetic Fields and Electric Current

Like gravitational and electric fields, a magnetic field is a force field. (Recall that force fields are vector quantities, meaning that they have both magnitude and direction.) The strength of a magnetic field is measured in teslas (T), named after the Serbian-American physicist Nikola Tesla.

$$1\,\text{T} = 1\,\frac{\text{N}}{\text{A·m}}$$

In the 1830s, English physicists Michael Faraday and Joseph Henry each independently discovered that an electric current could be produced by moving a magnet through a coil of wire, or by moving a wire through a magnetic field. This process is called electromagnetic induction.

If we move a conductive rod or wire that has length ℓ at a velocity *v* through a magnetic field of strength *B*, the magnetic forces send positive charges to one end of the rod and negative charges to the other. This creates a potential difference (emf) between the ends of the rod:

$$\mathcal{E} = vB\ell$$

If the rod or wire is part of a closed loop (circuit), then the induced ε produces a current around the closed loop. From Ohm's Law, $\mathcal{E} = IR$, we get:

$$I = \frac{\mathcal{E}}{R} = \frac{vB\ell}{R}$$

This production of emf is called the Hall effect, after American physicist Edwin Hall, who discovered this relationship in 1879.

EMF Produced by Changing Magnetic Flux

A changing magnetic field produces an electromotive force (emf) in a loop of wire. This emf is given by the equation:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(BAcos\theta)}{\Delta t}$$

(calculus) (algebraic)

If we replace the loop of wire with a coil that has *n* turns, the equation becomes:

$$\mathcal{E} = -n\frac{d\Phi_B}{dt} = -n\frac{\Delta\Phi_B}{t}$$

(calculus) (algebraic)

Devices that Use Electromagnetism

Unit: Magnetism & Electromagnetism

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 12.2.B, 12.2.B.1, 12.2.B.2, 12.2.B.2.ii, 12.3.A.1, 12.3.A.1.i, 12.3.A.1.iii, 12.3.A.1.iv, 12.3.B.1, 12.3.B.1.ii

Mastery Objective(s): (Students will be able to…)

- Explain the basic design of solenoids, motors, generators, transformers and mass spectrometers.
- Calculate the voltage and power from a step-up or step-down transfromer.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Calculations are set up and executed correctly.

Language Objectives:

- Explain how various devices work including solenoids, electromagnets and electric motors.

Tier 2 Vocabulary: force, field

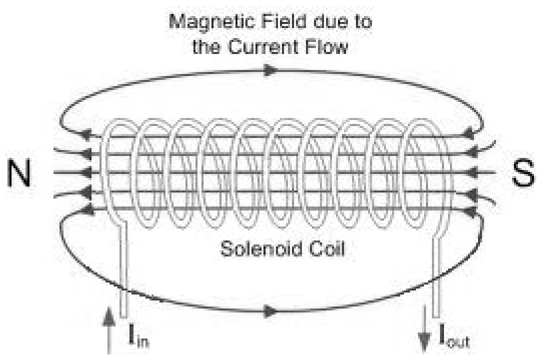
Labs, Activities & Demonstrations:

- build electromagnet
- build electric motor
- build speaker

Notes:

Solenoid

A solenoid is a coil made of fine wire. When a current is passed through the wire, it produces a magnetic field through the center of the coil.



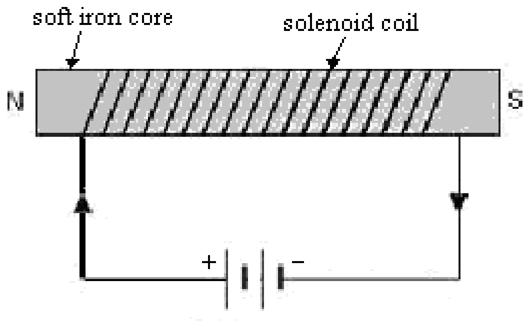
When a current is applied, a permanent magnet placed in the center of the solenoid will be attracted or repelled and will move.

One of the most common uses of a solenoid is for electric door locks.

Electromagnet

An electromagnet is a device that acts as magnet only when electric current is flowing through it.

An electromagnet is made by placing a soft iron core in the center of a solenoid. The high magnetic permeability of iron causes the resulting magnetic field to become thousands of times stronger:



Because the iron core is not a permanent magnet, the electromagnet only works when current is flowing through the circuit. When the current is switched off, the electromagnet stops acting like a magnet and releases whatever ferromagnetic objects might have been attracted to it.

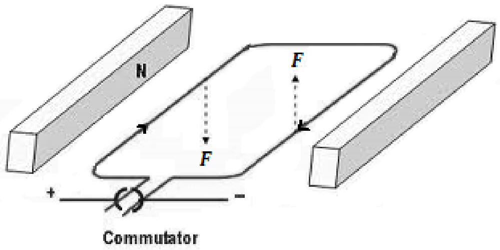
Of course, the above description is a simplification. Real ferromagnetic materials such as iron usually experience magnetic remanence, meaning that some of the electrons in the material remain aligned, and the material is weakly magnetized.

While magnetic remanence is undesirable in an electromagnet, it is the basis for magnetic computer storage media, such as audio and computer tapes and floppy and hard computer disks. To write information onto a disk, a disk head (an electromagnetic that can be moved radially) is pulsed in specific patterns as the disk spins. The patterns are encoded on the disk as locally magnetized regions.

When encoded information is read from the disk, the moving magnetic regions produce a changing electric field that causes an electric current in the disk head.

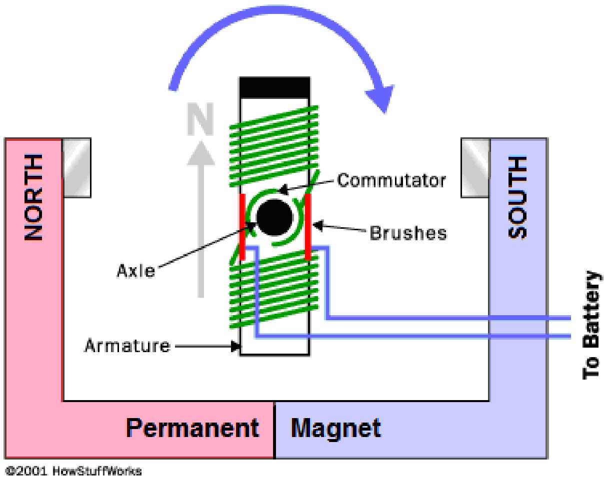
Motor

The force produced by a moving current in a magnetic field can be used to cause a loop of wire to spin:



A *commutator* is used to reverse the direction of the current as the loop turns, so that the combination of attraction and repulsion always applies force in the same direction.

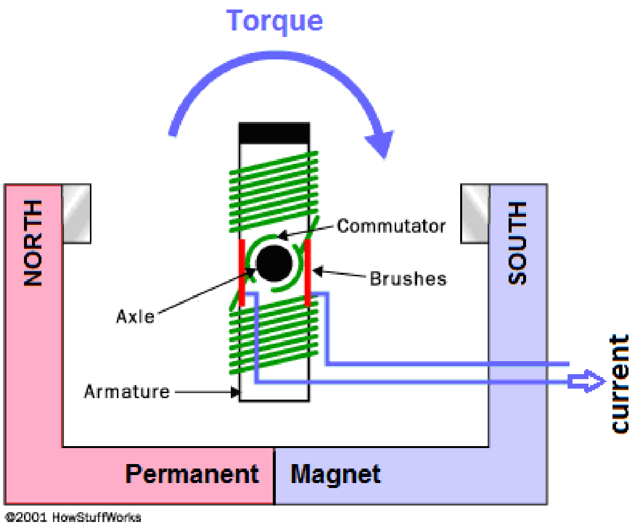
If we replace the loop of wire with an electromagnet (a coil of wire wrapped around a material such as iron that has both a high electrical conductivity and a high magnetic permeability), the electromagnet will spin with a strong force.



An electromagnet that spins because of its continuously switching attraction and repulsion to the magnetic field produced by a separate set of permanent magnets is called a *motor*. An electric motor turns electric current into rotational motion, which can be used to do work.

Generator

A *generator* uses the same components and operates under the same principle as a motor, except that a mechanical force is used to spin the coil. When the coil moves through the magnetic field, it produces an electric current. Thus a generator is a device that turns rotational motion (work) into an electric current.



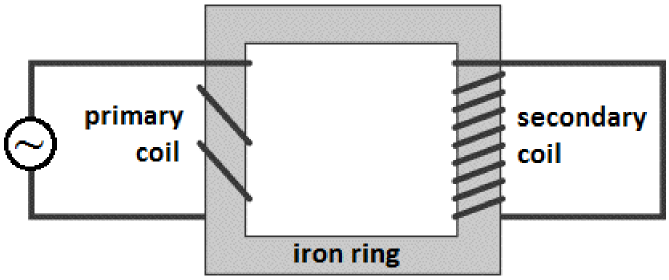
Recall from the previous section that Lenz's Law gives the emf produced by the generator. Because the coil is rotating through a uniform magnetic field, the magnetic flux through the coil is constantly changing, which means calculus is needed to calculate the emf produced.

$$\mathcal{E} = -n\frac{d\Phi_B}{dt}$$

Inductor (Transformer)

Because electric current produces a magnetic field, a ring made of a ferromagnetic material can be used to move an electric current. An inductor (transformer) is a device that takes advantage of this phenomenon in order to increase or decrease the voltage in an AC circuit.

The diagram below shows an inductor or transformer.



The current on the input side (primary) generates a magnetic field in the iron ring. The magnetic field in the ring generates a current on the output side (secondary).

In this particular transformer, the coil wraps around the output side more times than the input. This means that each time the current goes through the coil, the magnetic field adds to the electromotive force (voltage). This means the voltage will increase in proportion to the increased number of coils on the output side. However, the magnetic field on the output side will produce less current with each turn, which means the current will decrease in the same proportion:

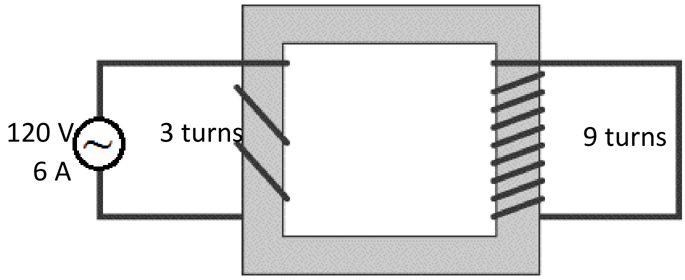
$$\frac{\#turns_{in}}{\#turns_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$$

$$P_{in} = P_{out}$$

A transformer like this one, which produces an increase in voltage, is called a ***step-up transformer***; a transformer that produces a decrease in voltage is called a ***step-down transformer***.

Sample Problem:

If the input voltage to the following transformer is 120 V, and the input current is 6 A, what are the output voltage and current?



The voltage on either side of a transformer is proportional to the number of turns in the coil on that side. In the above transformer, the primary has 3 turns, and the secondary coil has 9 turns. This means the voltage on the right side will be $\frac{9}{3}=3$ times as much as the voltage on the left, or 360 V. The current will be $\frac{3}{9}=\frac{1}{3}$ as much, or 2 A.

We can also use:

$$\frac{\#turns_{in}}{\#turns_{out}}=\frac{V_{in}}{V_{out}}$$
$$\frac{3}{9}=\frac{120\text{ V}}{V_{out}}$$
$$V_{out}=360\text{ V}$$

$$\frac{\#turns_{in}}{\#turns_{out}}=\frac{I_{out}}{I_{in}}$$
$$\frac{3}{9}=\frac{I_{out}}{6\text{ A}}$$
$$I_{out}=2\text{ A}$$

Mass Spectrometer

A mass spectrometer is a device uses the path of a charged particle in a magnetic field to determine its mass.

The particle is first selected for the desired velocity, as described on page 294. Then the particle enters a region where the only force on it is from the applied magnetic field. (In the example below, the magnetic field is directed out of the page.)

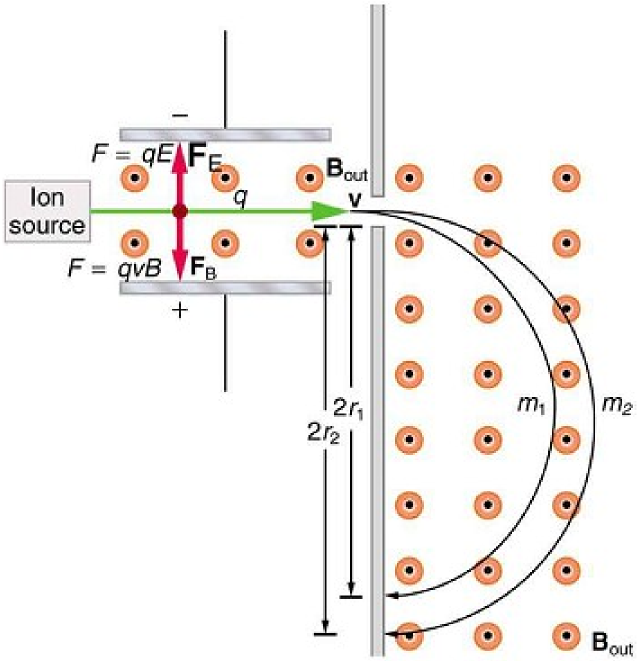


Image © 2016 by Douglas College Physics. Used with permission.

The magnetic field applies a force on the particle perpendicular to its path. As the particle's direction changes, the direction of the applied force changes with it, causing the particle to move in a circular path.

The path of the particle is the path for which the centripetal force (which you may recall from physics 1) is equal to the magnetic force:

$$F_B = F_c$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv^{\cancel{2}}}{q\cancel{v}B} = \frac{mv}{qB}$$

Thus if the particles are all ions with the same charge and are selected for having the same speed, the radius of the path will be directly proportional to the mass of the particle.

Introduction: Mechanical Waves

Unit: Mechanical Waves

Topics covered in this chapter:

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *Sound Level* describes the decibel scale and how loudness is measured.
- *The Doppler Effect* describes the change in pitch due to motion of the source or receiver (listener).
- *Exceeding the Speed of Sound* describes the Mach scale and sonic booms.

Standards addressed in this chapter:

NGSS Standards/MA Curriculum Frameworks (2016):

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling within various media. Recognize that electromagnetic waves can travel through empty space (without a medium) as compared to mechanical waves that require a medium.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

14.1.A: Describe the physical properties of waves and wave pulses.

14.1.A.1: Waves transfer energy between two locations without transferring matter between those locations.

14.1.A.1.i: A wave pulse is a single disturbance that transfers energy without transferring matter between two locations.

AP® only

14.1.A.1.ii: A wave is modeled as a continuous, periodic disturbance with well-defined wavelength and frequency.

14.1.A.2: Mechanical waves or wave pulses require a medium in which to propagate. Electromagnetic waves or wave pulses do not require a medium in which to propagate.

14.1.A.3: The speed at which a wave or wave pulse propagates through a medium depends on the type of wave and the properties of the medium.

14.1.A.3.i: The speed of all electromagnetic waves in a vacuum is a universal physical constant, $c = 3\times10^{8}\,\frac{\text{m}}{\text{s}}$.

14.1.A.3.ii: The speed at which a wave pulse or wave propagates along a string is dependent upon the tension in the string, F_T, and the mass per length of the string.

14.1.A.3.iii: In a given medium, the speed of sound waves increases with the temperature of the medium.

14.1.A.4: In a transverse wave, the direction of the disturbance is perpendicular to the direction of propagation of the wave.

14.1.A.5: In a longitudinal wave, the direction of the disturbance is parallel to the direction of propagation of the wave.

14.1.A.5.i: Sound waves are modeled as mechanical longitudinal waves.

14.1.A.5.ii: The regions of high and low pressure in a sound wave are called compressions and rarefactions, respectively.

14.1.A.6: Amplitude is the maximum displacement of a wave from its equilibrium position.

14.1.A.6.i: The amplitude of a longitudinal pressure wave may be determined by the maximum increase or decrease in pressure from equilibrium pressure.

14.1.A.6.ii: The loudness of a sound increases with increasing amplitude.

14.1.A.6.iii: The energy carried by a wave increases with increasing amplitude.

14.2.A: Describe the physical properties of a periodic wave.

14.2.A.1: Periodic waves have regular repetitions that can be described using period and frequency.

14.2.A.1.i: The period is the time for one complete oscillation of the wave.

14.2.A.1.ii: The frequency is the rate at which the wave repeats.

14.2.A.1.iii: The amplitude of a wave is independent of the period and the frequency of that wave.

14.2.A.1.iv: The energy of a wave increases with increasing frequency.

14.2.A.1.v: The frequency of a sound wave is related to its pitch.

AP® only

14.2.A.1.vi: Wavelength is the distance between successive corresponding positions (such as peaks or troughs) on a wave.

14.2.A.2: A sinusoidal wave can be described by equations for the displacement from equilibrium at a specific location as a function of time. A wave can also be described by an equation for the displacement from equilibrium at a specific time as a function of position.

14.2.A.3: For a periodic wave, the wavelength is proportional to the wave's speed and inversely proportional to the wave's frequency.

14.3.A: Describe the interaction between a wave and a boundary.

14.3.A.1: A wave that travels from one medium to another can be transmitted or reflected, depending on the properties of the boundary separating the two media.

14.3.A.1.i: A wave traveling from one medium to another (for example, a wave traveling between low-mass and high-mass strings) will result in reflected and transmitted waves.

14.3.A.1.ii: A reflected wave is inverted if the transmitted wave travels into a medium in which the speed of the wave decreases.

14.3.A.1.iii: A reflected wave is not inverted if the transmitted wave travels into a medium in which the speed of the wave increases.

14.3.A.1.iv: The frequency of a wave does not change when it travels from one medium to another.

Skills learned & applied in this chapter:

- Visualizing wave motion.

Waves

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.A.1.2, 6.A.2.2, 6.B.3.1

Mastery Objective(s): (Students will be able to…)

- Describe and explain properties of waves (frequency, wavelength, etc.)
- Differentiate between transverse, longitudinal and transverse waves.
- Calculate wavelength, frequency, period, and velocity of a wave.

Success Criteria:

- Parts of a wave are identified correctly.
- Descriptions & explanations account for observed behavior.

Language Objectives:

- Describe how waves propagate.

Tier 2 Vocabulary: wave, crest, trough, frequency, wavelength

Labs, Activities & Demonstrations:

- Show & tell: transverse waves in a string tied at one end, longitudinal waves in a spring, torsional waves.
- Buzzer in a vacuum.
- Tacoma Narrows Bridge collapse movie.
- Japan tsunami TV footage.

Notes:

wave: a disturbance that travels from one place to another.*

medium: a substance that a wave travels through.

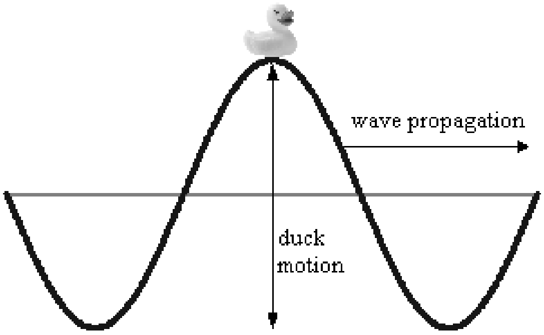
propagation: the process of a wave traveling through space.

* This is my favorite definition in these notes. I jokingly suggest that I nickname some of my students "wave" based on this definition.

honors (not AP®)

mechanical wave: a wave that propagates through a medium via contact between particles of the medium. Some examples of mechanical waves include ocean waves and sound waves.

1. The energy of the wave is transmitted via the particles of the medium as the wave passes through it.
2. The wave travels through the medium. The particles of the medium are moved by the wave passing through, and then return to their original position. (The duck sitting on top of the wave below is an example.)



3. Waves generally move fastest in solids and slowest in liquids. The velocity of a mechanical wave is dependent on characteristics of the medium:

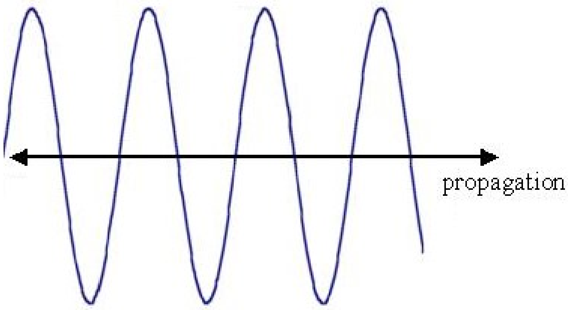
state	relevant factors	example	
		medium	**velocity of sound**
gas	density, pressure	air (20 °C and 1 atm)	343 $\frac{m}{s}$ (768 $\frac{mi}{hr}$)
liquid	density, compressibility	water (20 °C)	1 481 $\frac{m}{s}$ (3 317 $\frac{mi}{hr}$)
solid	stiffness	steel (longitudinal wave)	6 000 $\frac{m}{s}$ (13 000 $\frac{mi}{hr}$)

electromagnetic wave: a wave of electricity and magnetism interacting with each other. Electromagnetic waves can propagate through empty space, and are slowed down by interactions with a medium. Electromagnetic waves are discussed in more detail in the *Electromagnetic Waves* section starting on page 354.

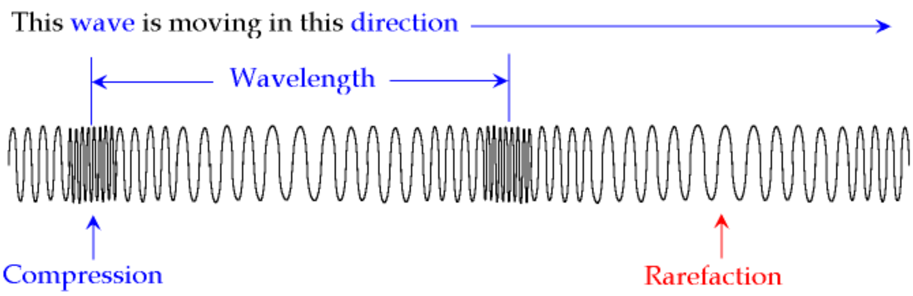
honors (not AP®)

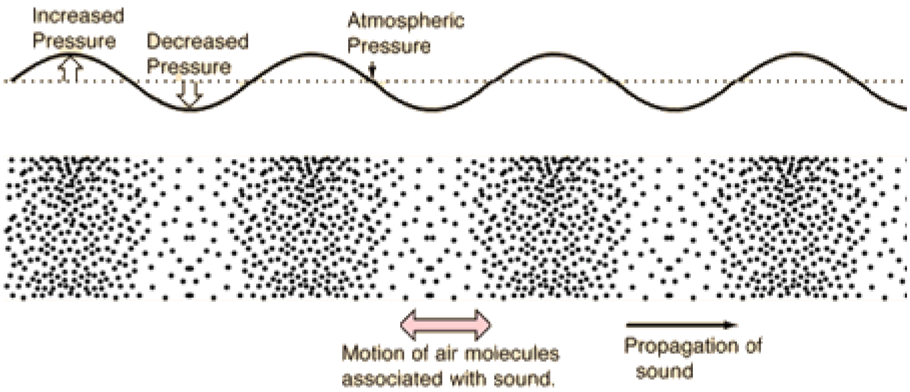
Types of Waves

transverse wave: moves its medium up & down (or back & forth) as it travels through. Examples: light, ocean waves



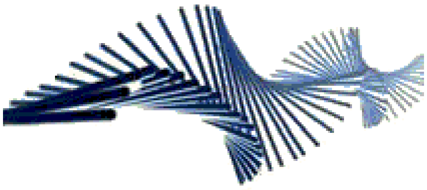
longitudinal wave (or compressional wave): compresses and decompresses the medium as it travels through. Examples: compression of a spring, sound.





honors (not AP®)

torsional wave: a type of transverse wave that propagates by twisting about its direction of propagation.



The most famous example of the destructive power of a torsional wave was the Tacoma Narrows Bridge, which collapsed on November 7, 1940. On that day, strong winds caused the bridge to vibrate torsionally. At first, the edges of the bridge swayed about eighteen inches. (This behavior had been observed previously, earning the bridge the nickname “Galloping Gertie”.) However, after a support cable snapped, the vibration increased significantly, with the edges of the bridge being displaced up to 28 feet! Eventually, the bridge started twisting in two halves, one half twisting clockwise and the other half twisting counterclockwise, and then back again. This opposing torsional motion eventually caused the bridge to twist apart and collapse.

The bridge’s collapse was captured on film. Video clips of the bridge twisting and collapsing are available on the internet. There is a detailed analysis of the bridge’s collapse at http://www.vibrationdata.com/Tacoma.htm

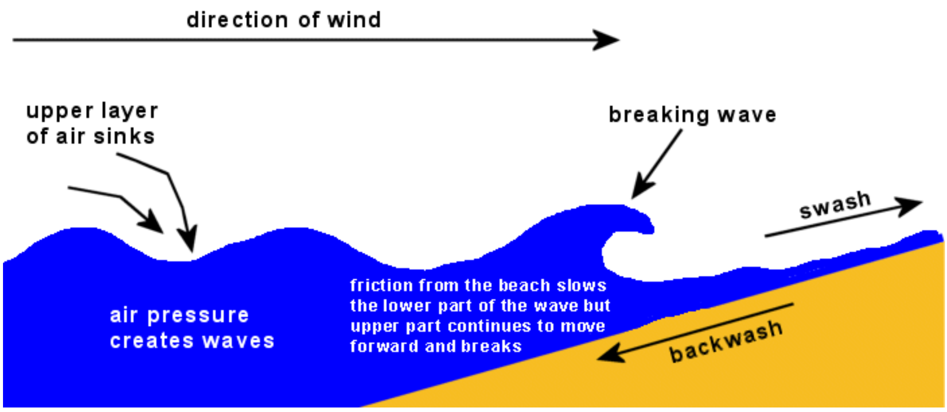
honors (not AP®)

Ocean Waves

Surface Waves

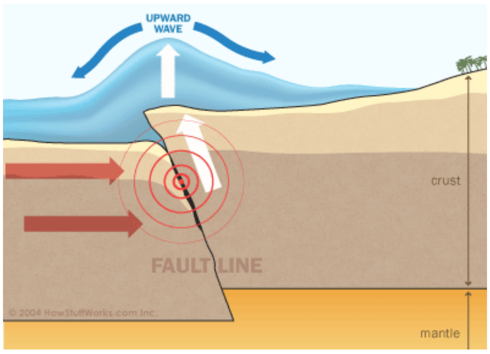
<u>surface wave</u>: a transverse wave that travels at the interface between two mediums.

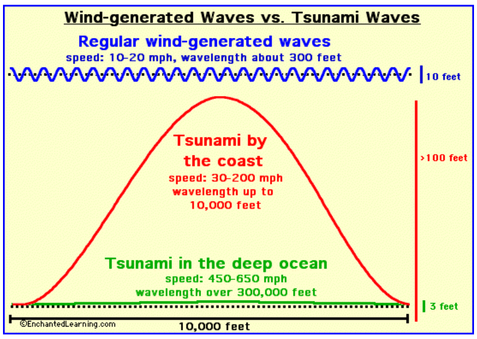
Ocean waves are an example of surface waves, because they travel at the interface between the air and the water. Surface waves on the ocean are caused by wind disturbing the surface of the water. Until the wave gets to the shore, surface waves have no effect on water molecules far below the surface.



Tsunamis

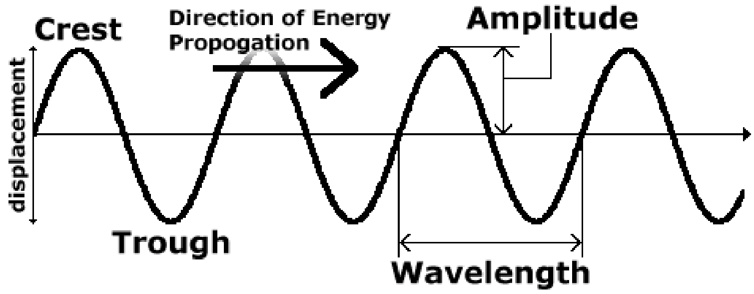
The reason tsunamis are much more dangerous than regular ocean waves is because tsunamis are created by earthquakes on the ocean floor. The tsunami wave propagates through the entire depth of the water, which means tsunamis carry many times more energy than surface waves.





This is why a 6–12 foot high surface wave breaks harmlessly on the beach; however, a tsunami that extends 6–12 feet above the surface of the water includes a significant amount of energy throughout the entire depth of the water, and can destroy an entire city.

Properties of Waves



crest: the point of maximum positive displacement of a transverse wave. (The highest point.)

trough: the point of maximum negative displacement of a transverse wave. (The lowest point.)

amplitude (*A*): the distance of maximum displacement of a point in the medium as the wave passes through it. (The maximum height or depth.)

wavelength (λ): the length of the wave, measured from a specific point in the wave to the same point in the next wave. Unit = distance (m, cm, nm, *etc.*)

frequency (*f* or ν): the number of waves that travel past a point in a given time. Unit = $^{1}/_{\text{time}}$ (Hz = $^{1}/_{s}$)

Note that while high school physics courses generally use the variable *f* for frequency, college courses usually use *ν* (the Greek letter "nu", which is different from but easy to confuse with the Roman letter "v").

period or time period (*T*): the amount of time between two adjacent waves. Unit = time (usually seconds)

$$T = {}^{1}/_{f}$$

<u>velocity</u>: the velocity of a wave depends on its frequency (f) and its wavelength (λ):

$$v = \lambda f$$

The velocity of electromagnetic waves (such as light, radio waves, microwaves, X-rays, *etc.*) is called the <u>speed of light</u>, which is $3.00\times10^{8}\,\frac{\text{m}}{\text{s}}$ in a vacuum. The speed of light is slower in a medium that has an index of refraction* greater than 1.

The velocity of a wave traveling through a string under tension (such as a piece of string, a rubber band, a violin/guitar string, *etc.*) depends on the tension and the ratio of the mass of the string to its length:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where F_T is the tension in the string, L is the length, and m is the mass.

Sample Problem:

Q: The Boston radio station WZLX broadcasts waves with a frequency of 100.7 MHz. If the waves travel at the speed of light, what is the wavelength?

A: $f = 100.7\,\text{MHz} = 100\,700\,000\,\text{Hz} = 1.007\times10^{8}\,\text{Hz}$

$$v = c = 3.00\times10^{8}\,\frac{\text{m}}{\text{s}}$$

$$v = \lambda f$$

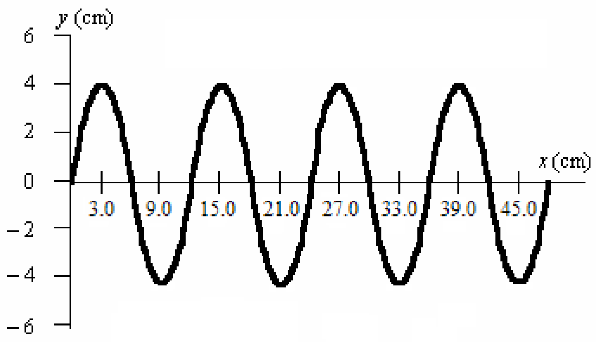
$$3.00\times10^{8} = \lambda\,(1.007\times10^{8})$$

$$\lambda = \frac{3.00\times10^{8}}{1.007\times10^{8}} = 2.98\,\text{m}$$

* The index of refraction is a measure of how much light bends when it moves between one medium and another. The sine of the angle of refraction is proportional to the speed of light in that medium. Index of refraction is part of the *Refraction* topic starting on page 465.

Homework Problems

1. **(M)** Consider the following wave:



   a. What is the amplitude of this wave?

   b. What is its wavelength?

   c. If the velocity of this wave is $30\,\frac{\text{m}}{\text{s}}$, what is its period?

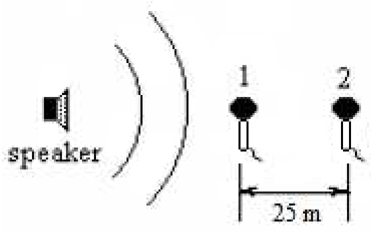
2. **(M)** What is the speed of wave with a wavelength of 0.25 m and a frequency of 5.5 Hz?

   Answer: $1.375\,\frac{\text{m}}{\text{s}}$

3. **(S)** A sound wave traveling in water at 10 °C has a wavelength of 0.65 m. What is the frequency of the wave.
   (*Note: you will need to look up the speed of sound in water at 10 °C in* Table W. Properties of Water and Air *on page 478 of your Physics Reference Tables*.)

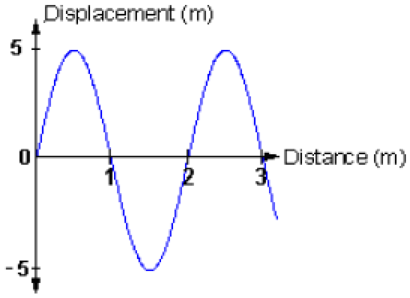
   Answer: 2 226Hz

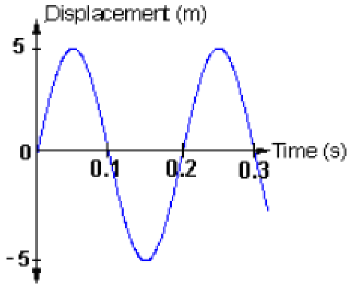
4. **(S)** Two microphones are placed in front of a speaker as shown in the diagram to the right. If the air temperature is 30 °C, what is the time delay between the two microphones?



Answer: 0.0716 s

5. **(M)** The following are two graphs of the same wave. The first graph shows the displacement *vs.* distance, and the second shows displacement *vs.* time.





a. What is the wavelength of this wave?

b. What is its amplitude?

c. What is its frequency?

d. What is its velocity?

Reflection and Superposition

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.C.1.1, 6.C.1.2

Mastery Objective(s): (Students will be able to…)

- Explain the behavior of waves when they pass each other in the same medium and when they reflect off something.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain what happens when two waves pass through each other.

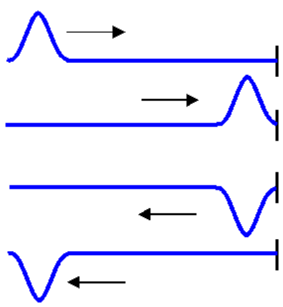
Tier 2 Vocabulary: reflection

Labs, Activities & Demonstrations:

- waves on a string or spring anchored at one end
- large Slinky with longitudinal and transverse waves passing each other

Notes:

Reflection of Waves



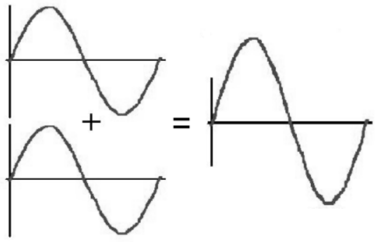
reflection: when a wave hits a fixed (stationary) point and "bounces" back.

Notice that when the end of the rope is fixed, the reflected wave is inverted. (If the end of the rope were free, the wave would not invert.)

Superposition of Waves

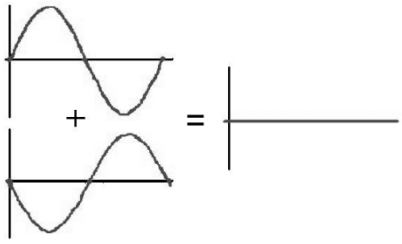
When waves are superimposed (occupy the same space), their amplitudes add.

<u>constructive interference</u>: when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.



Because the wavelengths are the same and the maximum, minimum, and zero points all coïncide (line up), the two component waves are said to be "in phase" with each other.

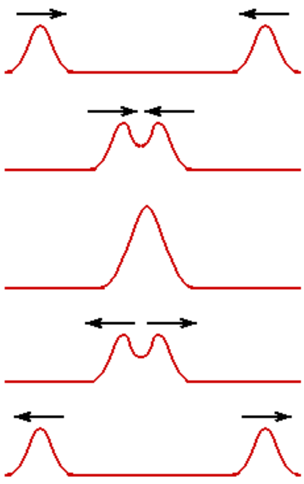
<u>destructive interference</u>: when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves. (Sometimes we say that the waves "cancel" each other.)



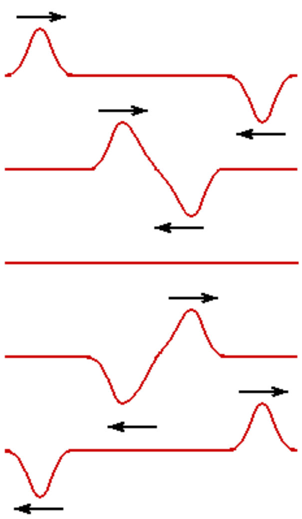
Because the wavelengths are the same but the maximum, minimum, and zero points do not coïncide, the waves are said to be "out of phase" with each other.

Note that waves can travel in two opposing directions at the same time. When this happens, the waves pass through each other, exhibiting constructive and/or destructive interference as they pass:

Constructive Interference



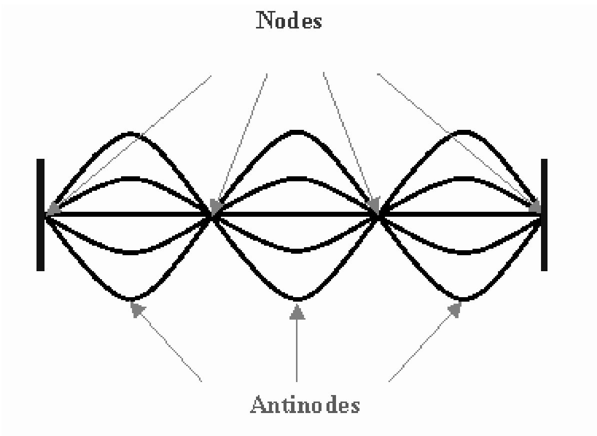
Destructive Interference



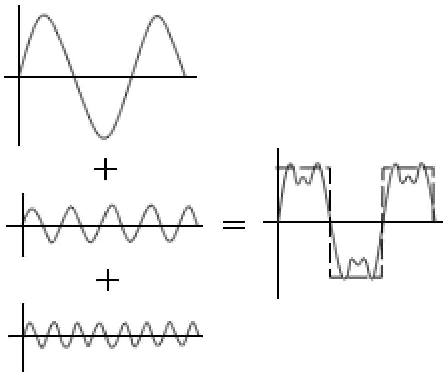
Standing Waves

standing wave: when half of the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary.

Points along the wave that are not moving are called "nodes". Points of maximum displacement are called "antinodes".



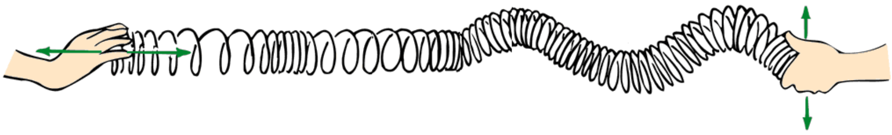
When we add waves with different wavelengths and amplitudes, the result can be complex:



This is how radio waves encode a signal on top of a "carrier" wave. Your radio's antenna receives ("picks up") radio waves within a certain range of frequencies. Imagine that the bottom wave (the one with the shortest wavelength and highest frequency) is the "carrier" wave. If you tune your radio to its frequency, the radio will filter out other waves that don't include the carrier frequency. Then your radio subtracts the carrier wave, and everything that is left is sent to the speakers.

Homework Problem

1. **(M)** A Slinky is held at both ends. The person on the left creates a longitudinal wave, while at same time the person on the right creates a transverse wave with the same frequency. Both people stop moving their ends of the Slinky just as the waves are about to meet.

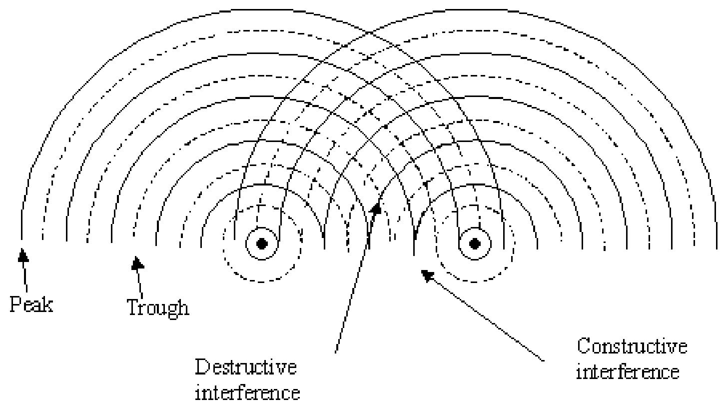


   a. Draw a picture of what the Slinky will look like when the waves completely overlap.

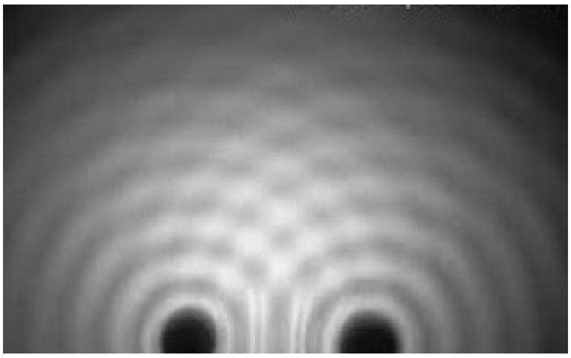
   b. Draw a picture of what the Slinky will look like just after the waves no longer overlap.

Two-Dimensional Interference Patterns

When two progressive waves propagate into each other's space, the waves produce interference patterns. This diagram shows how interference patterns form:



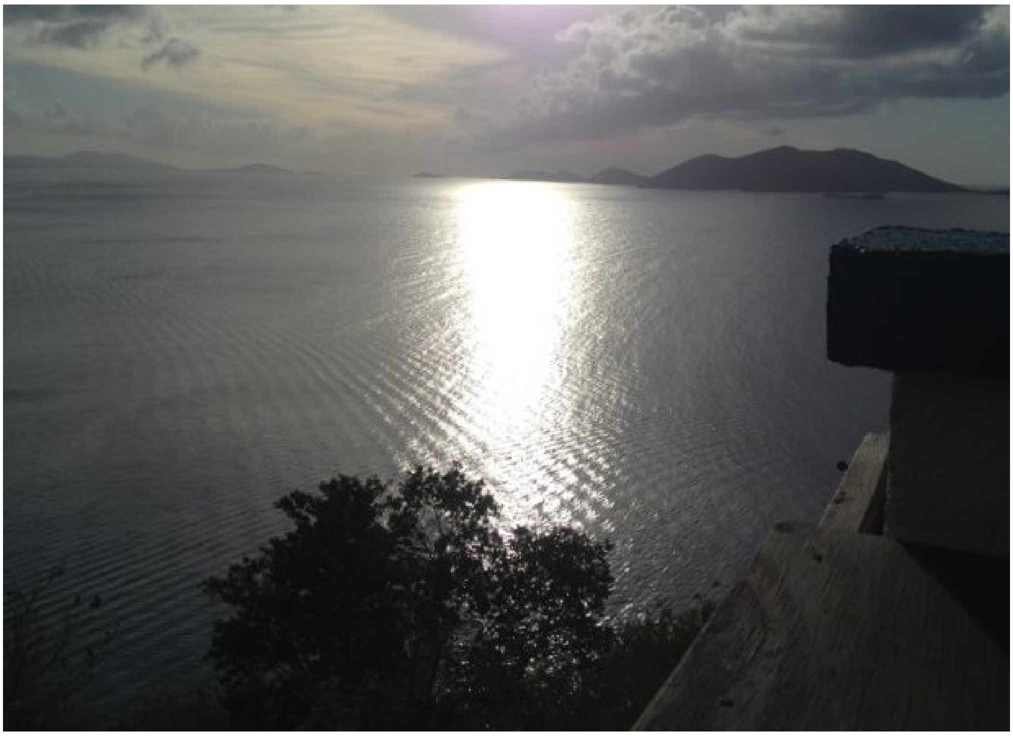
The resulting interference pattern looks like the following picture:



In this picture, the bright regions are wave peaks, and the dark regions are troughs. The brightest intersections are regions where the peaks interfere constructively, and the darkest intersections are regions where the troughs interfere constructively.

The following picture* shows an interference pattern created by ocean waves, one of which has been reflected off a point on the shore. The wave at the left side of the picture is traveling toward the right, and the reflected wave at the bottom right of the picture is traveling toward the top of the picture.

Because the sun is low in the sky (the picture was taken just before sunset), the light is reflected off the water, and the crests of the waves produce shadows behind them.



* Taken from Tortola in the British Virgin Islands, looking west toward Jost Van Dyke.

honors (not AP®)

Sound & Music

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-1

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Describe how musical instruments produce sounds.
- Describe how musical instruments vary pitch.
- Calculate frequencies of pitches produced by a vibrating string or in a pipe.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what produces the vibrations in various types of musical instruments.

Tier 2 Vocabulary: pitch

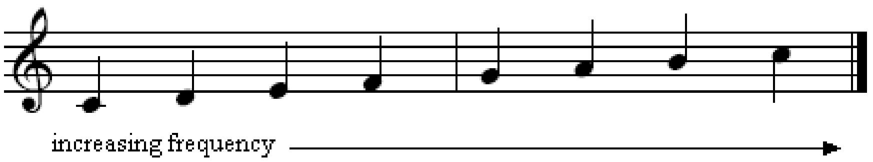
Labs, Activities & Demonstrations:

- Show & tell: violin, penny whistle, harmonica, Boomwhackers.
- Helmholtz resonators—bottles of different sizes/air volumes, slapping your cheek with your mouth open.
- Frequency generator & speaker.
- Rubens tube ("sonic flame tube").
- Measure the speed of sound in air using a resonance tube.

Notes:

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

pitch: how "high" or "low" a musical note is. The pitch is determined by the frequency of the sound wave.

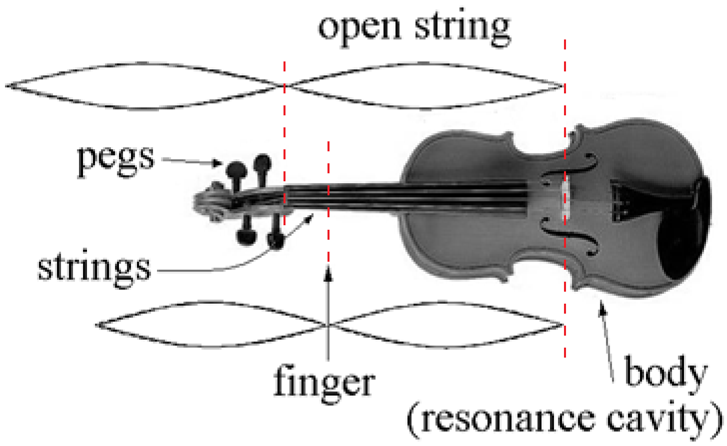


honors (not AP®)

resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, *i.e.,* λ = 2L. The musician changes the half-wavelength by using a finger to shorten the part of the string that vibrates. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string depends on the tension and the length and mass of the vibrating portion. The velocity is given by the equation:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where:

f = frequency (Hz) F_T = tension (N)

m = mass of string (kg) L = length of string (m) $= \frac{\lambda}{2}$

Given the velocity and wavelength, the frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

honors (not AP®)

Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. The design of the mouthpiece (or air inlet) causes the air to oscillate as it enters the pipe. This causes the air molecules to compress and spread out at regular intervals based on the dimensions of the closed section of the instrument, which determines the wavelength. The wavelength and speed of sound determine the frequency.

Most wind instruments use one of three ways of causing the air to oscillate:

Brass Instruments

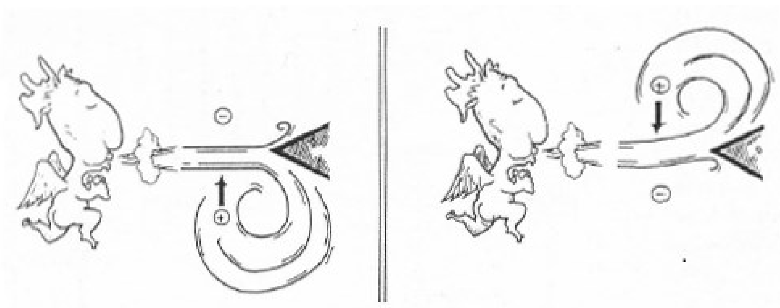
With brass instruments like trumpets, trombones, French horns, *etc.*, the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

Whistles (Instruments with Fipples)

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple results in a pressure difference on one side *vs.* the other. Air moves toward the lower pressure side, causing air to build up and the pressure to increase. When the pressure becomes greater than the other side, the air switches abruptly to the other side of the fipple. Then the pressure builds on the other side until the air switches back:



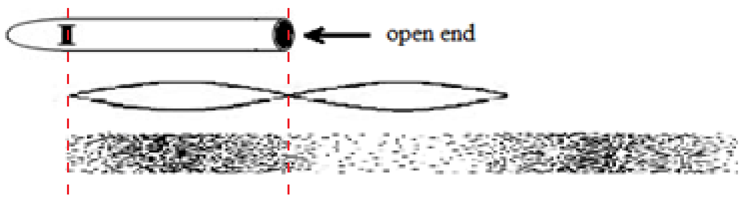
The frequency of this back-and-forth motion is what determines the pitch.

honors (not AP®)

Open vs. Closed-Pipe Instruments

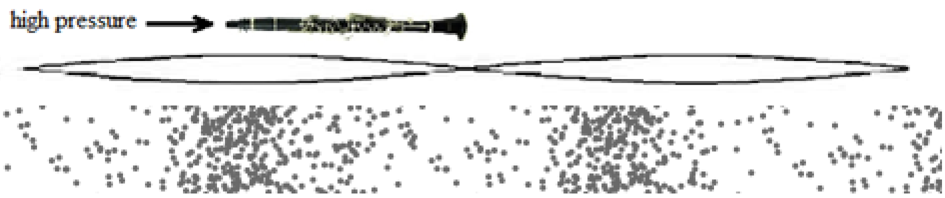
An open-pipe instrument has an opening at each end. A closed-pipe instrument has an opening at one end, and is closed at the other.

Examples of open-pipe instruments include uncapped organ pipes, whistles, recorders and flutes.



Notice that the two openings determine where the air pressure *must* be equal to atmospheric pressure (*i.e.,* the air is neither compressed nor expanded). This means that the length of the body of the instrument (*L*) is a half-wave, and that the wavelength (λ) of the sound produced must therefore be twice as long, *i.e.,* $\lambda = 2L$. (This is similar to string instruments, in which the length of the vibrating string is a half-wave.)

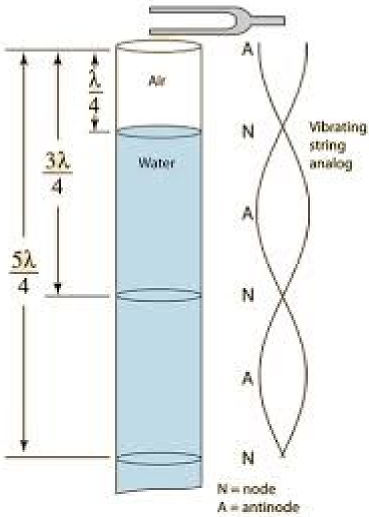
Examples of closed-pipe instruments include clarinets and all brass instruments. Air is blown in at high pressure via the mouthpiece, which means the mouthpiece is an antinode—a region of maximum displacement of the individual air molecules. This means that the body of the instrument is the distance from the antinode to a region of atmospheric pressure, *i.e.,* one-fourth of a wave. This means that for closed-pipe instruments, $\lambda = 4L$.



The difference in the resonant wavelength (4*L* *vs.* 2*L*) is why a closed-pipe instrument (*e.g.,* a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.,* a flute)—twice the wavelength results in half the frequency.

honors (not AP®)

The principle of a closed-pipe instrument can be used in a lab experiment to determine the frequency of a tuning fork (or the speed of sound) using a resonance tube—an open tube filled with water to a specific depth.



A tuning fork generates an oscillation of a precise frequency at the top of the tube. Because this is a closed pipe, the source (just above the tube) is an antinode (maximum amplitude).

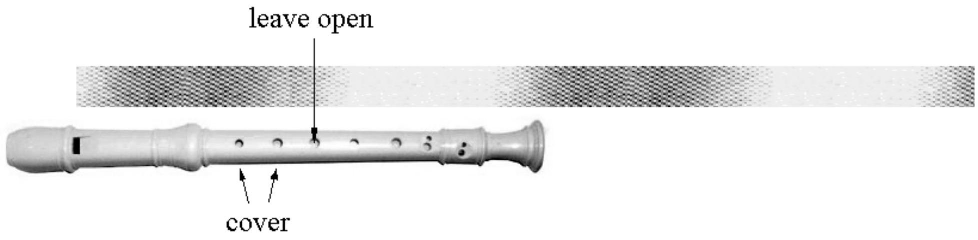
When the height of air above the water is exactly ¼ of a wavelength ($\frac{\lambda}{4}$), the waves that are reflected back have maximum constructive interference with the source wave, which causes the sound to be significantly amplified. This phenomenon is called resonance.

Resonance will occur at every antinode—*i.e.,* any integer plus ¼ of a wave ($\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, *etc.*)

honors (not AP®)

Playing Different Pitches (Frequencies)

For an instrument with holes, like a flute or recorder, the first open hole is the first place in the pipe where the pressure is equal to atmospheric pressure, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is v_s ($343\,\frac{\text{m}}{\text{s}}$ at 20 °C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$f = \frac{v_s}{2L}$ for an open-pipe instrument (*e.g.,* flute, recorder, whistle)

$f = \frac{v_s}{4L}$ for an closed-pipe instrument (*e.g.,* clarinet, brass instrument).

Note that the frequency is directly proportional to the speed of sound in air. The speed of sound increases as the temperature increases, which means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. Musicians claim that the instrument is going out of tune, but actually it's not the instrument that is out of tune, but the speed of sound!

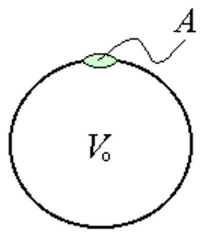
Note however, that the frequency is inversely proportional to the wavelength (which depends largely on the length of the instrument). This means that the extent to which the frequency changes with temperature will be different for different-sized instruments, which means the band will become more and more out of tune with itself as the temperature changes.

honors (not AP®)

Helmholtz Resonators

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate.

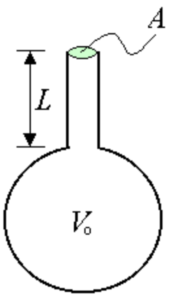
For an enclosed volume of air with a single opening, the resonant frequency depends on the resonant frequency of the air in the large cavity, and the cross-sectional area of the opening.



Resonant frequency:

$$f = \frac{v_s}{2\pi} \cdot \frac{A}{V_o}$$

For a bottle with a neck, the air in the neck behaves like a spring, with a spring constant that is proportional to the volume of air in the neck:



Resonant frequency:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_o L}}$$

where:

f = resonant frequency
v_s = speed of sound in air ($343\,\frac{m}{s}$ at 20 °C and 1 atm)
A = cross-sectional area of the neck of the bottle (m^2)
V_o = volume of the main cavity of the bottle (m^3)
L = length of the neck of the bottle (m)

(Note that it may be more convenient to use measurements in cm, cm^2, and cm^3, and use $v_s = 34\,300\,\frac{cm}{s}$.)

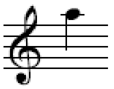
Blowing across the top of an open bottle is an example of a Helmholtz resonator.

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You can change the pitch by opening or closing your mouth a little, which changes the area of the opening (A).

honors (not AP®)

Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch	frequency (Hz)	pitch	frequency (Hz)
 A	440.0	 E	659.3
 B	493.9	 F	698.5
 C	523.3	 G	784.0
 D	587.3	 A	880.0

A note that is an octave above another note has exactly twice the frequency of the lower note. For example, the A in on the second line of the treble clef staff has a frequency of 440 Hz.[*] The A an octave above it (one ledger line above the staff) has a frequency of 440 × 2 = 880 Hz.

Harmonic Series

<u>harmonic series</u>: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves.

<u>fundamental frequency</u>: the natural resonant frequency of a particular pitch.

<u>harmonic</u>: a resonant frequency produced by vibrations that contain an integer number of half-waves that add up to the half-wavelength of the fundamental.

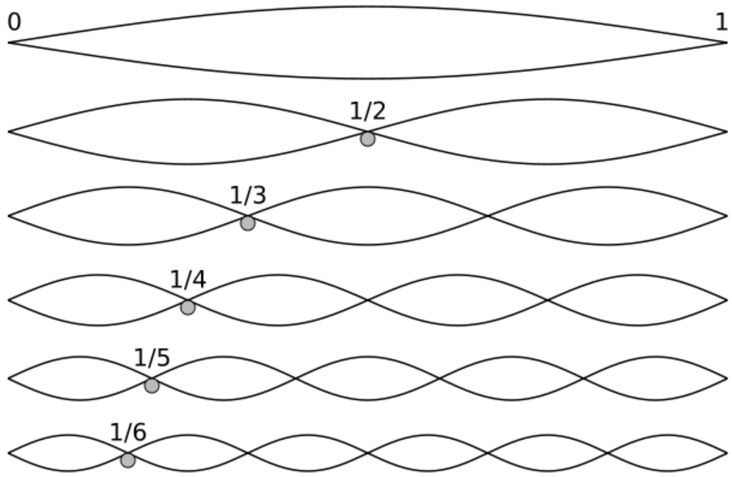
The harmonics are numbered based on their pitch relative to the fundamental frequency. The harmonic that is closest in pitch is the 1^{st} harmonic, the next closest is the 2^{nd} harmonic, *etc.*

Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

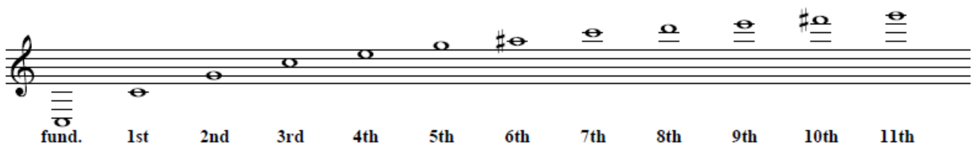
[*] Most bands and orchestras define the note "A" to be exactly 440 Hz, and use it for tuning.

honors (not AP®)

The following diagram shows the waves of the fundamental frequency and the first five harmonics in a pipe or a vibrating string:



Fraction of String	Wave-length	Harmonic	Frequency	Pitch (relative to fundamental)
1	$2L$	—	f_o	Fundamental.
$1/2$	$2L/2$	1st	$2f_o$	One octave above.
$1/3$	$2L/3$	2nd	$3f_o$	One octave + a fifth above.
$1/4$	$2L/4$	3rd	$4f_o$	Two octaves above.
$1/5$	$2L/5$	4th	$5f_o$	Two octaves + approximately a major third above.
$1/6$	$2L/6$	5th	$6f_o$	Two octaves + a fifth above.
$1/n$	$2L/n$	(n-1)th	nf_o	*etc.*

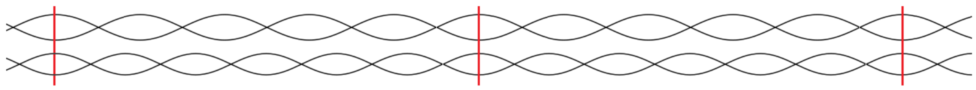


honors (not AP®)

Beats

When two or more waves are close but not identical in frequency, their amplitudes reinforce each other at regular intervals.

For example, when the following pair of waves travels through the same medium, the amplitudes of the two waves have maximum constructive interference every five half-waves (2½ full waves) of the top wave and every six half-waves (3 full waves) of the bottom wave.



If this happens with sound waves, you will hear a pulse or "beat" every time the two maxima coïncide.

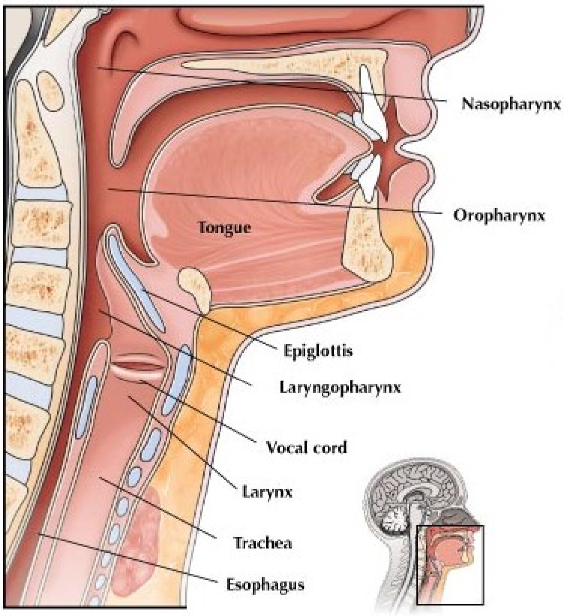
The closer the two wavelengths (and therefore also the two frequencies) are to each other, the more half-waves it takes before the amplitudes coïncide. This means that as the frequencies get closer, the time between beats gets longer.

Piano tuners listen for these beats, and adjust the tension of the string they are tuning until the time between beats gets longer and longer and finally disappears.

honors (not AP®)

The Biophysics of Sound

When a person speaks, abdominal muscles force air from the lungs through the larynx.

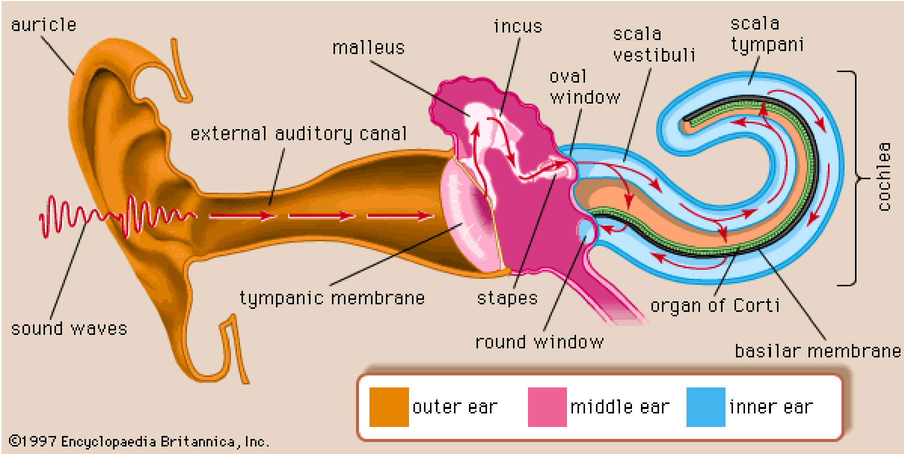


The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency at which it vibrates. Just like in a string instrument, the change in tension changes the pitch. Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

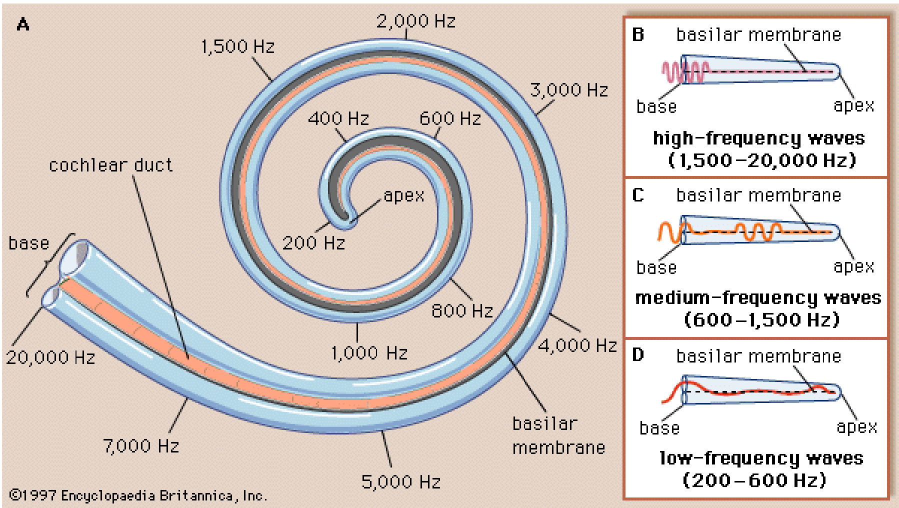
This process happens when you sing. Amateur musicians who sing a lot of high notes can develop laryngitis from tightening their laryngeal muscles too much for too long. Professional musicians need to train themselves to keep their larynx muscles relaxed and use other techniques (such as air pressure, which comes from breath support via the abdominal muscles) to adjust their pitch.

honors (not AP®)

When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



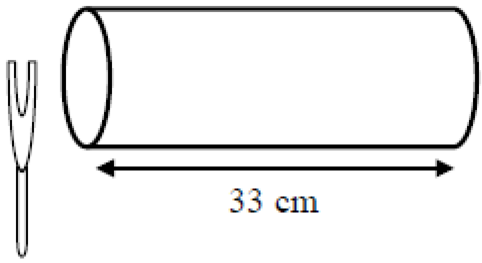
The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. As with a resonance tube, the wavelength determines exactly where the sound waves will vibrate the basilar membrane the most strongly, and the brain determines the pitch (frequency) of a sound based on the precise locations excited by these frequencies.



honors (not AP®)

Homework Problems

A tuning fork is used to establish a standing wave in an open ended pipe filled with air at a temperature of 20°C, where the speed of sound is $343\,\frac{\text{m}}{\text{s}}$, as shown below:



The sound wave resonates at the 3nd harmonic frequency of the pipe. The length of the pipe is 33 cm.

1. **(M)** Sketch the pipe with the standing wave inside of it. (For simplicity, you may sketch a transverse wave to represent the standing wave.)

2. **(M)** Determine the wavelength of the resonating sound wave.

   Answer: 22 cm

3. **(M)** Determine the frequency of the tuning fork.

   Answer: 1 559 Hz

4. **(M)** What is the next higher frequency that will resonate in this pipe?

   Answer: 2 079 Hz

honors (not AP®)

Sound Level (Loudness)

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain sound levels in decibels.
- Explain the Lombard Effect.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain how loudness is measured.

Tier 2 Vocabulary: level

Labs, Activities & Demonstrations:

- VU meter.

Notes:

sound level: the perceived intensity of a sound. Usually called "volume".

Sound level is usually measured in decibels (dB). One decibel is one tenth of one bel.

Sound level is calculated based on the logarithm of the ratio of the power (energy per unit time) causing a sound vibration to the power that causes some reference sound level.

You will not be asked to calculate decibels from an equation, but you should understand that because the scale is logarithmic, a difference of one bel (10 dB) represents a tenfold increase or decrease in sound level.

honors (not AP®)

The following table lists the approximate sound levels of various sounds:

sound level (dB)	Description
0	threshold of human hearing at 1 kHz
10	a single leaf falling to the ground
20	background in TV studio
30	quiet bedroom at night
36	whispering
40	quiet library or classroom
42	quiet voice
40–55	typical dishwasher
50–55	normal voice
60	TV from 1 m away
	normal conversation from 1 m away
60–65	raised voice
60–80	passenger car from 10 m away
70	typical vacuum cleaner from 1 m away
75	crowded restaurant at lunchtime
72–78	loud voice
85	hearing damage (long-term exposure)
84–90	shouting
80–90	busy traffic from 10 m away
100–110	rock concert, 1 m from speaker
110	chainsaw from 1 m away
110–140	jet engine from 100 m away
120	threshold of discomfort
	hearing damage (single exposure)
130	threshold of pain
140	jet engine from 50 m away
194	sound waves become shock waves

honors (not AP®)

Adjusting Sound Level in Conversation

In crowds, people unconsciously adjust the sound levels of their speech in order to be heard above the ambient noise. This behavior is called the Lombard effect, named for Étienne Lombard, the French doctor who first described it.

The Lombard coëfficient is the ratio of the increase in sound level of the speaker to the increase in sound level of the background noise:

$$L = \frac{\text{increase in speech level (dB)}}{\text{increase in background noise (dB)}}$$

Researchers have observed values of the Lombard coëfficient ranging from 0.2 to 1.0, depending on the circumstances.

When you are working in groups in a classroom, as the noise level gets louder, each person has to talk louder to be heard, which in turn makes the noise level louder. The Lombard effect creates a feedback loop in which the sound gets progressively louder and louder until your teacher complains and everyone resets to a quieter volume.

honors (not AP®)

Doppler Effect

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain the Doppler Effect and give examples.
- Calculate the apparent shift in wavelength/frequency due to a difference in velocity between the source and receiver.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how loudness is measured.

Tier 2 Vocabulary: shift

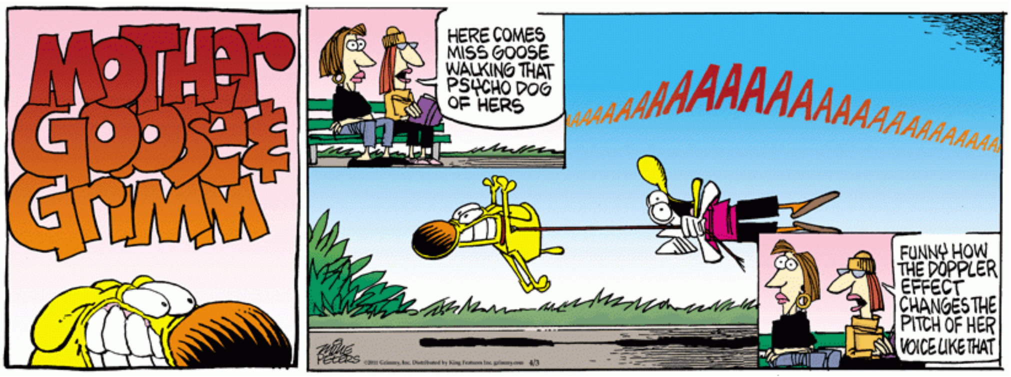
Labs, Activities & Demonstrations:

- Buzzer on a string.

Notes:

Doppler effect or Doppler shift: the apparent change in frequency/wavelength of a wave due to a difference in velocity between the source of the wave and the observer. The effect is named for the Austrian physicist Christian Doppler.

You have probably noticed the Doppler effect when an emergency vehicle with a siren drives by.



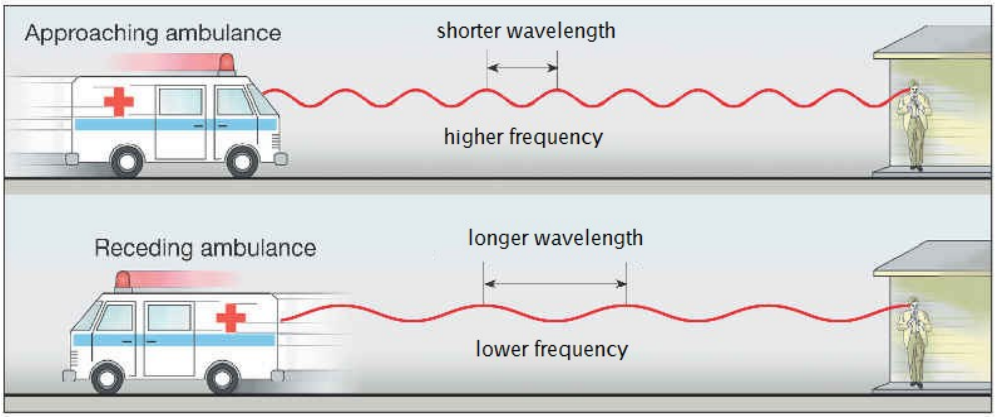
honors (not AP®)

Why the Doppler Shift Happens

The Doppler shift occurs because a wave is created by a series of pulses at regular intervals, and the wave moves at a particular speed.

If the source is approaching, each pulse arrives sooner than it would have if the source had been stationary. Because frequency is the number of pulses that arrive in one second, the moving source results in an increase in the frequency observed by the receiver.

Similarly, if the source is moving away from the observer, each pulse arrives later, and the observed frequency is lower.



honors (not AP®)

Calculating the Doppler Shift

The change in frequency is given by the equation:

$$f = f_o\left(\frac{v_w \pm v_r}{v_w \pm v_s}\right)$$

where:

f = observed frequency

f_o = frequency of the original wave

v_w = velocity of the wave

v_r = velocity of the receiver (you)

v_s = velocity of the source

The rule for adding or subtracting velocities is:

- The receiver's (your) velocity is in the numerator. If you are moving toward the sound, this makes the pulses arrive sooner, which makes the frequency higher. So if you are moving ***toward*** the sound, ***add*** your velocity. If you are moving ***away*** from the sound, ***subtract*** your velocity.
- The source's velocity is in the denominator. If the source is moving toward you, this makes the frequency higher, which means the denominator needs to be smaller. This means that if the source is moving ***toward*** you, ***subtract*** its velocity. If the source is moving ***away*** from you, ***add*** its velocity.

Don't try to memorize a rule for this—you will just confuse yourself. It's safer to reason through the equation. If something that's moving would make the frequency higher, that means you need to make the numerator larger or the denominator smaller. If it would make the frequency lower, that means you need to make the numerator smaller or the denominator larger.

honors (not AP®)

Sample Problem:

Q: The horn on a fire truck sounds at a pitch of 350 Hz. What is the perceived frequency when the fire truck is moving toward you at $20\,\frac{\text{m}}{\text{s}}$? What is the perceived frequency when the fire truck is moving away from you at $20\,\frac{\text{m}}{\text{s}}$? Assume the speed of sound in air is $343\,\frac{\text{m}}{\text{s}}$.

A: The observer is not moving, so $v_r = 0$.

The fire truck is the source, so its velocity appears in the denominator.

When the fire truck is moving toward you, that makes the frequency higher. This means we need to make the denominator smaller, which means we need to ***subtract*** v_s:

$$f = f_o\left(\frac{v_w}{v_w - v_s}\right) = 350\left(\frac{343}{343-20}\right) = 350(1.062) = 372\,\text{Hz}$$

When the fire truck is moving away, the frequency will be lower, which mean we need to make the denominator larger. This means we need to ***add*** v_s:

$$f = f_o\left(\frac{v_w}{v_w + v_s}\right) = 350\left(\frac{343}{343+20}\right) = 350(0.9449) = 331\,\text{Hz}$$

Note that the pitch shift in each direction corresponds with about one half-step on the musical scale.

honors (not AP®)

Exceeding the Speed of Sound

Unit: Mechanical Waves

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain the what a "sonic boom" is.
- Calculate Mach numbers.

Success Criteria:

- Explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how a sonic boom is produced.

Tier 2 Vocabulary: sonic boom

Labs, Activities & Demonstrations:

- Crack a bullwhip.

Notes:

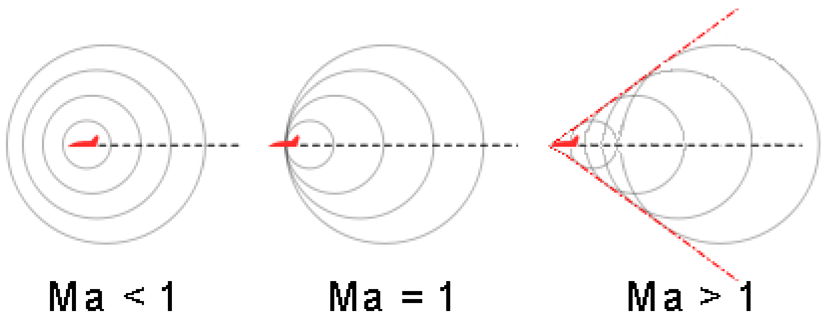
The speed of an object relative to the speed of sound in the same medium is called the Mach number (abbreviation Ma), named after the Austrian physicist Ernst Mach.

$$Ma = \frac{v_{object}}{v_{sound}}$$

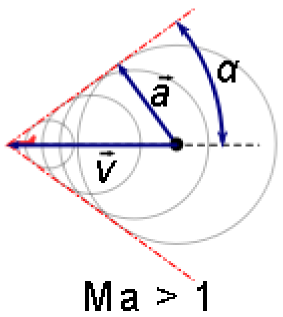
Thus "Mach 1" or a speed of Ma = 1 is the speed of sound. An object such as an airplane that is moving at 1.5 times the speed of sound would be traveling at "Mach 1.5" or Ma = 1.5.

honors (not AP®)

When an object such as an airplane is traveling slower than the speed of sound (Ma < 1) , the jet engine noise is Doppler shifted just like any other sound wave. When the airplane's velocity reaches the speed of sound (Ma = 1), the leading edge of all of the sound waves produced by the plane coincides. These waves amplify each other, producing a loud shock wave called a "sonic boom".



When an airplane is traveling faster than sound, the sound waves coincide at points behind the airplane at a specific angle, α:



The angle α is given by the equation:

$$\sin(\alpha) = \frac{1}{Ma}$$

Note that the airplane cannot be heard at points outside of the region defined by the angle α. Note also that the faster the airplane is traveling, the smaller the angle α, and the narrower the cone.

honors (not AP®)

The shock wave temporarily increases the temperature of the air affected by it. If the air is humid enough, when it cools by returning to its normal pressure, the water vapor condenses and forms a cloud, called a vapor cone:

The “crack” of a bullwhip is also a sonic boom—when a bullwhip is snapped sharply, the end of the bullwhip travels faster than sound and creates a miniature shock wave.

Introduction: Light & Optics

Unit: Light & Optics

Topics covered in this chapter:

This chapter discusses the behavior and our perception of light.

- *Electromagnetic Waves* discusses properties and equations that are specific to electromagnetic waves (including light).
- *Color* discusses properties of visible light and how we perceive it.
- *Reflection* and *Mirrors* discuss properties of flat and curved mirrors and steps for drawing ray tracing diagrams.
- *Refraction* and *Lenses* discuss properties of convex and concave lenses and steps for drawing ray tracing diagrams.
- *Polarization, Diffraction,* and *Scattering* discuss specific optical properties of light.

One of the new skills learned in this chapter is visualizing and drawing representations of how light is affected as it is reflected off a mirror or refracted by a lens. This can be challenging because the behavior of the light rays and the size and location of the image changes depending on the location of the object relative to the focal point of the mirror or lens. Another challenge is in drawing precise, to-scale ray tracing drawings such that you can use the drawings to accurately determine properties of the image, or of the mirror or lens.

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS4-5. Communicate technical information about how some technological devices use the principles of wave behavior and wave interactions with matter to transmit and capture information and energy.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

6.A.1.2: The student is able to describe representations of transverse and longitudinal waves. [**SP 1.2**]

6.A.1.3: The student is able to analyze data (or a visual representation) to identify patterns that indicate that a particular mechanical wave is polarized and construct an explanation of the fact that the wave must have a vibration perpendicular to the direction of energy propagation. [**SP 5.1**, **6.2**]

6.A.2.2: The student is able to contrast mechanical and electromagnetic waves in terms of the need for a medium in wave propagation. [**SP 6.4**, **7.2**]

6.B.3.1: The student is able to construct an equation relating the wavelength and amplitude of a wave from a graphical representation of the electric or magnetic field value as a function of position at a given time instant and vice versa, or construct an equation relating the frequency or period and amplitude of a wave from a graphical representation of the electric or magnetic field value at a given position as a function of time and vice versa. [**SP 1.5**]

6.C.1.1: The student is able to make claims and predictions about the net disturbance that occurs when two waves overlap. Examples should include standing waves. [**SP 6.4**, **7.2**]

6.C.1.2: The student is able to construct representations to graphically analyze situations in which two waves overlap over time using the principle of superposition. [**SP 1.4**]

6.C.2.1: The student is able to make claims about the diffraction pattern produced when a wave passes through a small opening, and to qualitatively apply the wave model to quantities that describe the generation of a diffraction pattern when a wave passes through an opening whose dimensions are comparable to the wavelength of the wave. [**SP 1.4**, **6.4**, **7.2**]

6.C.3.1: The student is able to qualitatively apply the wave model to quantities that describe the generation of interference patterns to make predictions about interference patterns that form when waves pass through a set of openings whose spacing and widths are small compared to the wavelength of the waves. [**SP 1.4**, **6.4**]

AP® only

6.C.4.1: The student is able to predict and explain, using representations and models, the ability or inability of waves to transfer energy around corners and behind obstacles in terms of the diffraction property of waves in situations involving various kinds of wave phenomena, including sound and light. [**SP 6.4**, **7.2**]

6.E.1.1: The student is able to make claims using connections across concepts about the behavior of light as the wave travels from one medium into another, as some is transmitted, some is reflected, and some is absorbed. [**SP 6.4**, **7.2**]

6.E.2.1: The student is able to make predictions about the locations of object and image relative to the location of a reflecting surface. The prediction should be based on the model of specular reflection with all angles measured relative to the normal to the surface. [**SP 6.4**, **7.2**]

6.E.3.1: The student is able to describe models of light traveling across a boundary from one transparent material to another when the speed of propagation changes, causing a change in the path of the light ray at the boundary of the two media. [**SP 1.1**, **1.4**]

6.E.3.2: The student is able to plan data collection strategies as well as perform data analysis and evaluation of the evidence for finding the relationship between the angle of incidence and the angle of refraction for light crossing boundaries from one transparent material to another (Snell's law). [**SP 4.1**, **5.1**, **5.2**, **5.3**]

6.E.3.3: The student is able to make claims and predictions about path changes for light traveling across a boundary from one transparent material to another at non-normal angles resulting from changes in the speed of propagation. [**SP 6.4**, **7.2**]

6.E.4.1: The student is able to plan data collection strategies, and perform data analysis and evaluation of evidence about the formation of images due to reflection of light from curved spherical mirrors. [**SP 3.2**, **4.1**, **5.1**, **5.2**, **5.3**]

6.E.4.2: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the reflection of light from surfaces. [**SP 1.4**, **2.2**]

6.E.5.1: The student is able to use quantitative and qualitative representations and models to analyze situations and solve problems about image formation occurring due to the refraction of light through thin lenses. [**SP 1.4**, **2.2**]

6.E.5.2: The student is able to plan data collection strategies, perform data analysis and evaluation of evidence, and refine scientific questions about the formation of images due to refraction for thin lenses. [**SP 3.2**, **4.1**, **5.1**, **5.2**, **5.3**]

6.F.1.1: The student is able to make qualitative comparisons of the wavelengths of types of electromagnetic radiation. [**SP 6.4**, **7.2**]

AP® only

6.F.2.1: The student is able to describe representations and models of electromagnetic waves that explain the transmission of energy when no medium is present. [**SP 1.1**]

Skills learned & applied in this chapter:

- Drawing images from mirrors and through lenses.

Electromagnetic Waves

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-1, HS-PS4-3, HS-PS4-5

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.F.1.1

Mastery Objective(s): (Students will be able to…)

- Describe the regions of the electromagnetic spectrum.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain why ultraviolet waves are more dangerous than infrared.

Tier 2 Vocabulary: wave, light, spectrum

Labs, Activities & Demonstrations:

- red *vs.* green *vs.* blue lasers on phosphorescent surface
- blue laser & tonic water
- wintergreen Life Savers™ (triboluminescence)

Notes:

electromagnetic wave: a transverse, traveling wave that is caused by oscillating electric and magnetic fields.

Electromagnetic waves travel through space and do not require a medium. The electric field creates a magnetic field, which creates an electric field, which creates another magnetic field, and so on. The repulsion from these induced fields causes the wave to propagate.

Electromagnetic waves (such as light, radio waves, etc.) travel at the speed of light. The speed of light depends on the medium it is traveling through, but it is a constant within its medium (or lack thereof), and is denoted by the letter "*c*" in equations. In a vacuum, the speed of light is:

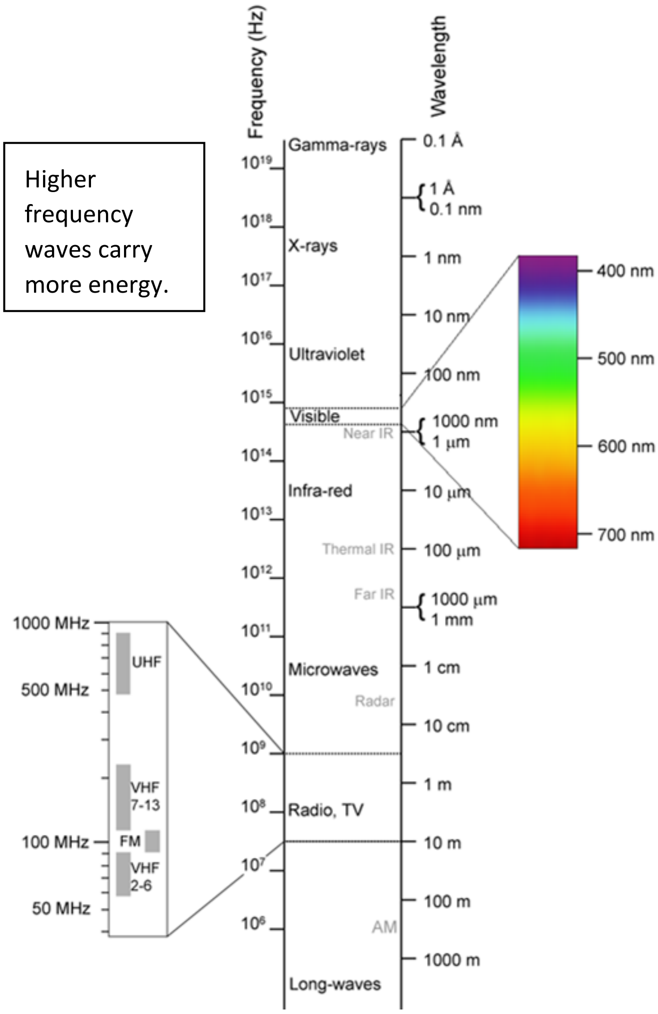
$$c = 3.00 \times 10^{8}\ {}^{\text{m}}\!/\!_{\text{s}} = 186{,}000 \text{ miles per second}$$

Recall that the speed of a wave equals its frequency times its wavelength:

$$c = \lambda f$$

<u>electromagnetic spectrum</u>: the entire range of possible frequencies and wavelengths for electromagnetic waves. The waves that make up the electromagnetic spectrum are shown in the diagram below:

Higher frequency waves carry more energy.



<u>visible spectrum</u>: wavelengths of light that can be detected by the human eye. These are wavelengths from roughly 400 nm to 700 nm.

The energy (E) that a wave carries is proportional to the frequency. (Think of it as the number of bursts of energy that travel through the wave every second.) For electromagnetic waves (including light), the constant of proportionality is Planck's constant (named after the physicist Max Planck), which is denoted by a script h in equations.

The energy of a wave is given by the Planck-Einstein equation:

$$E = hf = \frac{hc}{\lambda}$$

where E is the energy of the wave in Joules, f is the frequency in Hz, h is Planck's constant, which is equal to $6.626 \times 10^{-34}\ \mathrm{J \cdot s}$, c is the speed of light, and λ is the wavelength in meters.

Antennas

An antenna is a piece of metal that is affected by electromagnetic waves and is used to amplify waves of specific wavelengths. The optimum length for an antenna is either the desired wavelength, or some fraction of the wavelength such that one wave is an exact multiple of the length of the antenna. (*E.g.,* good lengths for an antenna could be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, *etc.* of the wavelength.)

Sample problem:

Q: What is the wavelength of a radio station that broadcasts at 98.5 MHz?

A:
$$c = \lambda f$$
$$3.00\times10^8 = \lambda(9.85\times10^7)$$
$$\lambda = \frac{3.00\times10^8}{9.85\times10^7} = 3.05\,\text{m}$$

Q: What would be a good length for an antenna that might be used to receive this radio station?

A: 3.05 m (about 10 feet) is too long to be practical for an antenna. Somewhere between half a meter and a meter is a good size.

$\frac{1}{4}$ wave would be 0.76 m (76 cm), which would be a good choice.

honors (not AP®)

Color

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain how colors are produced and mixed.
- Explain why we see colors the way we do.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain how someone who is red-green color blind might see a green object.

Tier 2 Vocabulary: color, mixing

Labs, Activities & Demonstrations:

- colored light box

Notes:

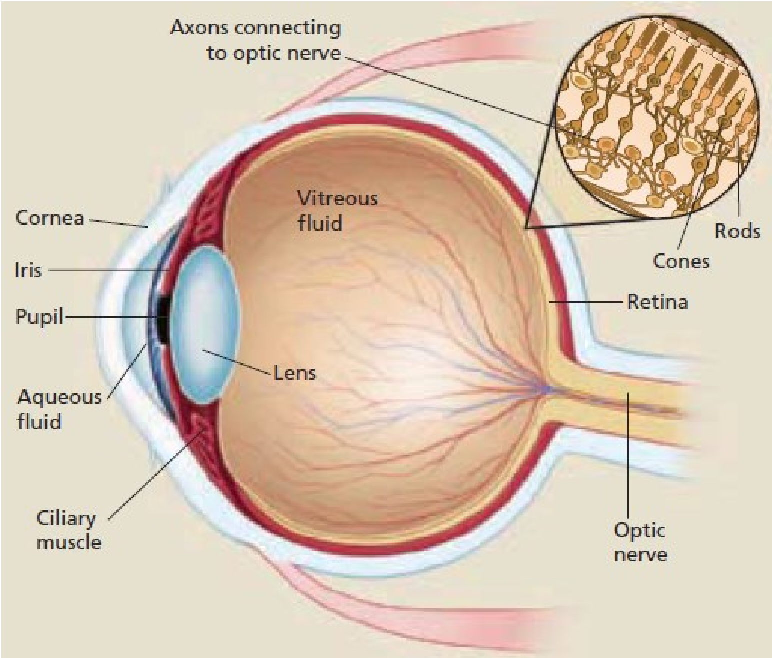
Light with frequencies/wavelengths in the part of the spectrum that the eye can detect is called visible light.

color: the perception by the human eye of how a light wave appears, based on its wavelength/frequency.

honors (not AP®)

How We See Color

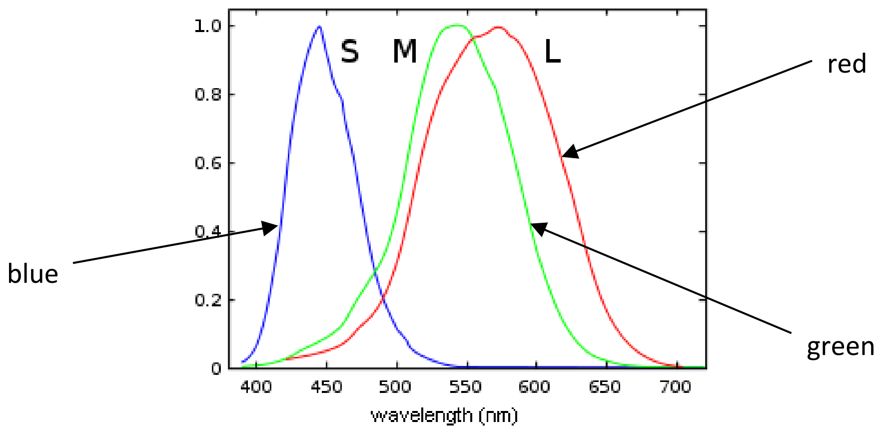
Humans (and other animals) have two types of cells in our retina that respond to light:



Rod cells resolve the physical details of images. Cone cells are responsible for distinguishing colors. Rod cells can operate in low light, but cone cells need much more light; this is why we cannot see colors in low light.

honors (not AP®)

There are three different types of cone cells in our eyes, called "S", "M", and "L", which stand for "short," "medium," and "long." Each type of cone cells responds to different wavelengths of light, having a peak (maximum) absorbance in a different part of the visible spectrum:



For example, light with a wavelength of 400–450 nm appears blue to us, because most of the response to this light is from the S cells, and our brains are wired to perceive this response as blue color. Light with a wavelength of around 500 nm would stimulate mostly the M cells and would appear green. Light with a wavelength of around 570 nm would stimulate the M and L cells approximately equally. When green and red receptors both respond, our brains perceive the color as yellow.

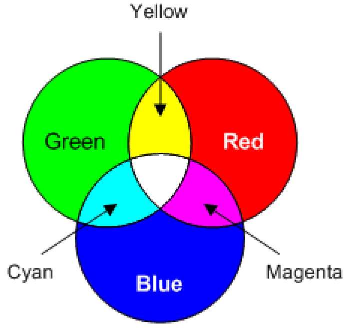
Colorblindness occurs when a genetic mutation causes a deficiency or absence of one or more types of cone cells. Most common is a deficiency in the expression of M cone cells, which causes red-green colorblindness. This means that a person with red-green colorblindness would see both colors as red.

Because colorblindness is recessive and the relevant gene is on the X-chromosome, red-green colorblindness is much more common in men than in women.

honors (not AP®)

Direct Light: Additive Mixing

Because our cone cells respond to red, green, and blue light, we call these colors the primary colors of light. Other colors can be made by mixing different amounts of these colors, thereby stimulating the different types of cone cells to different degrees. When all colors are mixed, the light appears white.



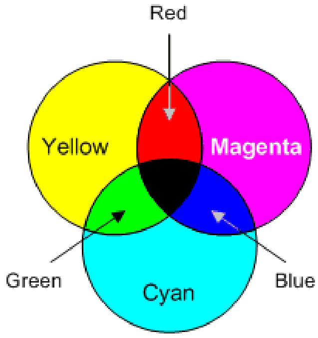
Additive (light)

primary color: light that excites only one type of cone cell. The primary colors of light are red, green, and blue.

secondary color: light that is a combination of exactly two primary colors. The secondary colors of light are cyan, magenta, and yellow.

Reflected Light: Subtractive Mixing

When light shines on an object, properties of that object cause it to absorb certain wavelengths of light and reflect others. The wavelengths that are reflected are the ones that make it to our eyes, causing the object to appear that color.



Subtractive (paint)

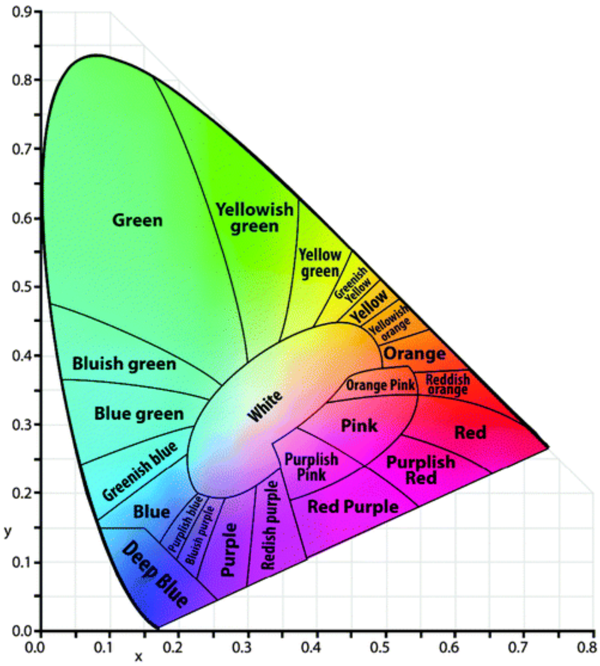
pigment: a material that changes the color of reflected light by absorbing light with specific wavelengths.

primary pigment: a material that absorbs light of only one primary color (and reflects the other two primary colors). The primary pigments are cyan, magenta, and yellow. Note that these are the secondary colors of light.

secondary pigment: a pigment that absorbs two primary colors (and reflects the other). The secondary pigments are red, green, and blue. Note that these are the primary colors of light.

honors (not AP®)

Of course, our perception of color is biological, so mixing primary colors is not a simple matter of taking a weighted average of positions on the color wheel. The relationship between the fractions of primary colors used to produce a color and the color perceived is called chromaticity. The following diagram shows the colors that would be produced by varying the intensities of red, green, and blue light.



On this graph, the *x*-axis is the fraction (from 0 – 1) of red light, the *y*-axis is the fraction of green light, and the fraction of blue is implicit [1 – (red + green)].

Notice that equal fractions (0.33) of red, green and blue light would produce white light.

To show the effects of mixing two colors, plot each color's position on the graph and connect them with a line. The linear distance along that line shows the proportional effects of mixing. (*E.g.,* the midpoint would represent the color generated by 50% of each of the source colors.) This method is how fireworks manufacturers determine the mixtures of different compounds that will produce the desired colors.

Reflection

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.E.2.1

Mastery Objective(s): (Students will be able to…)

- Explain why light is reflected off smooth surfaces.

Success Criteria:

- Descriptions & explanations account for observed behavior.

Language Objectives:

- Explain why light is reflected off smooth surfaces.

Tier 2 Vocabulary: light, reflection, virtual image, real image

Labs, Activities & Demonstrations:

- full length mirror on the wall (does amount of image visible change with distance?)
- Mirascope ("hologram maker")

Notes:

reflection: when a wave "bounces" off an object and changes direction.

specular reflection: reflection from a smooth surface.

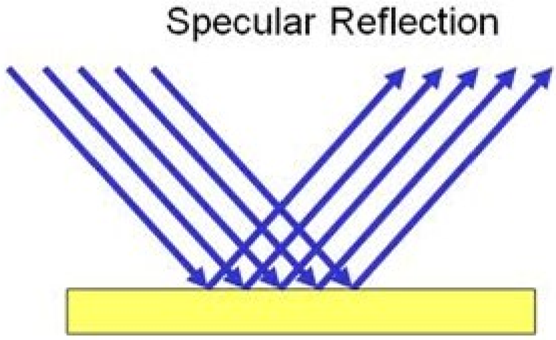
diffuse reflection: reflection from a rough surface.

virtual image: a perceived image that appears to be the point of origin of photons (rays of light) that diverge. Because light is reflected back from a mirror (*i.e.,* light cannot pass through it), a ***virtual image*** is one that appears ***behind*** (or "inside") ***the mirror***. A virtual image is what you are used to seeing in a mirror.

real image: a reflected image that is created by photons (rays of light) that converge. Because light is reflected back from a mirror (*i.e.,* light cannot pass through it), a ***real image*** is one that appears ***in front of the mirror***. A real image created by a mirror looks like a hologram.

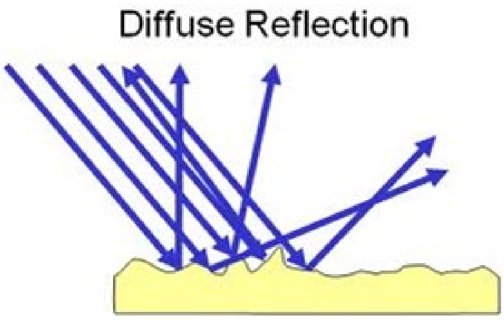
A rule of thumb that works for both mirrors and lenses is that a real image is produced by the convergence of actual rays of light. A virtual image is our perception of where the rays of light appear to have come from.

Specular reflection: reflection from a smooth surface, such as a mirror.



If the photons of light from the source are parallel when they strike the surface, they will also be parallel when they reflect from the surface. This results in a reflected image that appears to be the same size, shape, and distance from the surface as the original object.

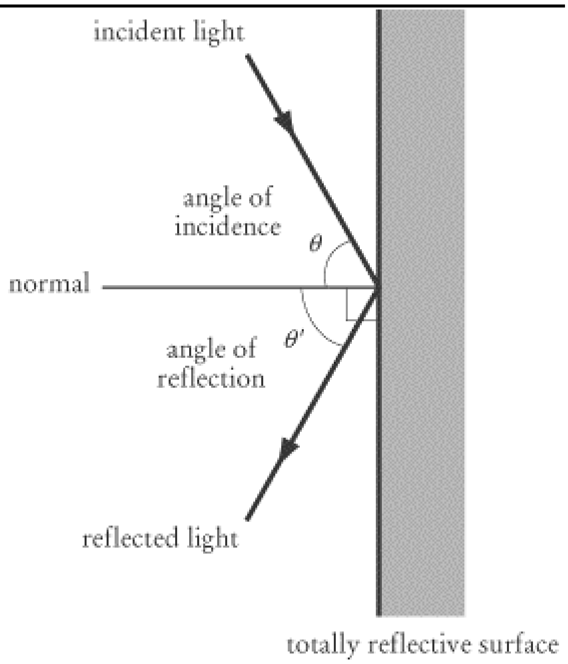
Diffuse reflection : reflection from a rough surface, such as a wall.



Light striking a rough surface will illuminate the surface. However, because the reflected light rays are not parallel, the reflected light does not create a reflected image of the object.

mirror: a surface that causes specular reflection. An object that was not made to be a mirror but behaves like one is often called a mirrored surface.

When light waves strike a mirrored surface at an angle (measured from the perpendicular or "normal" direction), they are reflected at the same angle away from the perpendicular. The most common statement of this concept is "The angle of incidence equals the angle of reflection."



This can be stated mathematically as either $\theta = \theta'$ or $\theta_i = \theta_r$.

Mirrors

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.E.4.1, 6.E.4.2

Mastery Objective(s): (Students will be able to…)

- Draw ray tracing diagrams for reflection from flat and curved (spherical) mirrors.
- Numerically calculate the distance from the mirror to its focus and the mirror to the image.

Success Criteria:

- Ray diagrams correctly show location of object, focus and image.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain when and why images are inverted (upside-down) *vs.* upright.

Tier 2 Vocabulary: light, reflection, virtual image, real image, mirror, focus

Labs, Activities & Demonstrations:

- Mirascope
- turn a glove inside-out

Notes:

<u>mirror</u>: a surface that light rays reflect from at the same angle the light rays came from.

<u>convex</u>: an object that curves outward.

<u>concave</u>: an object that curves inward.

<u>flat</u>: an object that is neither convex nor concave.

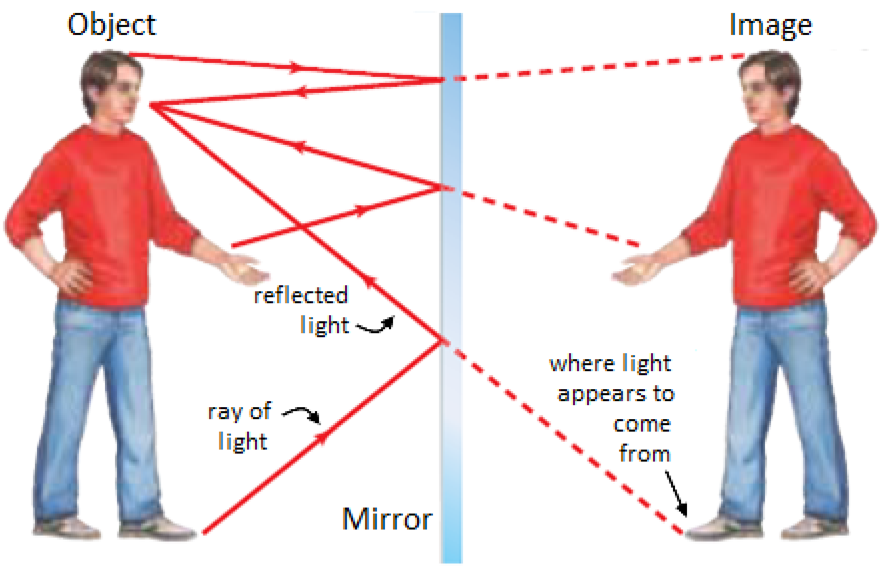
<u>focal point</u>: the point at which parallel rays striking a mirror converge.

<u>principal axis</u>: a line perpendicular to a mirror (*i.e.,* with an angle of incidence of 0°) such that a ray of light is reflected back along its incident (incoming) path.

The principal axis is often shown as a single horizontal line, but every point on a mirror has a principal axis.

Flat Mirrors

With a flat mirror, the light reflected off the object (such as the person in the picture below) bounces off the mirror and is reflected back. Because our eyes and the part of our brains that decode visual images can't tell that the light has been reflected, we "see" the reflection of the object in the mirror.

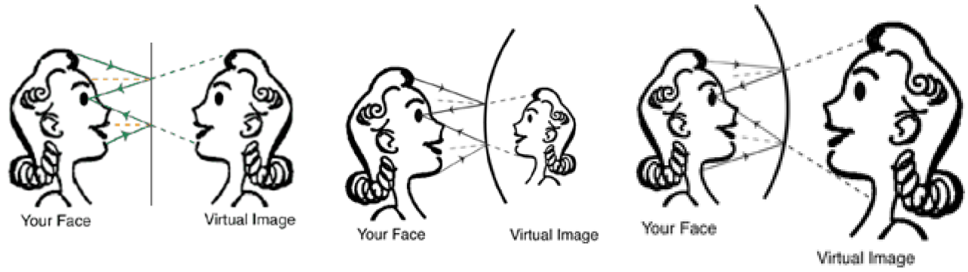


If the mirror is flat, the reflection is the same size and the same distance from the mirror as the actual object. However, the image looks like it is reversed horizontally, but not vertically.

It would seem that the mirror "knows" to reverse the image horizontally but not vertically. (Of course this is not true. If you want the mirror to reverse the image vertically, all you need to do is put the mirror on the floor.) What is actually happening is that light is reflected straight back from the mirror. Anything that is on your right will also be on the right side of the image (from your point of view; if the image were actually a person, this would be the other person's left). Anything that is on top of you will also be on top of the image as you look at it.

What the mirror is doing is the same transformation as flipping a polygon over the *y*-axis. ***The reversal is actually front-to-back*** (where "front" means closer to the mirror and "back" means farther away from it).

Convex and Concave Mirrors



flat mirror	convex mirror	concave mirror
same-size image	smaller image	larger image

With a convex mirror (curved outwards), the reflected rays diverge (get farther apart). When this happens, it makes the reflection appear smaller.

In a concave mirror (curved inwards), the reflected rays converge (get closer together). When this happens, it makes the reflection appear larger.

One place you have probably seen convex mirrors is the passenger-side mirrors in cars. The mirror is slightly convex in order to show a wider field of view. However, this makes the image smaller and appear farther away.

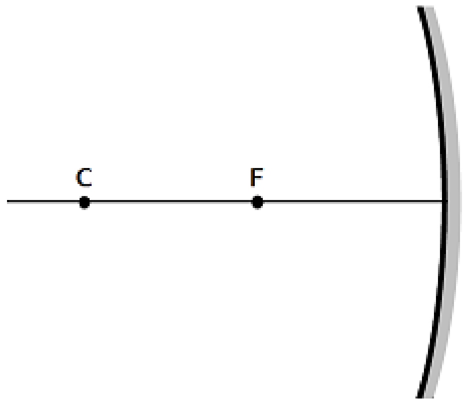
If you wear makeup, you may have used a concave mirror. The larger image makes it easier to see small details. (However, it is important to remember that those details are smaller than they appear!)

Focal Point

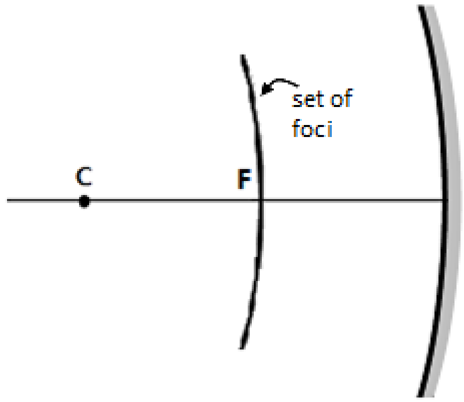
The focal point or focus of a mirror is the point where the rays of reflected light converge. For a spherical mirror (one in which the shape of the mirror is the surface of a sphere), the focus is halfway between the surface of the mirror and the center of the sphere. This means the distance from the mirror to the focus (f)* is half of the radius of curvature (r_c):

$$f = \frac{r_c}{2} \quad \text{or} \quad r_c = 2f$$

In an introductory physics class, the focus of a curved mirror is often described as a single point, as in the following diagram.



However, is important to remember that a principal axis (a line perpendicular to the mirror) can be drawn from any point along the surface of the mirror. This means that the focus is not a single point, but rather the ***set of all points*** that are halfway between the center of the sphere and the surface of the mirror:

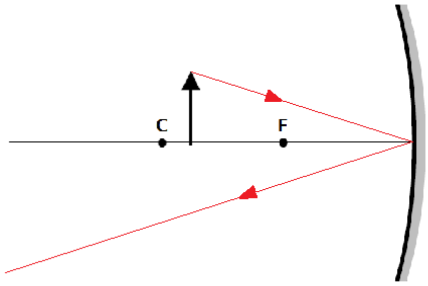


* Some physics textbooks use the variables d_o, d_i, and d_f for distances to the object, image, and focus, respectively. These notes use the variables s_o, s_i, and f in order to be consistent with the equation sheet provided by the College Board for the AP® Physics 2 exam.

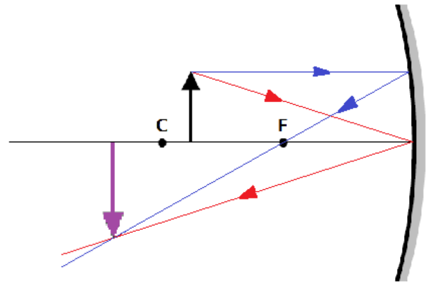
Ray Tracing

An intuitive way of finding the location, size and orientation of an image in a mirror is to draw (trace) the rays of light to see where they converge.

1. A ray of light that hits the mirror anywhere on a principal axis is reflected back at the same angle relative to that principal axis. (The angle of incidence relative to the principal axis equals the angle of reflection.)



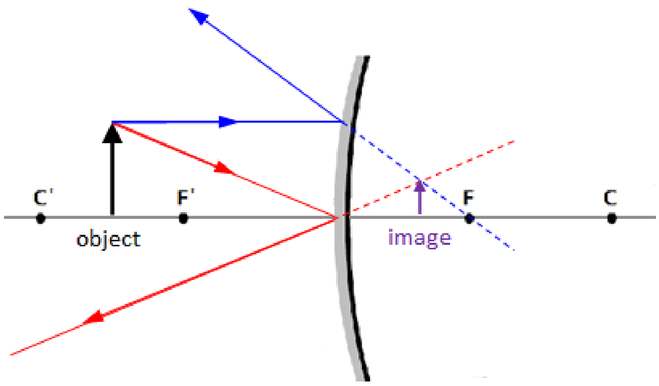
2. A ray of light that hits the mirror parallel to the principal axis is reflected directly toward or away from the focus.



3. If you draw a pair of rays from the top of the object as described by #1 and #2 above, the intersection will be at the top of the image of the object.

Convex Mirrors

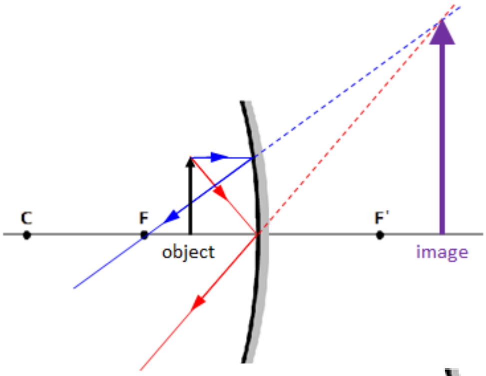
For a convex mirror, the image is always virtual (behind the mirror) and is always smaller than the object:



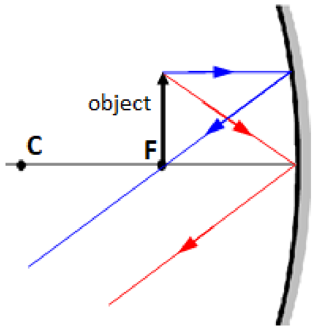
Concave Mirrors

For a concave mirror, what happens with the image changes depending on where the object is relative to the center of curvature and the focus, as shown by ray tracing in each the following cases.

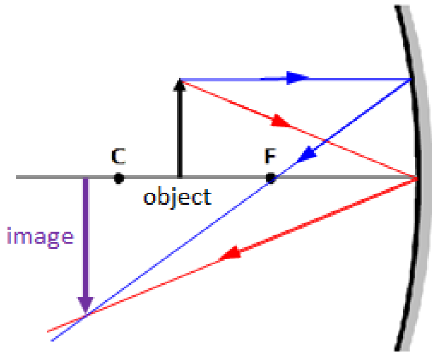
1. If the object is closer to the mirror than the focus, you see a virtual image (behind the mirror) that is upright (right-side-up), and larger than the original.



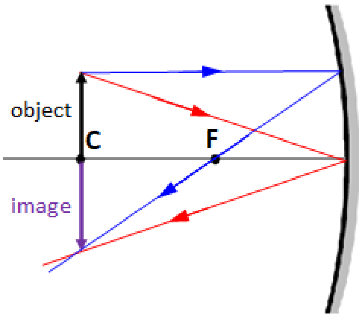
2. If the object is at the focus, there is no image because the rays do not converge.



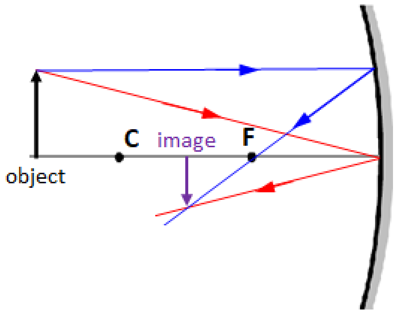
3. If the object is between the focus and the center of curvature, you see a real image (in front of the mirror) that is behind the object, inverted (upside-down), and larger.



4. If the object is at the center of curvature, you see a real, inverted image that is the same size and same distance from the mirror as the object.



5. If the object is farther from the mirror than the center of curvature, you see a real, inverted image that is smaller and closer to the mirror than the object.



Equations

The distance from the mirror to the focus (f) can be calculated from the distance to the object (s_o) and the distance to the image (s_i), using the following equation:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Distances for the image (s_i) and focus (f) are positive in front of the mirror (where a real image would be), and negative behind the mirror (where a virtual image would be).

The height of the image (h_i) can be calculated from the height of the object (h_o) and the two distances (s_i and s_o), using the following equation:

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

A positive value for h_i means the image is upright (right-side-up), and a negative value for h_i means the image is inverted (upside-down).

magnification: the ratio of the size of the image to the size of the object.

If $M > 1$, the image is larger than the object. (For example, if $M = 2$, then the image is twice as large as the object.) If $M = 1$, the object and image are the same size. If $M < 1$, the image is smaller. Finally, note that in a mirror, virtual images are always upright, and real images are always inverted.

Sample Problem:

Q: An object that is 5 cm high is placed 9 cm in front of a spherical convex mirror. The radius of curvature of the mirror is 10 cm. Find the height of the image and its distance from the mirror. State whether the image is real or virtual, and upright or inverted.

A: The mirror is convex, which means the focus is behind the mirror. This is the side where a ***virtual*** image would be, so the distance to the focus is therefore negative. The distance to the focus is half the radius of curvature, which means $f = -5$ cm. From this information, we can find the distance from the mirror to the image (s_i):

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$\frac{1}{9} + \frac{1}{s_i} = \frac{1}{-5}$$

$$\frac{5}{45} + \frac{1}{s_i} = -\frac{9}{45}$$

$$\frac{1}{s_i} = -\frac{14}{45}$$

$$s_i = -\frac{45}{14} = -3.2\,\text{cm}$$

The value of −3.2 cm means the image is a virtual image located 3.2 cm behind the mirror.

Now that we know the distance from the mirror to the image, we can calculate the height of the image (h_i):

$$\frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$$\frac{h_i}{5} = -\frac{-3.2}{9}$$

$$(5)(3.2) = 9\,h_i$$

$$h_i = \frac{16}{9} = +1.8\,\text{cm}$$

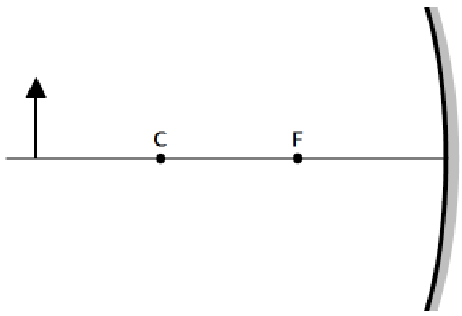
The image is 1.8 cm high. Because the height is a positive number, this means the image is upright (right-side-up).

Homework Problems

In each of the following problems, an object that is 12 cm tall is placed in front of a curved, spherical mirror with a focal length of 18 cm.

1. **(M)** The mirror is concave and the object is placed 58 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

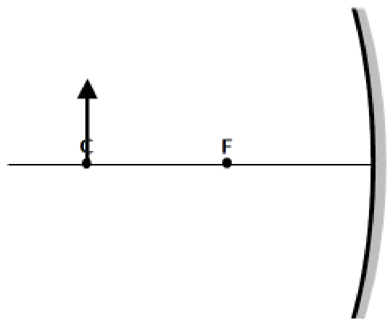


   b. Calculate the height and orientation (upright or inverted) of the image, and its distance from the mirror.

Answers: s_i = 26.1 cm; h_i = −5.4 cm

2. **(S)** The mirror is concave and the object is placed 36 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

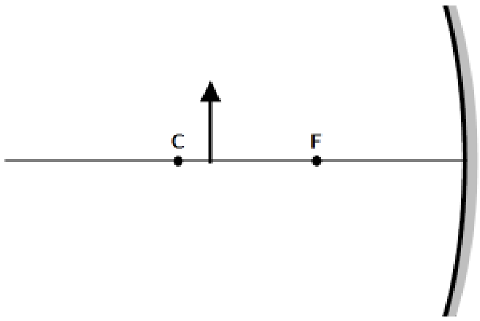


   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: s_i = 36 cm; h_i = −12 cm

3. **(S)** The mirror is concave and the object is placed 32 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

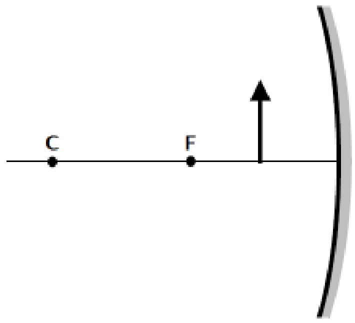


   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: s_i = 41.1 cm; h_i = −15.4 cm

4. **(M)** The mirror is concave and the object is placed 6 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

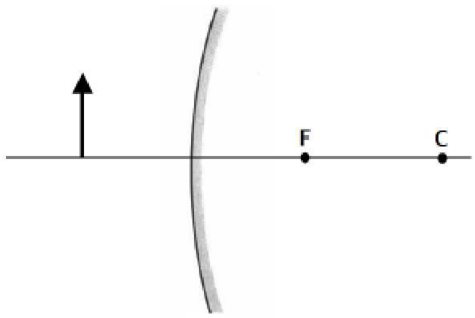


   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: $s_i = -9$ cm; $h_i = 18$ cm

5. **(M)** The mirror is convex and the object is placed 15 cm from the mirror.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



   b. Calculate the height and orientation of the image, and its distance from the mirror.

Answers: $s_i = -8.2$ cm; $h_i = 6.5$ cm

Refraction

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.E.1.1, 6.E.2.1, 6.E.3.1, 6.E.3.2, 6.E.3.3

Mastery Objective(s): (Students will be able to…)

- Explain how and why refraction happens.
- Solve problems using Snell's Law.

Success Criteria:

- Explanation accounts for the size, location and orientation of the image.
- Calculations are correct with correct algebra and trigonometry.

Language Objectives:

- Explain why we see the image of an object through a magnifying glass but not the object in its actual location.

Tier 2 Vocabulary: light, reflection, virtual image, real image, lens, focus

Labs, Activities & Demonstrations:

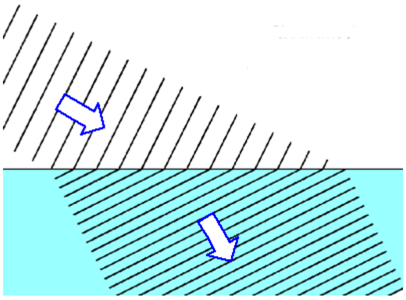
- laser through clear plastic
- laser through bent plastic (total internal reflection)
- laser through falling stream of water (with 1 drop milk)
- Pyrex stirring rod in vegetable oil (same index of refraction)
- penny in cup of water

Notes:

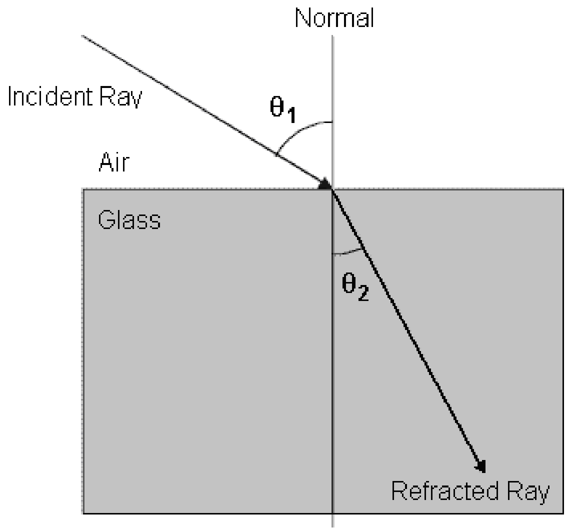
refraction: a change in the velocity and direction of a wave as it passes from one medium to another. The change in direction occurs because the wave travels at different velocities in the different media.

index of refraction: a number that relates the velocity of light in a medium to the velocity of light in a vacuum.

When light crosses from one medium to another, the difference in velocity of the waves causes the wave to bend. For example, in the picture below, the waves are moving faster in the upper medium. As they enter the lower medium, they slow down. Because the part of the wave that enters the medium soonest slows down first, the angle of the wave changes as it crosses the boundary.



When the waves slow down, they are bent toward the normal (perpendicular), as in the following diagram:



The index of refraction of a medium is the velocity of light in a vacuum divided by the velocity of light in the medium:

$$n = \frac{c}{v}$$

Thus the larger the index of refraction, the more the medium slows down light as it passes through.

The index of refraction for some substances is given below.

Substance	Index of Refraction	Substance	Index of Refraction
vacuum	1.00000	quartz	1.46
air (0°C and 1 atm)	1.00029	glass (typical)	1.52
water (20°C)	1.333	NaCl (salt) crystals	1.54
acetone	1.357	polystyrene (#6 plastic)	1.55
ethyl alcohol	1.362	diamond	2.42

These values are for yellow light with a wavelength of 589 nm.

For light traveling from one medium into another, the ratio of the speeds of light is related inversely to the ratio of the indices of refraction, as described by Snell's Law (named for the Dutch astronomer Willebrord Snellius):

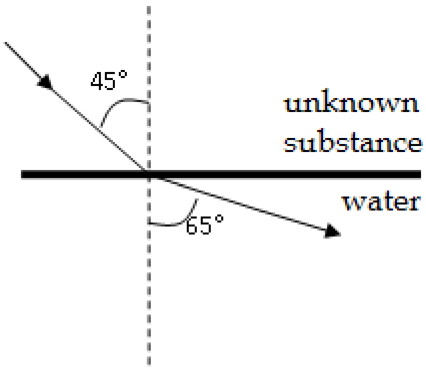
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

The more familiar presentation of Snell's Law is:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Sample Problem:

Q: Incident light coming from an unknown substance strikes water at an angle of 45°. The light refracted by the water at an angle of 65°, as shown in the diagram at the right. What is the index of refraction of the unknown substance?



A: Applying Snell's Law:

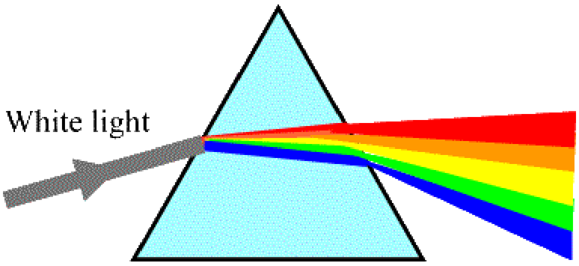
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_1 \sin(45°) = (1.33)\sin(65°)$$
$$n_1 = \frac{(1.33)\sin 65°}{\sin 45°} = \frac{(1.33)(0.906)}{0.707} = 1.70$$

Prisms

The index of refraction of a medium varies with the wavelength of light passing through it. The index of refraction is greater for shorter wavelengths (toward the violet end of the spectrum) and less (closer to 1) for longer wavelengths (toward the red end of the spectrum.

prism: an object that refracts light

If light passes through a prism (from air into the prism and back out) and the two interfaces are not parallel, the different indices of refraction for the different wavelengths will cause the light to spread out.

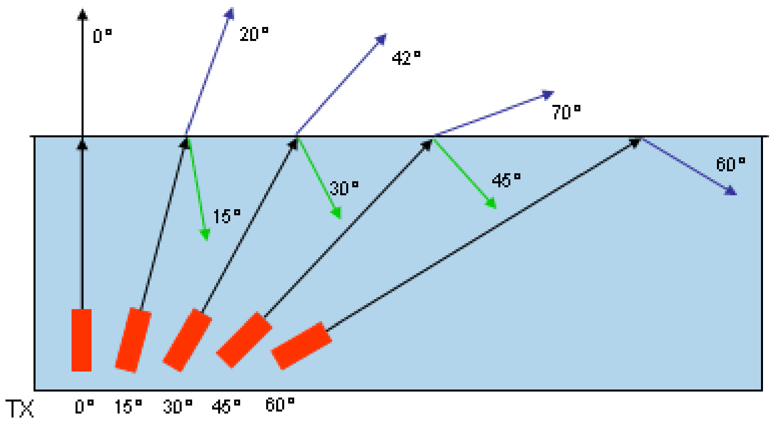


When light is bent by a prism, the ratio of indices of refraction is the inverse of the ratio of wavelengths. Thus we can expand Snell's Law as follows:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

Total Internal Reflection

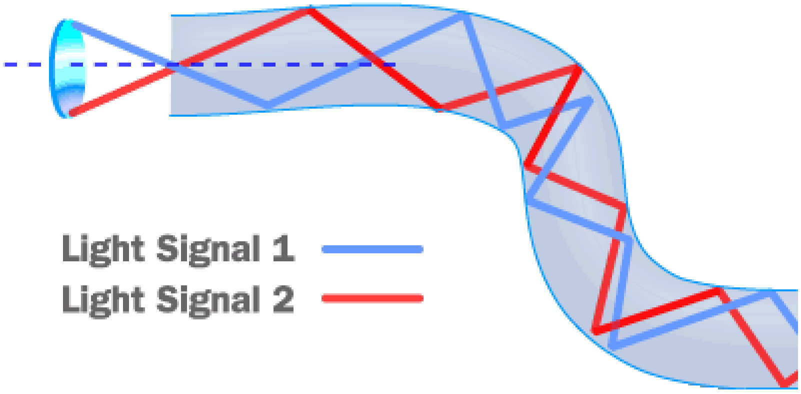
If a light wave is traveling from a slower medium to a faster one and the angle is so steep that the refracted angle would be 90° or greater, the boundary acts as a mirror and the light ray reflects off of it. This phenomenon is called *total internal reflection:*



critical angle (θ_c): the angle beyond which total internal reflection occurs.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Total internal reflection is how optical fibers (long strands of optically pure glass with a high index of refraction) are used to transmit information over long distances, using pulses of light.



Total internal reflection is also the principle behind speech teleprompters:

The speaker stands behind a clear piece of glass. The image of the speech is projected onto the glass. The text is visible to the speaker, but not to the audience.

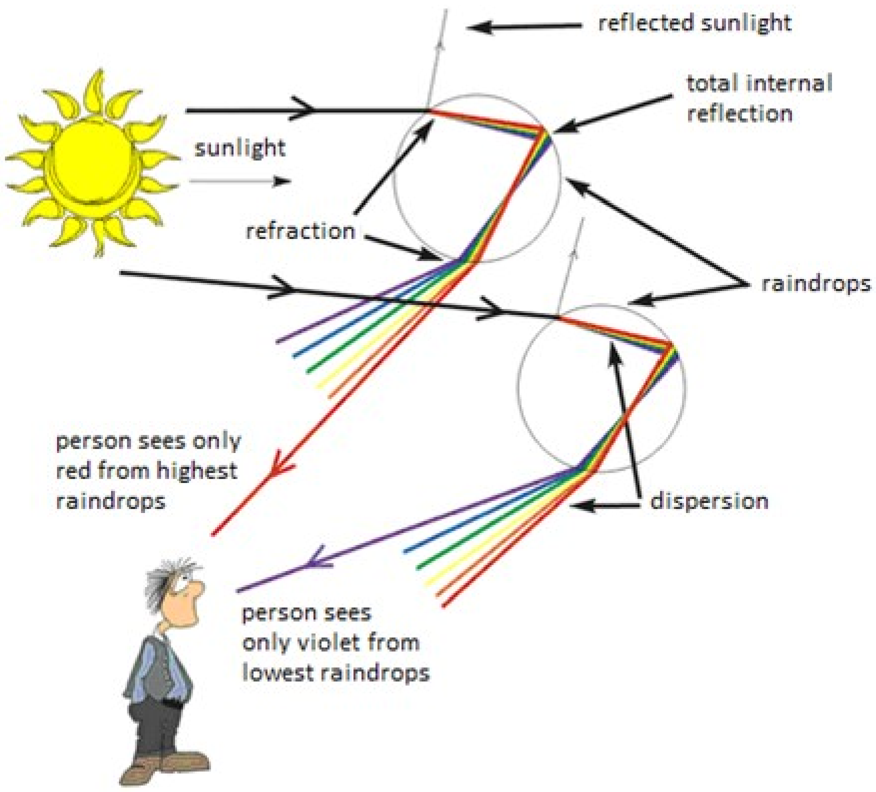
honors (not AP®)

Rainbows

A rainbow occurs from a combination of refraction, total internal reflection, and a second refraction, with raindrops acting as the prisms.

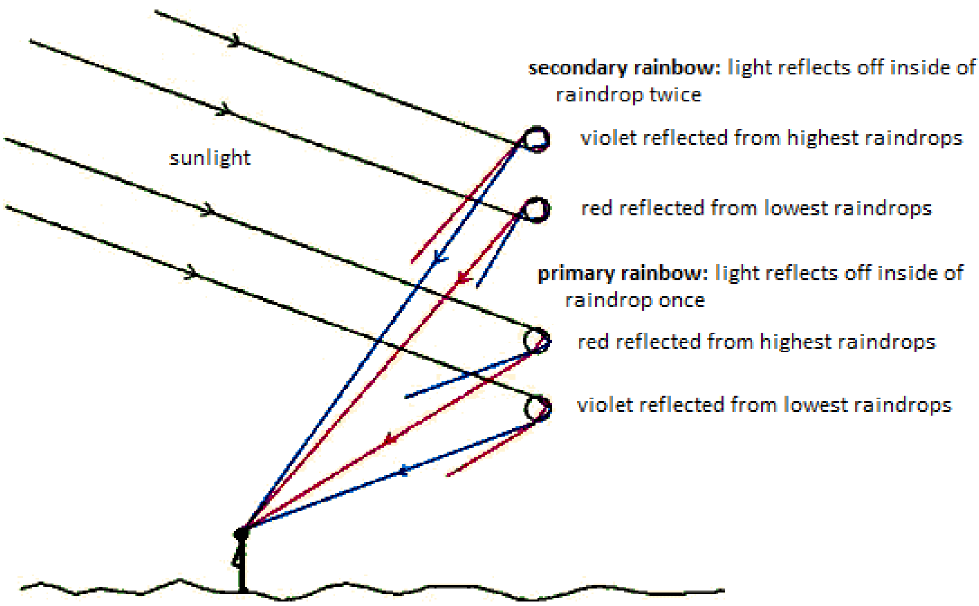
When this process occurs, different wavelengths of are refracted at different angles. Because colors near the red end of the spectrum have a lower index of refraction, the critical angle is shallower for these wavelengths, and they are reflected at a shallower angle than colors closer to the violet end of the spectrum.

The overall change in the direction of the light after this combination of refraction–reflection–refraction (including both refractions as well as the reflection) ranges from approximately 40° for violet light to approximately 42° for red light. This difference is what produces the spread of colors in a rainbow, and is why red is always on the outside of the rainbow and violet is always on the inside.



honors (not AP®)

When internal reflection occurs twice on the inside of a raindrop, the result is a second rainbow.



The second rainbow appears above the first because the angle of light exiting the raindrop is greater—varying from 50° for red light to 52.5° for violet light. The second internal reflection reverses the colors, which is why violet is on the outside and red is on the inside in the second rainbow.

honors (not AP®)

This is a picture of a double rainbow in Lynn, Massachusetts. Note that the order of the colors in the second rainbow is reversed.

Note also that the sky is brighter inside the primary rainbow. There are two reasons for this. First, it's not actually true that each band is only one color of light. Because red light reflects at *all* angles greater than or equal to 40°, red light is therefore a component of all of the colors inside the red band of the rainbow. The same is true for each of the other colors; inside of the violet band, all wavelengths of visible light are present, and the result is white light. Outside of the red band, no visible light is refracted, which causes the sky outside the rainbow to appear darker.

Second, raindrops scatter light at all wavelengths, and light scattering is also a significant contributing factor to the brightness inside. (See the *Scattering* topic starting on page 409 for more information.)

You may also notice that because the second rainbow is reversed, the sky is slightly brighter outside of the second rainbow.

Homework Problems

You will need to look up indices of refraction in Table Q on page 587 of your Physics Reference Tables in order to answer these questions.

1. **(M)** A ray of light traveling from air into borosilicate glass strikes the surface at an angle of 30°. What will be the angle of refraction?

   Answer: 19.8°

2. **(S)** Light traveling through air encounters a second medium which slows the light to $2.7\times 10^{8}\ \frac{m}{s}$. What is the index of refraction of the second medium?

   Answer: 1.11

3. **(M)** What is the velocity of light as it passes through a diamond?

   Answer: $1.24\times 10^{8}\ \frac{m}{s}$

4. **(M)** A diver in a freshwater lake shines a flashlight toward the surface of the water. What is the minimum angle (from the vertical) that will cause beam of light to be reflected back into the water (total internal reflection)?

   Answer: 48.6°

5. **(S)** A graduated cylinder contains a layer of silicone oil floating on water. A laser beam is shone into the silicone oil from above (in air) at an angle of 25° from the vertical. What is the angle of the beam in the water?

   Answer: 18.5°

6. **(S)** A second graduated cylinder contains only a layer of water. The same laser beam is shone into the water from above (in air) at the same angle of 25° from the vertical. What is the angle of the beam in the water?

   Answer: 18.5°

Polarization

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.A.1.3

Mastery Objective(s): (Students will be able to…)

- Explain how and under which circumstances light can be polarized.

Success Criteria:

- Explanation accounts for the filtering of waves of other orientations and for the specific direction.

Language Objectives:

- Explain how polarized sunglasses work.

Tier 2 Vocabulary: polarized

Labs, Activities & Demonstrations:

- polarizing filters

Notes:

Normally, light (and other electromagnetic waves) propagate in all directions. When the light is passed through a special filter, called a polarizer, it blocks light waves in all but one plane (direction), as shown in the following diagram:

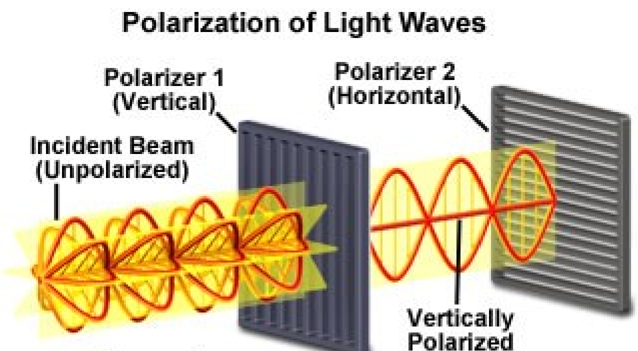


Figure 1

Light that is polarized in this manner is called <u>plane-polarized</u> <u>light</u>.

Note that if you place two polarizers on top of each other and turn them so they polarize in different directions, no light can get through. This is called crossed polarization.

A flat surface can act as a polarizer at certain angles. The Scottish physicist Sir David Brewster derived a formula for the angle of maximum polarization based on the indices of refraction of the two substances:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where:

θ_B = Brewster's angle, the angle of incidence at which unpolarized light striking a surface is perfectly polarized when reflected.

n_1 & n_2 = indices of refraction of the two substances

The two pictures below were taken with the same camera and a polarizing filter. In the picture on the left, the polarizing filter is aligned with the light reflected off the window. In the picture on the right, the polarizing filter is rotated 90° so that none of the reflected light from the window can get to the camera lens.

Another example is light reflecting off a wet road. When the sun shines on a wet road at a low angle, the reflected light is polarized parallel to the surface (*i.e.,* horizontally). Sunglasses that are polarized vertically (*i.e.,* that allow only vertically polarized light to pass through) will effectively block most or all of the light reflected from the road.

Yet another example is the light that creates a rainbow. When sunlight reflects off the inside of a raindrop, the angle of incidence is very close to Brewster's angle. This causes the light that exits the raindrop to be polarized in the same direction as the bows of the rainbow (*i.e.,* horizontally at the top). This is why you cannot see a rainbow through polarized sunglasses!

Lenses

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.E.5.1, 6.E.5.2

Mastery Objective(s): (Students will be able to…)

- Draw ray tracing diagrams for refraction through convex and concave lenses.
- Numerically calculate the distance from the lens to its focus and the lens to the image.

Success Criteria:

- Ray diagrams correctly show location of object, focus and image.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain when and why images are inverted (upside-down) *vs.* upright.

Tier 2 Vocabulary: light, refraction, virtual image, real image, lens, focus

Labs, Activities & Demonstrations:

- Fresnel lens
- optics bench lab

Notes:

Lenses are similar to curved mirrors in that they change the direction of light rays to produce an image of an object that can have a different size, orientation and distance from the mirror relative to the object.

Lenses are different from mirrors in that light passes through them, which means they operate by refraction instead of reflection.

<u>lens</u>: a usually-symmetrical optical device which refracts light in a way that makes the rays of light either converge or diverge.

<u>convex lens</u>: a lens that refracts light so that it converges as it passes through.

<u>concave lens</u>: a lens that refracts light so that it diverges as it passes through.

<u>focus or focal point</u>: the point at which light rays converge after passing through the lens.

principal axis: a line perpendicular to the surface of the lens, such that light passing through it is refracted at an angle of 0° (*i.e.*, the direction is not changed).

The principal axis is often shown as a single horizontal line, but every point on the surface of a lens has a principal axis. Note also that if a lens is asymmetrical, its principal axis may be different on each side.

vertex: the point where the principal axis passes through the center of the lens.

real image: an image produced by light rays that pass through the lens. A ***real image*** will appear on the ***opposite side of the lens*** from the object. A real image is what you are used to seeing through a magnifying glass.

virtual image: an apparent image produced at the point where diverging rays appear to originate. A ***virtual image*** will appear on the ***same side of the lens*** as the object.

A rule of thumb that works for both mirrors and lenses is that a ***real image*** is produced by the convergence of the ***actual rays of light***. A virtual image is our perception of where the rays of light would have come from.

upright image: an image that is oriented in the same direction as the object. ("right-side-up")

inverted image; an image that is oriented in the opposite direction from the object. ("upside-down")

Calculations

The equations for lenses are the same as the equations for curved mirrors. Distances are measured from the vertex.

The magnification (M) is the ratio of the height of the image (h_i) to the height of the object (h_o), which is equal to the ratio of the distance of the image (s_i) to distance of the object (s_o).

$$M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

As with mirrors, the distance to the image is defined to be positive for a real image, and negative for a virtual image. However, note that with lenses the real image is caused by the rays of light that pass through the lens, which means a real image is behind a lens, where as a real image is in front of a mirror.

Note also that for lenses, this means that the positive direction for the object and the positive for the image are opposite.

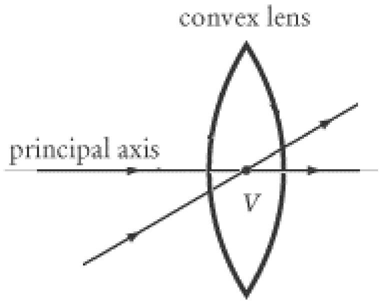
As with mirrors, the distance from the vertex of the lens to the focus (f) is defined by the equation:

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

Ray Tracing

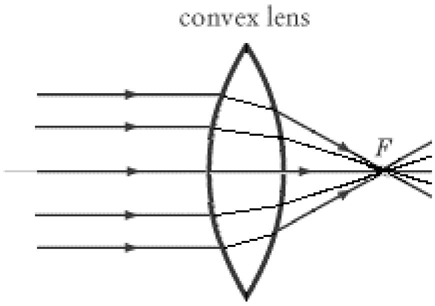
In all lenses:

1. Light passing through the *vertex* of the lens comes out of the lens in the same direction as it entered, as if the lens were not there.

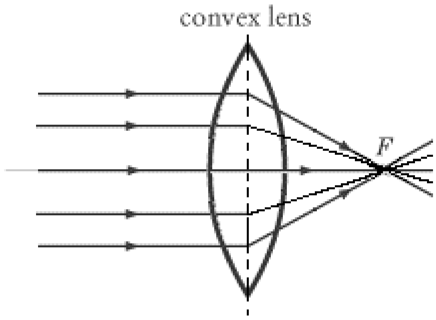


2. Light passing through any part of the lens other than the vertex is refracted through the *focus*.

   Notice that the light is refracted twice, once upon entering the lens and a second time upon entering the air when it exits. For convenience, we usually draw the ray trace as if the light is refracted once when it crosses the center of the lens.



what actually happens

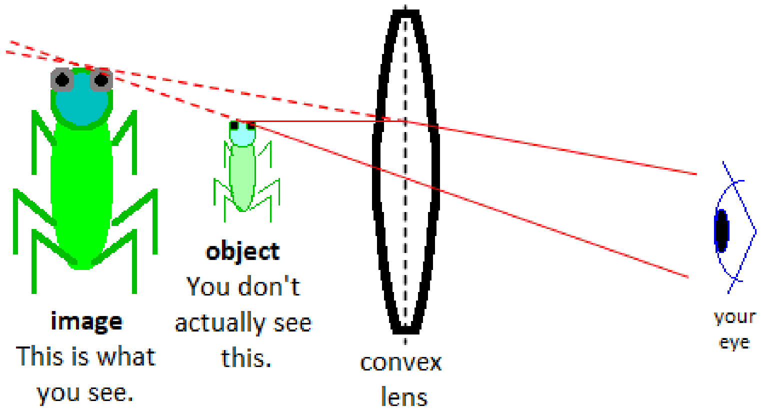


how we usually draw it

Convex Lenses

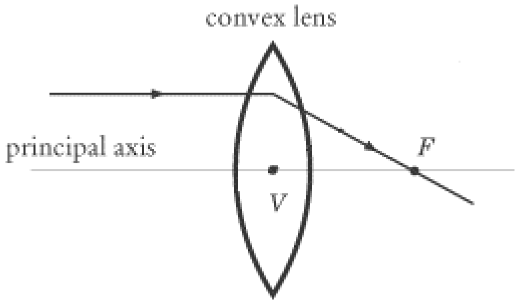
A convex lens causes light rays to converge (bend towards each other) as they pass through.

The most familiar use of convex lenses is as a magnifying glass. Note how the bending of the light rays makes the object appear larger

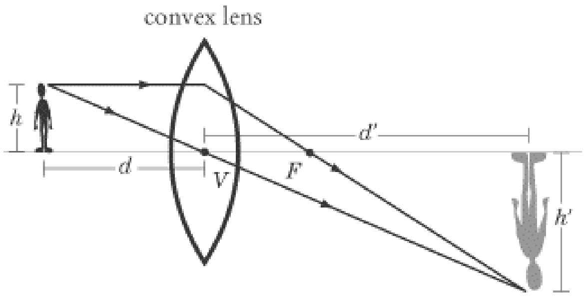


Note also that the lens bends ***all*** of the light. Your eyes cannot see the unbent light rays, which means you cannot see the actual object in its actual location; you *only* see the image.

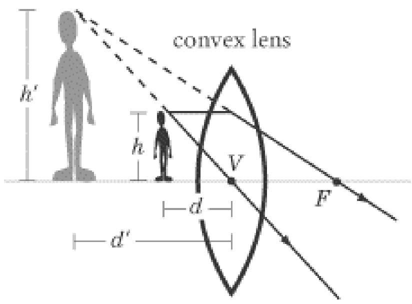
For a convex lens, the focus is always on the opposite side of the lens from the object:



1. If the object is farther away from the lens than the focus, the image is real (on the opposite side of the lens) and inverted (upside-down).

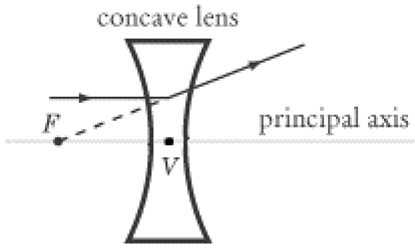


2. If the object is closer than the focus, the image is virtual (on the same side of the lens) and upright (right-side-up).

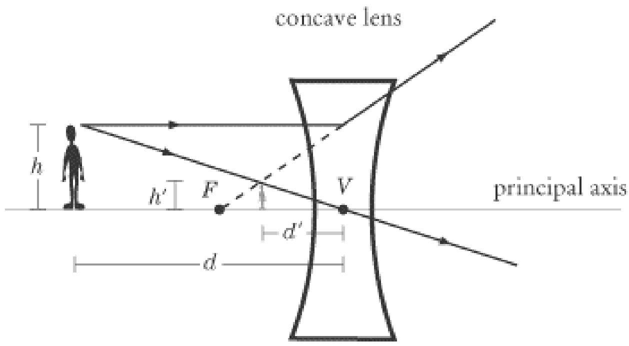


Concave Lenses

A concave lens causes light rays to diverge (bend away from each other). For a concave lens, the focus is on the same side of the lens as the object.

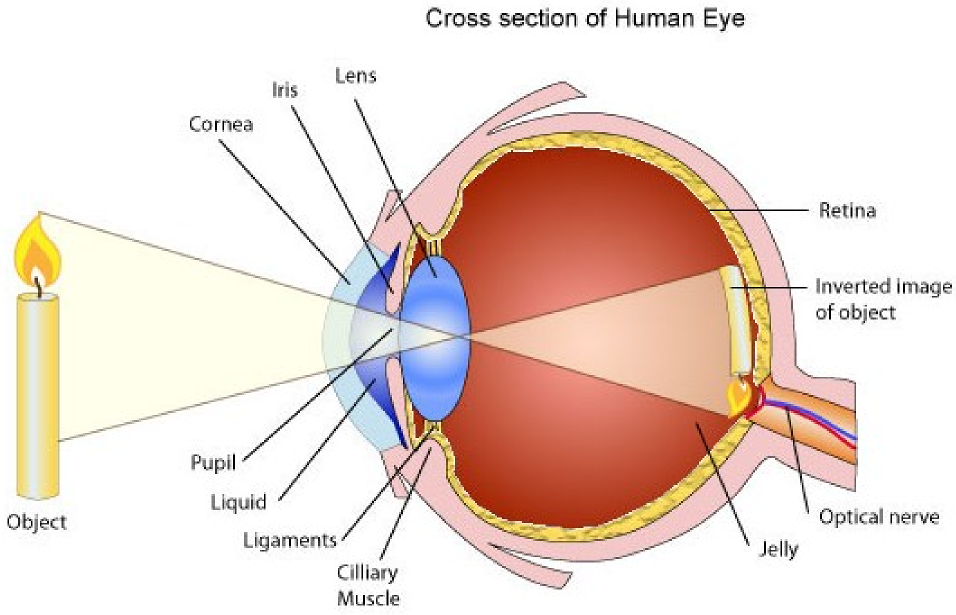


For a concave lens, the image is always virtual (on the same side of the lens) and upright (right-side-up):



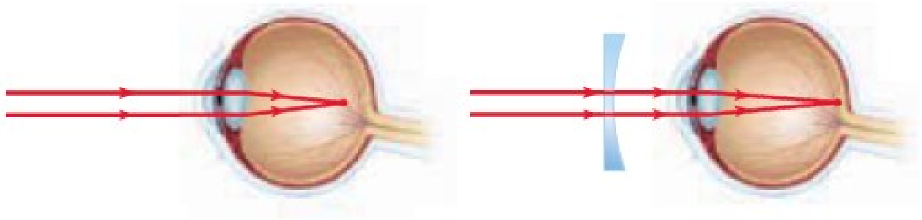
Physiology

In the human eye, the cornea and lens both act as lenses. However, because the action of the ciliary muscles changes the shape of the lens, the lens is responsible for the exact focal point, which determines what we are focusing our eyes on. When the ciliary muscles relax, the images of distant objects are focused on the retina. When these muscles contract, the focal point moves and closer objects come into focus.



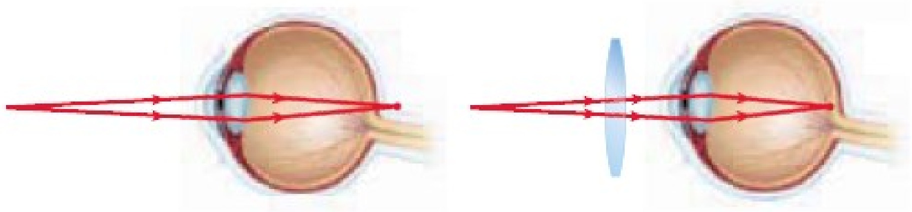
Nearsightedness and Farsightedness

"Nearsighted" means only objects near the eye are in focus; the viewer is unable to focus on distant objects. This happens because the focus of the lens when the ciliary muscles are fully relaxed is in front of the retina. Nearsightedness is corrected by eyeglasses with concave lenses, which move the focal point back to the retina.



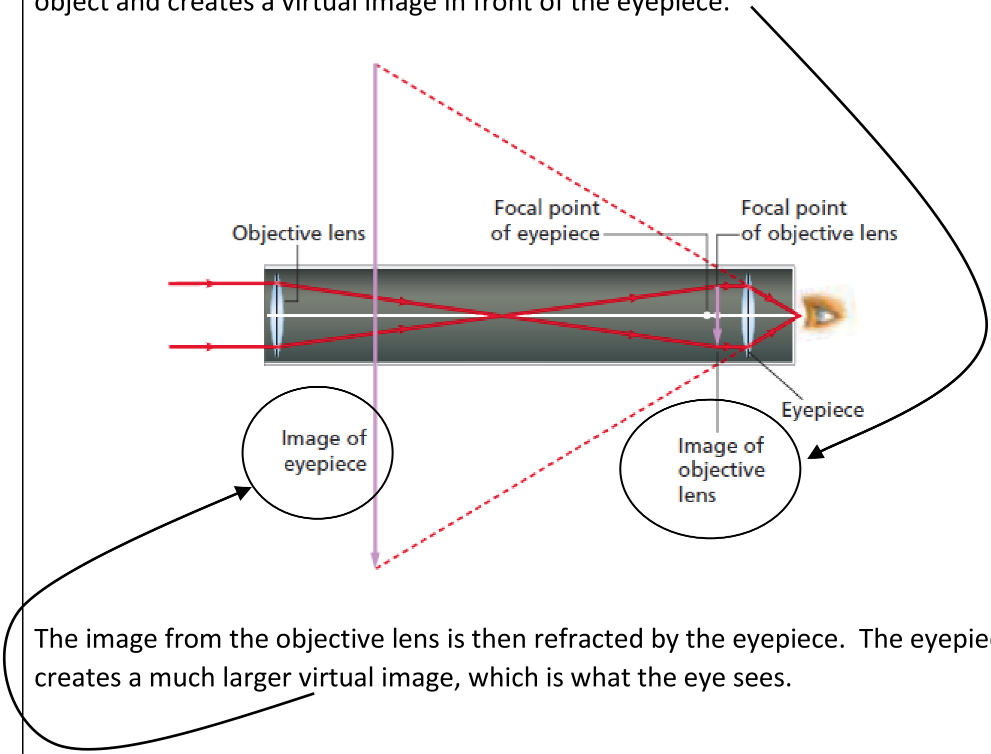
Notice that lenses that correct nearsightedness are concave only on the inside. This helps the lenses avoid the "Coke bottle" look.

"Farsighted" means only objects far away from the eye are in focus; the viewer is unable to focus on close objects. This happens because the ciliary muscles cannot contract enough to bring the focal point of the lens for light coming from nearby objects onto the retina. Farsightedness is corrected by eyeglasses with convex lenses, which move the focal point forward to the retina.



Telescopes

A telescope performs two tasks. The objective lens focuses light from a distant object and creates a virtual image in front of the eyepiece.

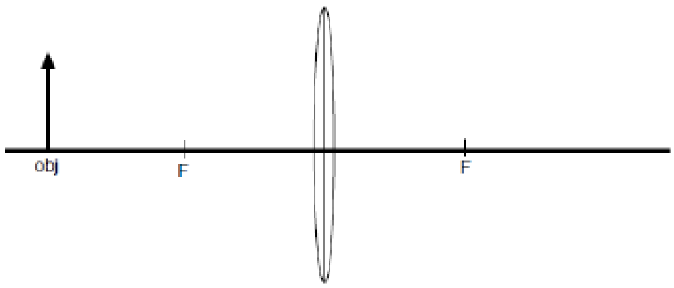


The image from the objective lens is then refracted by the eyepiece. The eyepiece creates a much larger virtual image, which is what the eye sees.

Homework Problems

1. **(M)** A 4.2 cm tall object is placed 12 cm from a convex lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

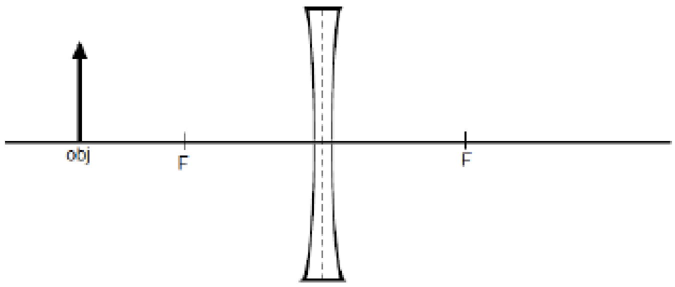


   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = +12$ cm; $h_i = -4.2$ cm

2. **(M)** A 4.2 cm tall object is placed 12 cm from a concave lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

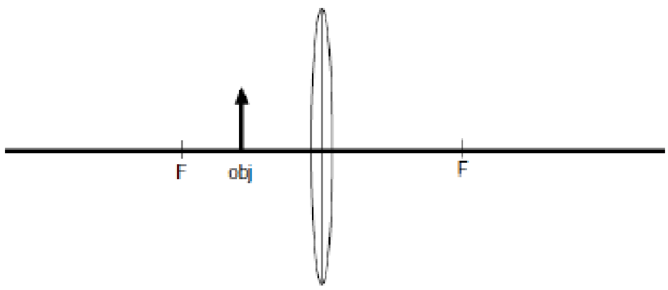


   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = -4$ cm; $h_i = +1.4$ cm

3. **(M)** A 2.7 cm tall object is placed 3.4 cm from a convex lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.

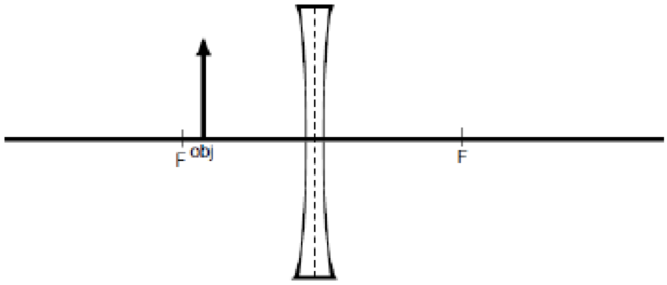


   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: s_i = −7.84 cm; h_i = +6.23 cm

4. **(M)** A 2.7 cm tall object is placed 5.1 cm from a concave lens that has a focal length of 6.0 cm.

   a. Show the location and orientation of the image by accurately drawing a ray diagram on the image below.



   b. Calculate the height and orientation of the image, and its distance from the lens.

Answers: $s_i = -2.76$ cm; $h_i = +1.46$ cm

Diffraction

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.C.2.1, 6.C.3.1, 6.C.4.1

Mastery Objective(s): (Students will be able to…)

- Explain how light "spreads" beyond an opening or around an obstacle.
- Perform calculations relating to the location of bright and dim regions when light passes through a diffraction grating.

Success Criteria:

- Explanations account for observed behavior.
- Calculations are correct with correct algebra.

Language Objectives:

- Explain why looking through a diffraction grating produces a "rainbow".

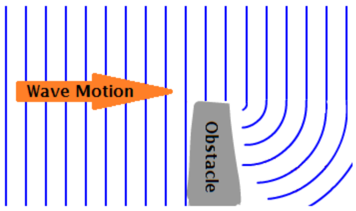
Tier 2 Vocabulary: light, diffraction, slit

Labs, Activities & Demonstrations:

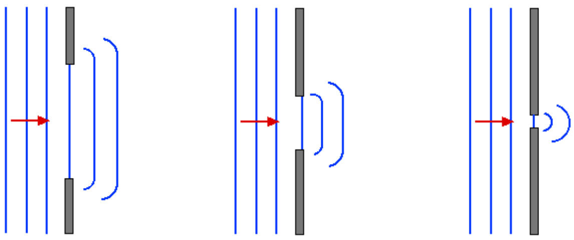
- thickness of human hair
- double slit experiment with laser & diffraction grating

Notes:

diffraction: the slight bending of a wave as it passes around the edge of an object or through a slit:

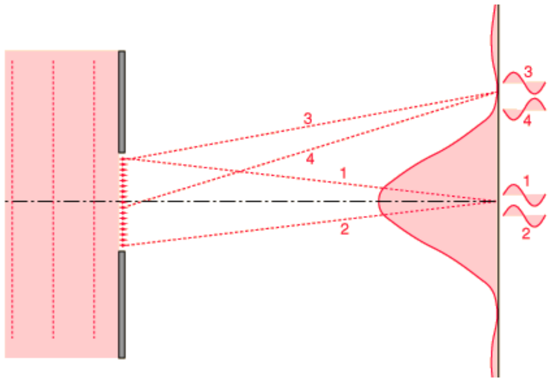


When light passes straight through a wide opening, the rays continue in a straight line. However, if we make the slit so narrow that the width is approximately equal to the wavelength, then the slit effectively becomes a point, and diffraction occurs in all directions from it.



If we shine light through a slit whose thickness is approximately the same order of magnitude as the wavelength, the light can only hit the wall in specific locations.

In the this diagram, light travels the same distance for paths 1 and 2—the same number of wavelengths. Light waves hitting this point will add constructively, which makes the light brighter.



However, for paths 3 and 4, path 4 is ½ wavelength longer than path 3. Light taking path 4 is ½ wavelength out of phase with light from path 3. The waves add destructively (cancel), and there is no light:

Farther up or down on the right side will be alternating locations where the difference in path length results in waves that are different by an exact multiple of the wavelength (in phase = constructive interference = bright spots), *vs.* by a multiple of the wavelength plus ½ (out-of-phase = destructive interference = dark spots).

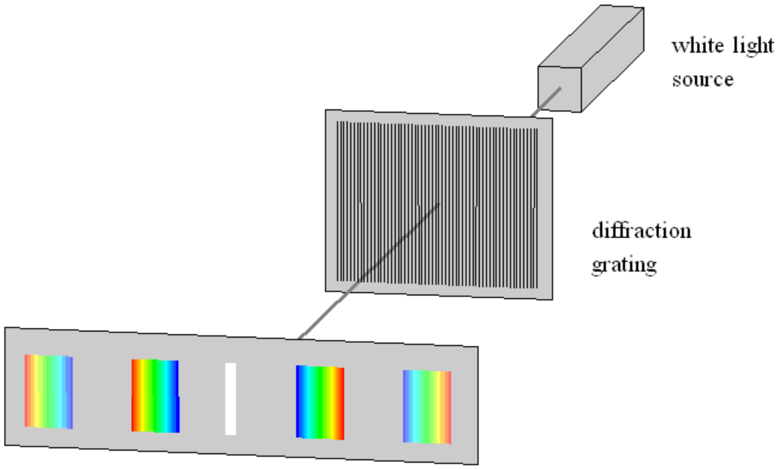
The equation that relates the distance between these regions of constructive interference to the distance between the slits in a diffraction grating is:

$$d \sin\theta_m = m\lambda$$

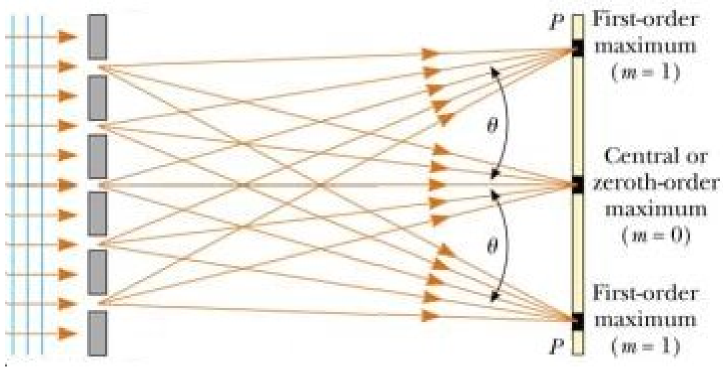
where:

- m = the number of waves that equals the difference in the lengths of the two paths (integer)
- θ_m = the angle of emergence (or angle of deviation) in order for light from one slit to add constructively to light from a neighboring slit that is *m* wavelengths away.
- d = the distance between the slits
- λ = the wavelength of the light

<u>diffraction grating</u>: a screen with a series of evenly-spaced slits that scatters light in a repeating, predictable pattern.



When light shines through a diffraction grating, the following happens:



The patterns of light surrounding the center are the points where the waves of light add constructively.

Notice that blue and violet light (with the shortest wavelengths) is diffracted the least and appears closest to the center, whereas red light is diffracted more and appears farther away. This is because shorter waves bend more around the edges of a slit, because they need to turn less far to fit through the slits than longer waves do.

Sample Problem:

Q: Consider three laser pointers: a red laser with a wavelength of 650 nm, a green laser with a wavelength of 532 nm, and a blue laser with a wavelength of 405 nm. If each of these is shone through a diffraction grating with 5 000 lines per cm, what will be the angle of emergence for each color?

A: For our diffraction grating, 5 000 lines per cm equals 500 000 lines per meter.

$$d = \frac{1}{500\,000} = 2\times10^{-6}\text{ m}$$

For the red laser, 650 nm equals $\lambda = 650\text{ nm} = 6.50\times10^{-7}\text{m}$

The equation is:

$$d\sin\theta_m = m\lambda$$

For the red laser at m = 1, this becomes:

$$(2\times10^{-6})\sin\theta = (1)(6.50\times10^{-7})$$

$$\sin\theta = \frac{6.50\times10^{-7}}{2\times10^{-6}} = 0.325$$

$$\theta = \sin^{-1}(0.325) = 19.0°$$

For the green laser $(\lambda = 532\text{ nm} = 5.32\times10^{-7}\text{m})$ and the blue laser also at m = 1 $(\lambda = 405\text{ nm} = 4.05\times10^{-7}\text{m})$:

$$\sin\theta = \frac{5.32\times10^{-7}}{2\times10^{-6}} = 0.266$$

$$\theta = \sin^{-1}(0.266) = 15.4°$$

and

$$\sin\theta = \frac{4.05\times10^{-7}}{2\times10^{-6}} = 0.203$$

$$\theta = \sin^{-1}(0.203) = 11.7°$$

Homework Problems

In a Young's double slit experiment using yellow light of wavelength 550 nm, the fringe separation (separation between bright spots) is 0.275 mm.

1. **(M)** Find the slit separation if the fringes are 2.0 m from the slit.

   Answer: 0.004 m (= 4 mm)

2. **(M)** The yellow lamp is replaced with a purple one whose light is made of two colors, red light with a wavelength of 700 nm, and violet light with a wavelength of 400 nm.

   a. Find the distance between the violet fringes.

      Answer: 0.000 2 m (= 0.2 mm)

   b. Find the distance between the red fringes.

      Answer: 0.000 35 m (= 0.35 mm)

honors (not AP®)

Scattering

Unit: Light & Optics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain why the sky is blue and the sun looks red at sunset.

Success Criteria:

- Explanations account for observed behavior.

Language Objectives:

- Explain why the beaker in the "sunset in a beaker" demo looks light blue, but the light coming through it looks yellow or red.

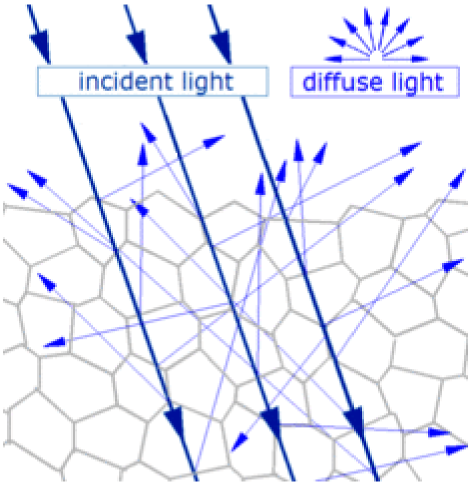
Tier 2 Vocabulary: light, scatter

Labs, Activities & Demonstrations:

- sunset in a beaker

Notes:

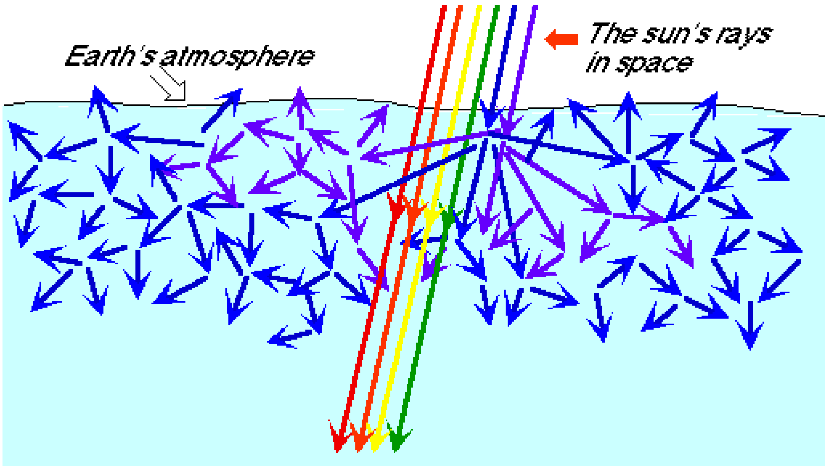
scattering: a change in the direction of rays of light caused by irregularities in the propagation medium, collisions with small particles, or at the interface between two media.



honors (not AP®)

Rayleigh scattering: scattering of light because of collisions with small particles in the medium. Rayleigh scattering is named after the British physicist Lord Rayleigh.

Rayleigh scattering is responsible for the color of the sky. Small particles (0.5–1 micron) scatter visible light as it passes through Earth's atmosphere. Because light at the blue and violet end of the spectrum is about five times as likely to be scattered as light at the red end of the spectrum, the majority of the scattered light in our atmosphere is blue.



This is also why the sun appears yellow during the day—the combination of red, orange, yellow and green light appears yellow to us.

Water vapor molecules are much larger—ranging in size from 2–5 microns. For these larger particles, the probability of scattering is approximately the same for all wavelengths, which is why clouds appear white.

At sunset, because the angle of the sun is much lower, the light must pass through much more of the atmosphere before we see it. By the time the light gets to our eyes, all of the colors are removed by scattering except for the extreme red end of the spectrum, which is why the sun appears red when it sets.

Introduction: Quantum and Particle Physics

Unit: Quantum and Particle Physics

Topics covered in this chapter:

This chapter discusses the particles that atoms and other matter are made of, how those particles interact, and the process by which radioactive decay can change the composition of a substance from one element into another.

- *Photoelectric Effect* describes the observation that light of a sufficiently high frequency can remove electrons from an atom.
- *Bohr Model of the Hydrogen Atom* describes the development of quantum theory to describe the behavior of the electrons in an atom.
- *Wave-Particle Duality* and *Quantum Mechanical Model of the Atom* describe the idea that matter can behave like a wave as well as a particle, and the application of that idea to the modern quantum mechanical model of the atom.
- *Fundamental Forces* describes the four natural forces that affect everything in the universe: the strong and weak nuclear forces, the electromagnetic force, and the gravitational force.
- *The Standard Model* describes and classifies the particles that make up atoms.
- *Particle Interactions* describes interactions between subatomic particles.

One of the challenging aspects of this chapter is that it describes process that happen on a scale that is much too small to observe directly. Another challenge is the fact that the Standard Model continues to evolve. Many of the connections between concepts that make other topics easier to understand have yet to be made in the realm of quantum & particle physics.

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS4-3. Evaluate the claims, evidence, and reasoning behind the idea that electromagnetic radiation can be described by either a wave model or a particle model, and that for some situations involving resonance, interference, diffraction, refraction, or the photoelectric effect, one model is more useful than the other.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

1.A.2.1: The student is able to construct representations of the differences between a fundamental particle and a system composed of fundamental particles and to relate this to the properties and scales of the systems being investigated. [**SP 1.1**, **7.1**]

1.A.4.1: The student is able to construct representations of the energy-level structure of an electron in an atom and to relate this to the properties and scales of the systems being investigated. [**SP 1.1**, **7.1**]

1.C.4.1: The student is able to articulate the reasons that the theory of conservation of mass was replaced by the theory of conservation of mass-energy. [**SP 6.3**]

1.D.1.1: The student is able to explain why classical mechanics cannot describe all properties of objects by articulating the reasons that classical mechanics must be refined and an alternative explanation developed when classical particles display wave properties. [**SP 6.3**]

1.D.3.1: The student is able to articulate the reasons that classical mechanics must be replaced by special relativity to describe the experimental results and theoretical predictions that show that the properties of space and time are not absolute. [Students will be expected to recognize situations in which nonrelativistic classical physics breaks down and to explain how relativity addresses that breakdown, but students will not be expected to know in which of two reference frames a given series of events corresponds to a greater or lesser time interval, or a greater or lesser spatial distance; they will just need to know that observers in the two reference frames can "disagree" about some time and distance intervals.] [**SP 6.3**, **7.1**]

3.G.3.1: The student is able to identify the strong force as the force that is responsible for holding the nucleus together. [**SP 7.2**]

4.C.4.1: The student is able to apply mathematical routines to describe the relationship between mass and energy and apply this concept across domains of scale. [**SP 2.2**, **2.3**, **7.2**]

AP® only

5.B.8.1: The student is able to describe emission or absorption spectra associated with electronic or nuclear transitions as transitions between allowed energy states of the atom in terms of the principle of energy conservation, including characterization of the frequency of radiation emitted or absorbed. [**SP 1.2**, **7.2**]

5.B.11.1: The student is able to apply conservation of mass and conservation of energy concepts to a natural phenomenon and use the equation $E = mc^2$ to make a related calculation. [**SP 2.2**, **7.2**]

5.D.1.6: The student is able to make predictions of the dynamical properties of a system undergoing a collision by application of the principle of linear momentum conservation and the principle of the conservation of energy in situations in which an elastic collision may also be assumed. [**SP 6.4**]

5.D.1.7: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]

5.D.2.5: The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum as the appropriate solution method for an inelastic collision, recognize that there is a common final velocity for the colliding objects in the totally inelastic case, solve for missing variables, and calculate their values. [**SP 2.1**, **2.2**]

5.D.2.6: The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy. [**SP 6.4**, **7.2**]

5.D.3.2: The student is able to make predictions about the velocity of the center of mass for interactions within a defined one-dimensional system. [**SP 6.4**]

5.D.3.3: The student is able to make predictions about the velocity of the center of mass for interactions within a defined two-dimensional system. [**SP 6.4**]

6.F.3.1: The student is able to support the photon model of radiant energy with evidence provided by the photoelectric effect. [**SP 6.4**]

6.F.4.1: The student is able to select a model of radiant energy that is appropriate to the spatial or temporal scale of an interaction with matter. [**SP 6.4**, **7.1**]

6.G.1.1: The student is able to make predictions about using the scale of the problem to determine at what regimes a particle or wave model is more appropriate. [**SP 6.4**, **7.1**]

AP® only

6.G.2.1: The student is able to articulate the evidence supporting the claim that a wave model of matter is appropriate to explain the diffraction of matter interacting with a crystal, given conditions where a particle of matter has momentum corresponding to a de Broglie wavelength smaller than the separation between adjacent atoms in the crystal. [**SP 6.1**]

6.G.2.2: The student is able to predict the dependence of major features of a diffraction pattern (*e.g.,* spacing between interference maxima), based upon the particle speed and de Broglie wavelength of electrons in an electron beam interacting with a crystal. (de Broglie wavelength need not be given, so students may need to obtain it.) [**SP 6.4**]

7.C.1.1: The student is able to use a graphical wave function representation of a particle to predict qualitatively the probability of finding a particle in a specific spatial region. [**SP 1.4**]

7.C.2.1: The student is able to use a standing wave model in which an electron orbit circumference is an integer multiple of the de Broglie wavelength to give a qualitative explanation that accounts for the existence of specific allowed energy states of an electron in an atom. [**SP 1.4**]

7.C.4.1: The student is able to construct or interpret representations of transitions between atomic energy states involving the emission and absorption of photons. [For questions addressing stimulated emission, students will not be expected to recall the details of the process, such as the fact that the emitted photons have the same frequency and phase as the incident photon; but given a representation of the process, students are expected to make inferences such as figuring out from energy conservation that since the atom loses energy in the process, the emitted photons taken together must carry more energy than the incident photon.] [**SP 1.1**, **1.2**]

Photoelectric Effect

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 1 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Explain the photoelectric effect.
- Calculate the work function of an atom and the kinetic energy of electrons emitted.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct with correct units and reasonable rounding.

Language Objectives:

- Explain why a minimum amount of energy is needed in order to emit an electron.

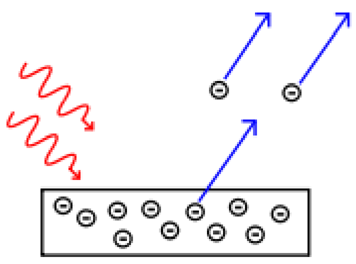
Tier 2 Vocabulary: work function

Labs, Activities & Demonstrations:

- threshold voltage to light an LED
- glow-in-the-dark substance and red *vs.* green *vs.* blue laser pointer

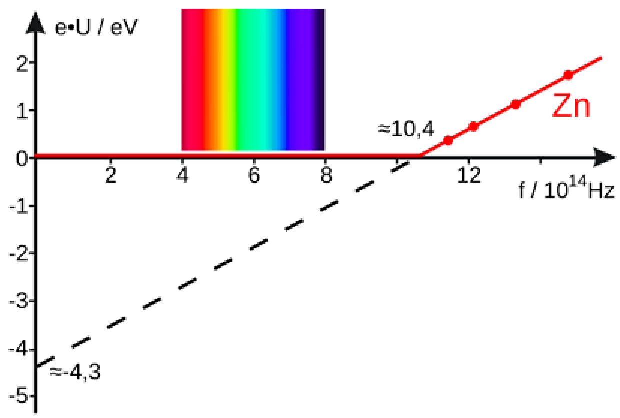
Notes:

The photoelectric effect was discovered in 1887 when Heinrich Hertz discovered that electrodes emitted sparks more effectively when ultraviolet light was shone on them. We now know that the particles are electrons, and that ultraviolet light of sufficiently high frequency (which varies from element to element) causes the electrons to be emitted from the surface of the element:



The photoelectric effect requires light with a sufficiently high frequency, because the frequency of the light is related to the amount of energy it carries. The energy of the photons needs to be above a certain threshold frequency in order to have enough energy to ionize the atom.

For example, a minimum frequency of 10.4×10^{14} Hz is needed to dislodge electrons from a zinc atom:



The maximum kinetic energy of the emitted electron is equal to Planck's constant times the difference between the frequency of incident light (f) and the minimum threshold frequency of the element (f_o):

$$K_{max} = h(f - f_o)$$

The quantity hf_o is called the "work function" of the atom, and is denoted by the variable ϕ. Thus the kinetic energy equation can be rewritten as:

$$K_{max} = hf - \phi$$

Values of the work function for different elements range from about 2.3–6 eV. $(1\text{ eV} = 1.6 \times 10^{-19}\text{ J})$

The importance of this discovery was that it gave rise to the idea that light can behave both as a wave and as a particle.

In 1905, Albert Einstein published a paper explaining that the photoelectric effect was evidence that energy from light was carried in discrete, quantized packets. This discovery, for which Einstein was awarded the Nobel prize in physics in 1921, led to the birth of the field of quantum physics.

Homework Problems

1. **(M)** The work function for gold is 4.8 eV.

   a. What is the minimum frequency of light required to remove electrons from gold?

   Answer: 1.16×10^{15} Hz

   b. What is the wavelength of this frequency of light?

   Answer: 2.58×10^{-7} m or 258 nm

   c. In what part of the spectrum is light of this frequency & wavelength?

   Answer: ultraviolet

2. **(S)** A beam of light from a 445 nm blue laser pointer contains how much energy?

   Answer: 4.45×10^{-19} J or 2.78 eV

3. **(M)** Photons of energy 6 eV cause electrons to be emitted from an unknown metal with a kinetic energy of 2 eV. If photons of twice the wavelength are incident on this metal, what will be the energy of the emitted electrons? (If no electrons are emitted, explain why.)

   Answer: no electrons will be emitted.

honors (not AP®)

Bohr Model of the Hydrogen Atom

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 5.B.8.1

Mastery Objective(s): (Students will be able to…)

- Explain how Bohr's model unified recent developments in the fields of spectroscopy, atomic theory and early quantum theory.
- Calculate the frequency/wavelength of light emitted using the Rydberg equation.
- Calculate the energy associated with a quantum number using Bohr's equation.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct with correct rounding and reasonable units.

Language Objectives:

- Explain why the Bohr Model was such a big deal.

Tier 2 Vocabulary: model, quantum

Notes:

Significant Developments Prior to 1913

Discovery of the Electron (1897): English physicist J.J. Thomson determined that cathode rays were actually particles emitted from atoms that the cathode was made of. These particles had an electrical charge, so they were named "electrons" (though Thomson called them "corpuscles").

Planetary Model of the Atom (1903): Japanese physicist Hantaro Nagaoka first proposed a model of the atom in which a small nucleus was surrounded by a ring of electrons.

Discovery of the Atomic Nucleus (1909): English physicist Ernest Rutherford's famous "gold foil experiment" determined that atoms contained a dense, positively-charged nucleus that comprised most of the atom's mass.

honors (not AP®)

Spectroscopy

Balmer Formula (1885): Swiss mathematician Johann Balmer devised an empirical equation to relate the emission lines in the visible spectrum for the hydrogen atom.

Rydberg Formula (1888): Swedish physicist Johannes Rydberg developed a generalized formula that could describe the wave numbers of all of the spectral lines in hydrogen (and similar elements).

There are several series of spectral lines for hydrogen, each of which converge at different wavelengths. Rydberg described the Balmer series in terms of a pair of integers (n_1 and n_2, where $n_1 < n_2$), and devised a single formula with a single constant (now called the Rydberg constant) that relates them.

$$\frac{1}{\lambda_{vac}} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The value of Rydberg's constant is $\dfrac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10\,973\,731.6\text{ m}^{-1} \approx 1.1\times10^7\text{ m}^{-1}$

where m_e is the rest mass of the electron, e is the elementary charge, ε_o is the permittivity of free space, h is Planck's constant, and c is the speed of light in a vacuum.

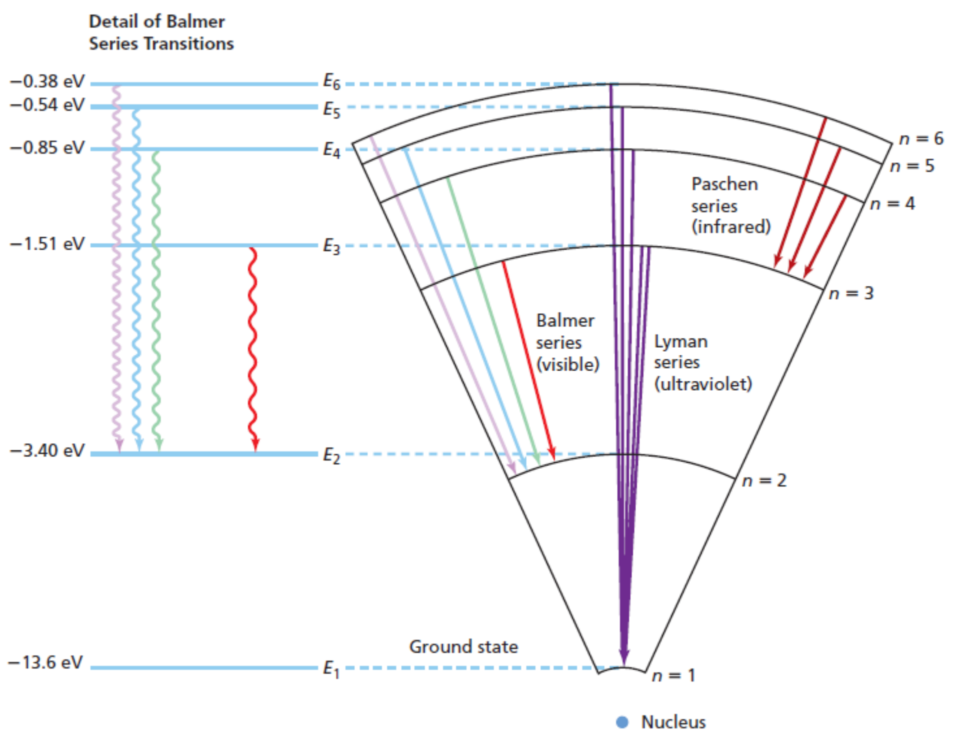
Rydberg's equation was later found to be consistent with other series discovered later, including the Lyman series (in the ultraviolet region; first discovered in 1906) and the Paschen series (in the infrared region; first discovered in 1908).

Those series and their converging wavelengths are:

Series	Wavelength	n_1	n_2
Lyman	91 nm	1	$2 \rightarrow \infty$
Balmer	365 nm	2	$3 \rightarrow \infty$
Paschen	820 nm	3	$4 \rightarrow \infty$

honors (not AP®)

The following diagram shows Lyman, Balmer and Paschen series transitions for the hydrogen atom, from higher energy levels (n = 2 through n = 6) back to lower ones (n = 1 through n = 3):



Early Quantum Theory

quantum: an elementary unit of energy.

In 1900, German physicist Max Planck published the Planck postulate, stating that electromagnetic energy could be emitted only in quantized form, *i.e.,* only certain "allowed" energy states are possible.

Plack determined the constant that bears his name as the relationship between the frequency of one quantum unit of electromagnetic wave and its energy. This relationship is the equation:

$$E = hf$$

where:

E = energy (J)

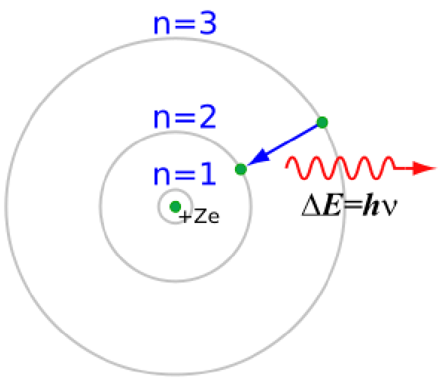
h = Planck's constant = $6.626\times10^{-34}\,\text{J}\cdot\text{s}$

f = frequency* (Hz ≡ s^{-1})

* Most physics texts use the Greek letter *v* (nu) as the variable for frequency. However, high school texts and the College Board use *f*, presumably to avoid confusion with the letter "v".

Bohr's Model of the Atom (1913)

In 1913, Danish physicist Niels Bohr combined atomic, quantum and spectroscopy theories into a single unified theory. Bohr hypothesized that electrons moved around the nucleus as in Rutherford's model, but that these electrons had only certain allowed quantum values of energy, which could be described by a quantum number (n). The value of that quantum number was the same n as in Rydberg's equation, and that using quantum numbers in Rydberg's equation could predict the wavelengths of light emitted when the electrons gained or lost energy by moved from one quantum level to another.



Bohr's model gained wide acceptance, because it related several prominent theories of the time. He received a Nobel Prize in physics in 1922 for his work.

honors (not AP®)

The theory worked well for hydrogen, giving a theoretical basis for Rydberg's equation. Bohr defined the energy associated with a quantum number (n) in terms of Rydberg's constant:

$$E_n = -\frac{R_H}{n^2}$$

Although the Bohr model worked well for hydrogen, the equations could not be solved exactly for atoms with more than one electron, because of the additional effects that electrons exert on each other (*e.g.,* the Coulomb force, $F_e = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$).

Wave-Particle Duality

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.G.1.1, 6.G.2.1, 6.G.2.2, 7.C.1.1, 7.C.2.1

Mastery Objective(s): (Students will be able to…)

- Explain the de Broglie model of the atom.
- Calculate the de Broglie wavelength of a moving particle such as an electron.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

Notes:

In 1924, French physicist Louis de Broglie determined that quanta of light could be considered to particles with very small mass moving at relativistic speeds (*i.e.,* close to the speed of light.)

From this, de Broglie concluded that any moving particle or object must therefore be able to be characterized with some periodic frequency, $f=\frac{E}{h}$, from Planck's equation. This means that the wavelength of any moving object is therefore:

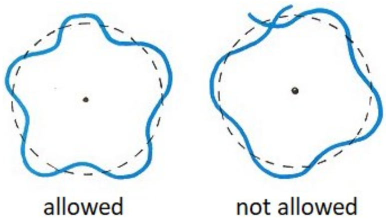
$$\lambda=\frac{h}{p}=\frac{h}{mv}$$

where:

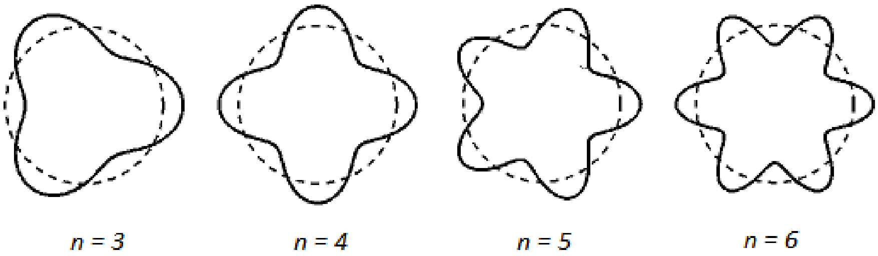
λ = de Broglie wavelength (m)
h = Planck's constant = $6.626\times10^{-34}\ \text{J}\cdot\text{s}$
p = momentum (N·s)
m = mass (kg)
v = velocity $\left(\frac{\text{m}}{\text{s}}\right)$

Every moving object has a de Broglie wavelength, though wavelengths of large objects are too small to be detectable.

de Broglie theorized that if waves were considered to be electrons, then the reason that only certain wavelengths were possible was because the wave produced by an electron would only be stable if the path length as it orbited the nucleus was an integer multiple of the wavelength.



Different quantum amounts of energy were possible with de Broglie's theory, but were restricted to amounts that produced an integer number of wavelengths.



Homework Problems

1. **(M)** What is the de Broglie wavelength associated with an electron moving at 0.5*c*? (You will need to look up the mass of the electron and the speed of light in a vacuum in *Table FF. Constants Used in Nuclear Physics* on page 482 of your Physics Reference Tables.)

   Answer: $4.8\times10^{-12}\,\text{m} = 0.0048\,\text{nm}$

2. **(M)** How fast would that same electron need to be moving in order to produce a wavelength of visible light of 500 nm (which equals 5×10^{-7} m)?

   Answer: $1450\,\frac{\text{m}}{\text{s}}$

Quantum Mechanical Model of the Atom

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS4-3

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 6.G.1.1, 6.G.2.1, 6.G.2.2, 7.C.1.1, 7.C.2.1

Mastery Objective(s): (Students will be able to…)

- Explain the de Broglie model of the atom.
- Explain the Schrödinger model of the atom.
- Explain the wave-particle duality of nature.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

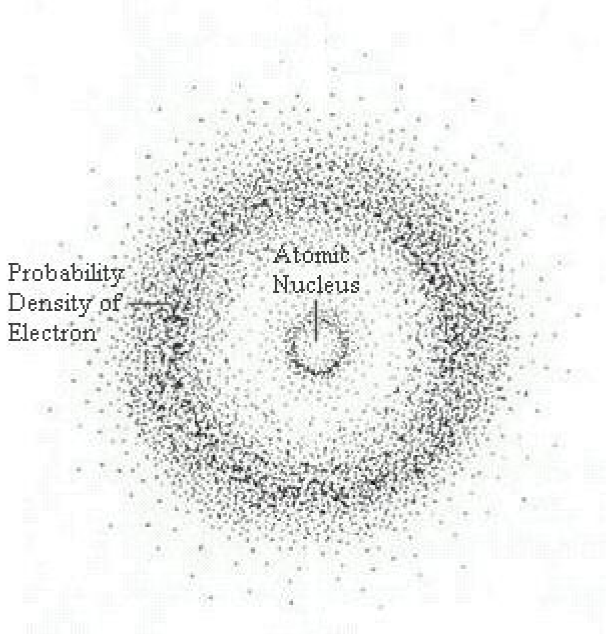
Tier 2 Vocabulary: model, quantum

Notes:

In 1925, following de Broglie's research, Austrian physicist Erwin Schrödinger found that by treating each electron as a unique wave function, the energies of the electrons could be predicted by the mathematical solutions to the wave equation*. Schrödinger used the wave equation to construct a probability map for where the electrons can be found in an atom. Schrödinger's work is the basis for the modern quantum-mechanical model of the atom.

* The wave equation in physics is a second-order partial differential equation that mathematically describes the behavior of waves in space and time. The mathematics required are well beyond the scope of a high school physics class.

To understand the probability map, it is important to realize that because the electron acts as a wave, it is detectable when the amplitude of the wave is nonzero, but not detectable when the amplitude is zero. This makes it appear as if the electron is teleporting from place to place around the atom. If you were somehow able to take a time-lapse picture of an electron as it moves around the nucleus, the picture might look something like the diagram to the right.



Notice that there is a region close to the nucleus where the electron is unlikely to be found, and a ring a little farther out where there is a high probability of finding the electron.

As you get farther and farther from the nucleus, Schrödinger's equation predicts different shapes for these probability distributions. These regions of high probability are called "orbitals," because of their relation to the orbits originally suggested by the planetary model.

Schrödinger was awarded the Nobel prize in physics in 1933 for this discovery.

The implications of quantum theory are vast. Among other things, the energies, shapes and numbers of orbitals in an atom is responsible for each atom's chemical and physical properties and its location on the Periodic Table of the Elements, which means quantum mechanics is responsible for pretty much all of chemistry!

Some principles of quantum theory that are studied explicitly in chemistry include:

- atomic & molecular orbitals
- electron configurations
- the aufbau principle

Fundamental Forces

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 3.G.3.1

Mastery Objective(s): (Students will be able to…)

- Name, describe, and give relative magnitudes of the four fundamental forces of nature.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain why the gravitational force is more relevant than the electromagnetic force in astrophysics.

Tier 2 Vocabulary: model, quantum

Notes:

All forces in nature ultimately come from one of the following four forces:

strong force (or "strong nuclear force" or "strong interaction"): an attractive force between quarks. The strong force holds the nuclei of atoms together. The energy comes from converting mass to energy.
Effective range: about the size of the nucleus of an average-size atom.

weak force (or "weak nuclear force" or "weak interaction"): the force that causes protons and/or neutrons in the nucleus to become unstable and leads to beta nuclear decay. This happens because the weak force causes an up or down quark to change its flavor. (This process is described in more detail in the section on the *Standard Model* of Particle Physics, starting on page 428.)
Relative Strength: 10^{-6} to 10^{-7} times the strength of the strong force.
Effective range: about $^1/_3$ the diameter of an average nucleus.

electromagnetic force: the force between electrical charges. If the charges are the same ("like charges")—both positive or both negative—the particles repel each other. If the charges are different ("opposite charges")—one positive and one negative—the particles attract each other.
Relative Strength: about $^1/_{137}$ as strong as the strong force.
Effective range: ∞, but gets smaller as $(\text{distance})^2$.

gravitational force: the force that causes masses to attract each other. Usually only observable if one of the masses is very large (like a planet).
Relative Strength: only 10^{-39} times as strong as the strong force.
Effective range: ∞, but gets smaller as $(\text{distance})^2$.

Standard Model

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 1.A.2.1

Mastery Objective(s): (Students will be able to…)

- Name and describe the particles of the Standard Model.
- Describe interactions between particles, according to the Standard Model.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain the important features of each model of the atom.

Tier 2 Vocabulary: model, quantum

Notes:

The Standard Model is a theory of particle physics that:

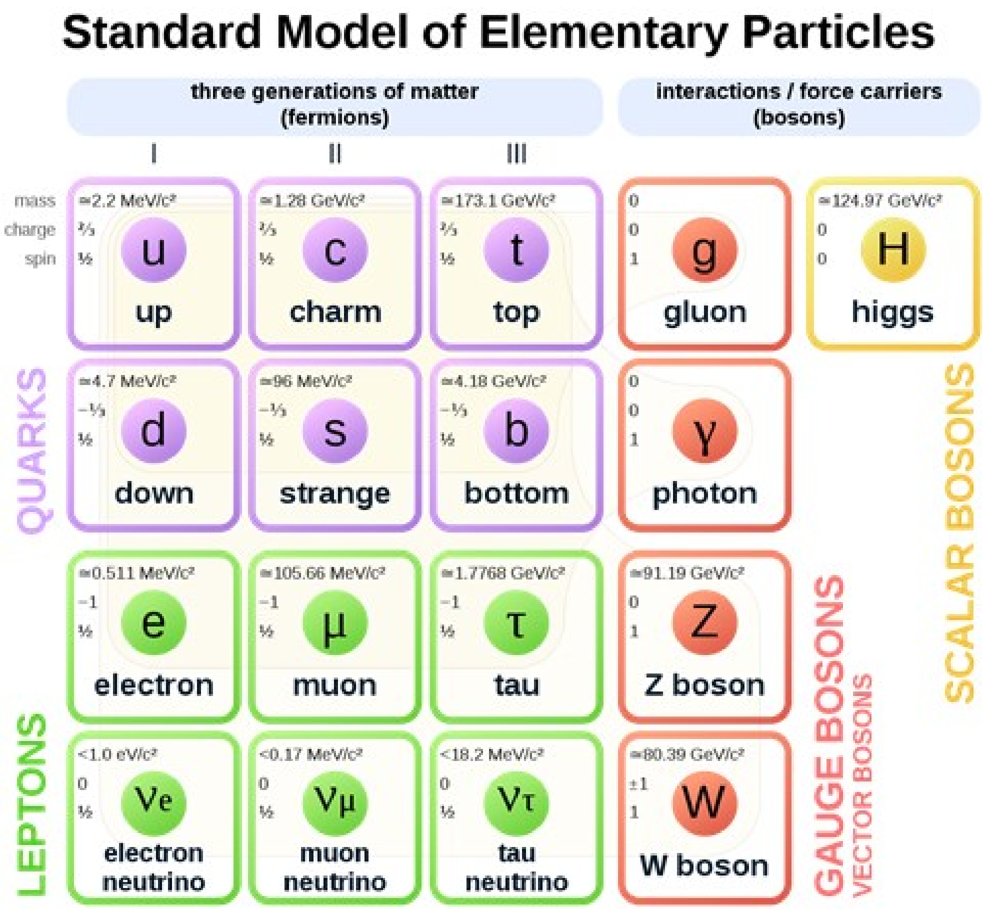
- identifies the particles that matter is ultimately comprised of
- describes properties of these particles, including their mass, charge, and spin
- describes interactions between these particles

The Standard Model dates to the mid-1970s, when the existence of quarks was first experimentally confirmed. Physicists are still discovering new particles and relationships between particles, so the model and the ways it is represented are evolving, much like atomic theory and the Periodic Table of the Elements was evolving at the turn of the twentieth century. The table and the model described in these notes represent our understanding, as of 2024. By the middle of this century, the Standard Model may evolve to a form that is substantially different from the way we represent it today.

The Standard Model in its present form does not incorporate dark matter, dark energy, or gravitational attraction.

The Standard Model is often presented in a table, with rows, columns, and color-coded sections used to group subsets of particles according to their properties.

As of 2021, the Standard Model is represented by a table similar to this one:



Properties shown in the table include mass, charge, and spin.

- **Mass** is shown in units of electron volts divided by the speed of light squared $\left(\frac{\text{eV}}{c^2}\right)$. An electron volt (eV) is the energy acquired by an electric potential difference of one volt applied to one electron. (Recall that the metric prefix "M" stands for mega (10^6) and the metric prefix "G" stands for giga (10^9).) The c^2 in the denominator comes from Einstein's equation, $E = mc^2$, solved for m.
- **Charge** is the same property that we studied in the electricity unit. The magnitude and sign of charge is relative to the charge of an electron, which is defined to be −1.
- **Spin** is the property that is believed to be responsible for magnetism. (The name is because magnetism was previously thought to come from a magnetic field produced by electrons spinning within their orbitals.)

Fundamental Particles

Quarks

Quarks are particles that participate in strong interactions (sometimes called the "strong force") through the action of "color charge" (which will be described later). Because protons and neutrons (which make up most of the mass of an atom) are made of three quarks each, quarks are the subatomic particles that make up most of the ordinary matter[*] in the universe.

- quarks have color charge (*i.e.,* they interact via the strong force)
- quarks have spin of $\pm\frac{1}{2}$
- "up-type" quarks carry a charge of $+\frac{2}{3}$; "down-type" quarks carry a charge of $-\frac{1}{3}$.

There are six flavors[†] of quarks: up and down, charm and strange, and top and bottom. (Originally, top and bottom quarks were called truth and beauty.)

Leptons

Leptons are the smaller particles that make up most matter. The most familiar lepton is the electron. Leptons participate in "electroweak" interactions, meaning combinations of the electromagnetic and weak forces.

- leptons do not have color charge (*i.e.,* they do not interact via the strong force)
- leptons have spins of $+\frac{1}{2}$
- electron-type leptons have a charge of −1; neutrinos do not have a charge.
- neutrinos oscillate, which makes their mass indefinite.

Gauge Bosons

Gauge bosons are the particles that carry force—their interactions are responsible for the fundamental forces of nature: the strong force, the weak force, the electromagnetic force and the gravitational force. The hypothetical particle responsible for the gravitational force is the graviton, which has not yet been detected (as of 2024).

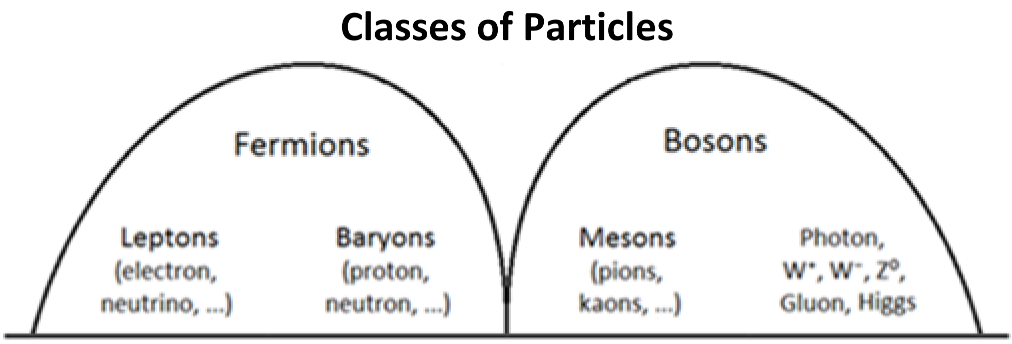
- photons are responsible for the electromagnetic force.
- gluons are responsible for the strong interaction (strong force)
- W and Z bosons are responsible for the weak interaction (weak force)

[*] Matter that is not "ordinary matter" is called "dark matter", whose existence is theorized but not yet proven.

[†] Yes, "flavors" really is the correct term.

Scalar Bosons

At present, the only scalar boson we know of is the Higgs boson, discovered in 2012, which is responsible for mass.



Fermions

Quarks and leptons (the left columns in the table of the Standard Model) are fermions. Fermions are described by Fermi-Dirac statistics and obey the Pauli exclusion principle (which states that no two particles in an atom may have the same exact set of quantum numbers—numbers that describe the energy states of the particle).

Fermions are the building blocks of matter. They have a spin of ½, and each fermion has its own antiparticle (see below).

Bosons

Bosons (the right columns in the table of the Standard Model) are described by Bose-Einstein statistics, have integer spins and do not obey the Pauli Exclusion Principle. Interactions between boson are responsible for forces and mass.

Antiparticles

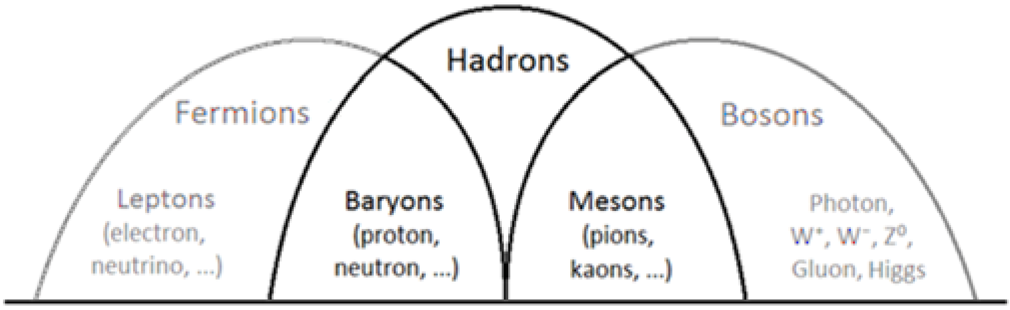
Each particle in the Standard Model has a corresponding antiparticle. Like chemical elements in the Periodic Table of the Elements, fundamental particles are designated by their symbols in the table of the Standard Model. Antiparticles are designated by the same letter, but with a line over it. For example, an up quark would be designated "u", and an antiup quark would be designated " $\overline{\text{u}}$ ".

The antiparticle of a fermion has the same name as the corresponding particle, with the prefix "anti-", and has the opposite charge. *E.g.,* the antiparticle of a tau neutrino is a tau antineutrino. (However, for historical reasons an antielectron is usually called a positron.) *E.g.,* up quark carries a charge of $+\frac{2}{3}$, which means an antiup quark carries a charge of $-\frac{2}{3}$.

Each of the fundamental bosons ***is*** its own antiparticle, except for the W^- boson, whose antiparticle is the W^+ boson.

When a particle collides with its antiparticle, the particles annihilate each other, and their mass is converted to energy ($E = mc^2$) and released.

Composite Particles



Hadrons

Hadrons are a special class of strongly-interacting composite particles (meaning that they are comprised of multiple individual particles). Hadrons can be bosons or fermions. Hadrons composed of strongly-interacting fermions are called baryons; hadrons composed of strongly-interacting bosons are called mesons.

Baryons

The most well-known baryons are protons and neutrons, which each comprised of three quarks. Protons are made of two up quarks and one down quark ("uud"), and carry a charge of +1. Neutrons are made of one up quark and two down quarks ("udd"), and carry a charge of zero.

Some of the better-known baryons include:

- nucleons (protons & neutrons).
- hyperons, *e.g.,* the Λ, Σ, Ξ, and Ω particles. These contain one or more strange quarks, and are much heavier than nucleons.
- various charmed and bottom baryons.
- pentaquarks, which contain four quarks and an antiquark.

Mesons

Ordinary mesons are comprised of a quark plus an antiquark. Examples include the pion, kaon, and the J/Ψ. Mesons mediate the residual strong force between nucleons.

Some of the exotic mesons include:

- tetraquarks, which contain two quarks and two antiquarks.
- glueball, a bound set of gluons with no quarks.
- hybrid mesons, which contain one or more quark/antiquark pairs and one or more gluons.

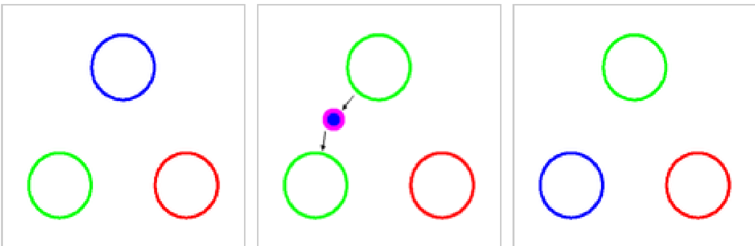
Color Charge

Color charge is the property that is responsible for the strong nuclear interaction. All electrons and fermions (particles that have half-integer spin quantum numbers) must obey the Pauli Exclusion Principle, which states that no two particles within the same larger particle (such as a hadron or atom) can have identical sets of quantum numbers.

For electrons, (as you should have learned in chemistry), if two electrons share the same orbital, they need to have opposite spins. In the case of quarks, all quarks have a spin of $+\frac{1}{2}$, so in order to satisfy the Pauli Exclusion Principle, if a proton or neutron contains three quarks, there has to be some other quantum property that has different values for each of those quarks. This property is called "color charge" (or sometimes just "color*").

The "color" property has three values, which are called "red," "green," and "blue" (named after the primary colors of light). When there are three quarks in a subatomic particle, the colors have to be different, and have to add up to "colorless". (Recall that combining each of the primary colors of light produces white light, which is colorless.)

Quarks can exchange color charge by emitting a gluon that contains one color and one anticolor. Another quark absorbs the gluon, and both quarks undergo color change. For example, suppose a blue quark emits a blue antigreen gluon:



You can imagine that the quark sent away its own blue color (the "blue" in the "blue antigreen" gluon). Because it also sent out antigreen, it was left with green so it became a green quark. Meanwhile, the antigreen part of the gluon finds the green quark and cancels its color. The blue from the blue antigreen gluon causes the receiving quark to become blue. After the interaction, the particle once again has one red, one green, and one blue quark, which means color charge is conserved.

* Just like "spin" is the name of a property of energy that has nothing to do with actual spinning, "color" is a property that has nothing to do with actual color. In fact, quarks couldn't possibly have actual color—the wavelengths of visible light are thousands of times larger than quarks!

honors (not AP®)

Particle Interactions

Unit: Quantum and Particle Physics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Fully describe an interaction between particles based on a Feynman diagram.
- Draw a Feynman diagram representing an interaction between particles.

Success Criteria:

- Descriptions & explanations are accurate.
- Diagrams correctly show all parts of the interaction.

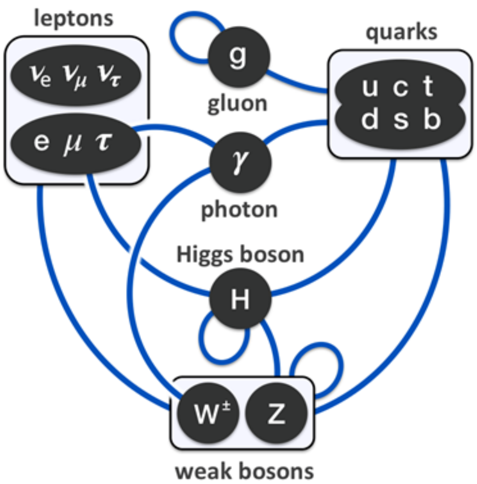
Language Objectives:

- Describe a particle interaction as a narrative.

Tier 2 Vocabulary: interaction, particle

Notes:

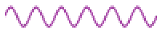
In particle physics, the Standard Model describes the types of particles found in nature, their properties, and how they interact. The following diagram shows which types of particles can interact with which other types.



honors (not AP®)

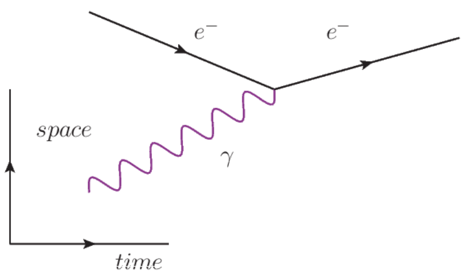
The interactions between particles can be shown pictorially in diagram called a Feynman diagram, named for American physicist Richard Feynman. The Feynman diagram tells the "story" of the interaction.

The characteristics of a Feynman diagram are:

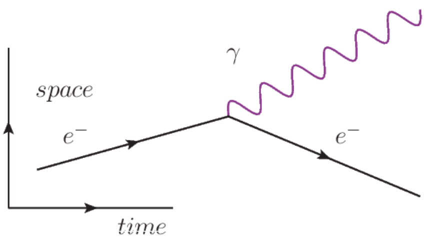
1. Straight lines represent the motion of a particle. The arrow points to the right for a negatively-charged particle, and to the left for a positively-charged particle.
2. A wavy line represents a photon (γ). 
3. A coiled line (like a spring) represents a gluon (g). 
4. The x-axis is time. The interaction starts (in terms of time) on the left and proceeds from left to right.
5. The y-axis represents space. Lines coming together represent particles coming together. Lines moving apart represent particles moving away from each other. (Note that ***the diagram is not a map***; particles can move together or apart in any direction.)
6. Each vertex, where two or more lines come together, represents an interaction.

Probably the best way to explain the diagrams is with examples.

In this diagram, we start (at the left) with an electron (e^{-}) and photon (γ). The two come together, and the electron absorbs the photon.

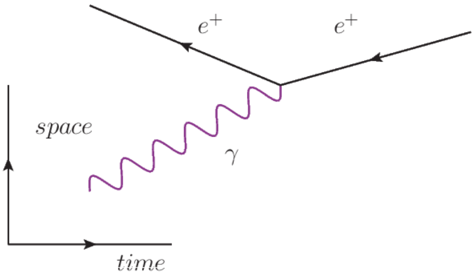


In this diagram, we start (at the left) with an electron (e^{-}) by itself. The electron emits a photon (γ), but is otherwise unchanged.

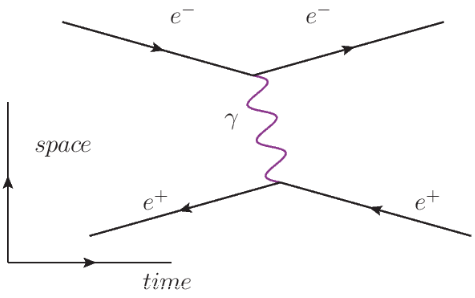


honors (not AP®)

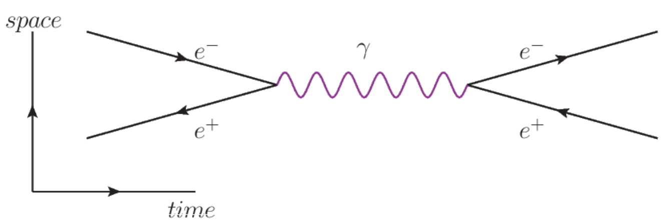
In this diagram, we start (at the left) with a positron (e^+) and photon (γ). (Note that the arrow pointing to the left indicates a positively-charged particle.) The two come together, and the positron absorbs the photon.



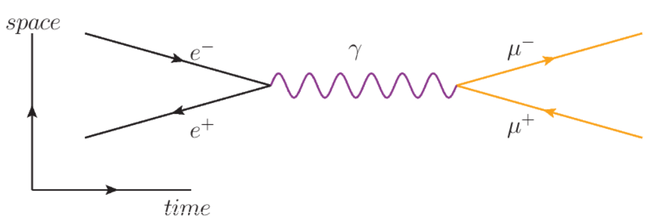
In this diagram, we start with an electron (e^-) and positron (e^+) (coming in from the left). They exchange a photon (γ) between them. (Note that the diagram does not make it clear which particle emits the photon and which one absorbs it.) Then the two particles exit.



In the following diagram, we start with an electron (e^-) and positron (e^+). They come together and annihilate each other, producing a photon (γ). (You can tell this because for a length of time, nothing else exists except for the photon.) Then the photon pair-produces a new electron/positron pair.

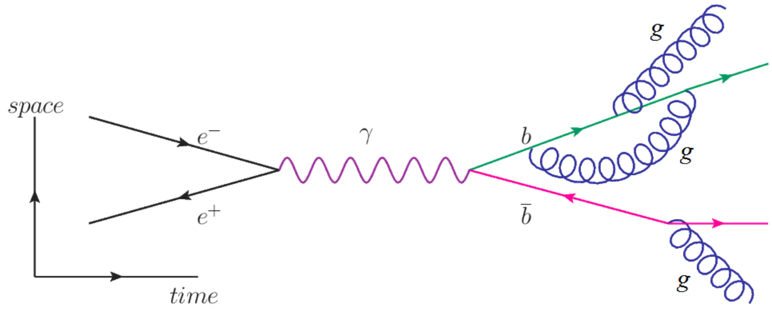


In the following diagram, an electron (e^-) and positron (e^+) annihilate each other as above, but this time the photon produces a muon (μ^-)/antimuon (μ^+) pair. (Again, note that the muon, which has a negative charge, has the arrow pointing to the right. The antimuon, which has a positive charge, has the arrow pointing to the left.)



honors (not AP®)

Finally, in the following diagram, an electron (e^-) and positron (e^+) annihilate each other, producing a photon (γ). The photon pair-produces a bottom quark (*b*) and an antibottom quark ($\bar{b}$), which radiate gluons (*g*).



Introduction: Atomic and Nuclear Physics

Unit: Atomic and Nuclear Physics

Topics covered in this chapter:

This chapter discusses the particles that atoms and other matter are made of, how those particles interact, and the process by which radioactive decay can change the composition of a substance from one element into another.

- *Radioactive Decay* and *Nuclear Equations* describe the process of radioactive decay and how to predict the results.
- *Mass Defect & Binding Energy* uses Einstein's equation $E = mc^2$ to determine the energy that was converted to mass in order to hold the nucleus of an atom together.
- *Half-Life* explains how to calculate the rate at which radioactive decay happens and the amount of material remaining.
- *Nuclear Fission & Fusion* and *Practical Uses for Nuclear Radiation* describe ways that radioactive materials are used to produce energy or otherwise provide benefits to society.

One of the challenges of this chapter is remembering concepts from chemistry, including numbers of protons, neutrons and electrons, and how to use the Periodic Table of the Elements.

Standards addressed in this chapter:

Massachusetts Curriculum Frameworks (2016):

HS-PS1-8: Develop a model to illustrate the energy released or absorbed during the processes of fission, fusion, and radioactive decay.

AP® only

AP® Physics 2 Learning Objectives/Essential Knowledge (2024):

5.C.1.1: The student is able to analyze electric charge conservation for nuclear and elementary particle reactions and make predictions related to such reactions based upon conservation of charge. [**SP 6.4**, **7.2**]

5.G.1.1: The student is able to apply conservation of nucleon number and conservation of electric charge to make predictions about nuclear reactions and decays such as fission, fusion, alpha decay, beta decay, or gamma decay. [**SP 6.4**]

7.C.3.1: The student is able to predict the number of radioactive nuclei remaining in a sample after a certain period of time, and also predict the missing species (alpha, beta, gamma) in a radioactive decay. [**SP 6.4**]

Radioactive Decay

Unit: Atomic, Particle, and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 7.C.3.1

Mastery Objective(s): (Students will be able to…)

- Explain the causes of nuclear instability.
- Explain the processes of α, $\beta-$, and $\beta+$ decay and electron capture.

Success Criteria:

- Descriptions & explanations are accurate and account for observed behavior.

Language Objectives:

- Explain what happens in each of the four types of radioactive decay.

Tier 2 Vocabulary: decay, capture

Labs, Activities & Demonstrations:

- (old) smoke detector & Geiger counter

Notes:

nuclear instability: When something is unstable, it is likely to change. If the nucleus of an atom is unstable, changes can occur that affect the number of protons and neutrons in the atom.

Note that when this happens, the nucleus ends up with a different number of protons. This causes the atom to literally turn into an atom of a different element. When this happens, the physical and chemical properties instantaneously change into the properties of the new element!

radioactive decay: the process by which the nucleus of an atom changes, transforming the element into a different element or isotope.

nuclear equation: an equation describing (through chemical symbols) what happens to an atom as it undergoes radioactive decay.

Causes of Nuclear Instability

Two of the causes of nuclear instability are:

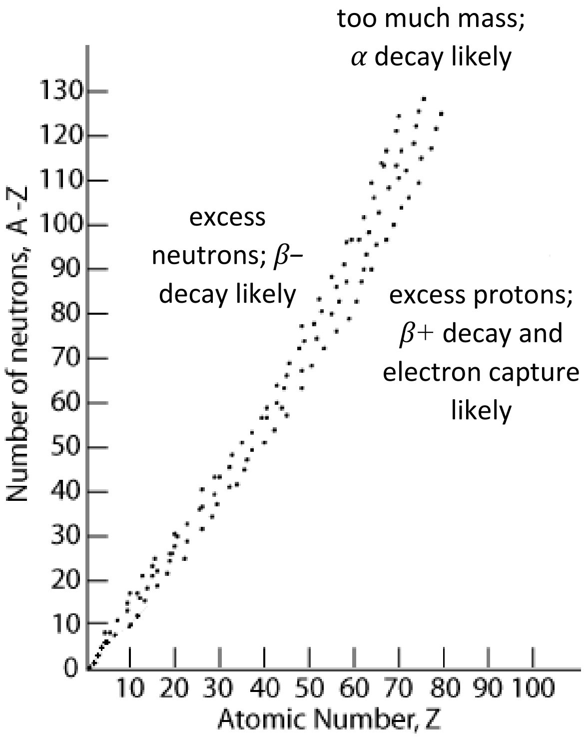
Size

Because the strong force acts over a limited distance, when nuclei get too large (more than 82 protons), it is no longer possible for the strong force to keep the nucleus together indefinitely. The form of decay that results from an atom exceeding its stable size is called alpha (α) decay.

The Weak Nuclear Force

The weak force is caused by the exchange (absorption and/or emission) of W and Z bosons. This causes a down quark to change to an up quark or vice-versa. The change of quark flavor has the effect of changing a proton to a neutron, or a neutron to a proton. (Note that the action of the weak force is the only known way of changing the flavor of a quark.) The form of decay that results from the action of the weak force is called beta (β) decay.

band of stability: isotopes with a ratio of protons to neutrons that results in a stable nucleus (one that does not spontaneously undergo radioactive decay). This observation suggests that the ratio of up to down quarks within the nucleus is somehow involved in preventing the weak force from causing quarks to change flavor.



alpha (α) decay: a type of radioactive decay in which the nucleus loses two protons and two neutrons (an alpha particle). An alpha particle is a ${}^{4}_{2}He^{2+}$ ion (the nucleus of a helium-4 atom), with two protons, a mass of 4 amu, and a charge of +2. For example:

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

Atoms are most likely to undergo alpha decay if they have an otherwise stable proton/neutron ratio but a large atomic number.

Alpha decay has never been observed in atoms with an atomic number less than 52 (tellurium), and is rare in elements with an atomic number less than 73 (tantalum).

Net effects of α decay:

- Atom loses 2 protons and 2 neutrons (atomic number goes down by 2 and mass number goes down by 4)
- An α particle (a ${}^{4}_{2}He^{+2}$ ion) is ejected from the nucleus at high speed.

beta minus ($\beta-$) decay: a type of radioactive decay in which a neutron is converted to a proton and the nucleus ejects a high speed electron (${}^{0}_{-1}e$).

Note that a neutron consists of one up quark and two down quarks (udd), and a proton consists of two up quarks and one down quark (uud). When $\beta-$ decay occurs, the weak force causes one of the quarks changes its flavor from down to up, which causes the neutron (uud) to change into a proton (udd). Because a proton was gained, the atomic number increases by one. However, because the proton used to be a neutron, the mass number does not change. For example:

$${}^{32}_{15}P \rightarrow {}^{32}_{16}S + {}^{0}_{-1}e$$

Atoms are likely to undergo $\beta-$ decay if they have too many neutrons and not enough protons to achieve a stable neutron/proton ratio. Almost all isotopes that are heavier than isotopes of the same element within the band of stability (because of the "extra" neutrons) undergo $\beta-$ decay.

Net effects of $\beta-$ decay:

- Atom loses 1 neutron and gains 1 proton (atomic number goes up by 1; mass number does not change)
- A $\beta-$ particle (an electron) is ejected from the nucleus at high speed.

Note that a $\beta-$ particle is assigned an atomic number of −1. *This does not mean an electron is some sort of "anti-proton".* The −1 is just used to make the equation for the number of protons work out in the nuclear equation.

<u>beta plus (*β+*) decay</u>: a type of radioactive decay in which a proton is converted to a neutron and the nucleus ejects a high speed antielectron (positron, ${}^{0}_{+1}e$).

With respect to the quarks, *β+* decay is the opposite of *β−* decay When *β+* decay occurs, one of the quarks changes its flavor from up to down, which changes the proton (uud) into a neutron (udd). Because a proton was lost, the atomic number decreases by one. However, because the neutron used to be a proton, the mass number does not change. For example:

$${}^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}e$$

Atoms are likely to undergo *β+* decay if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability ("not enough neutrons") undergo *β+* decay.

<u>Net effects of *β+* decay:</u>

- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)
- A *β+* particle (an antielectron or positron) is ejected from the nucleus at high speed.

<u>electron capture</u> (sometimes called "K-capture"): when the nucleus of the atom "captures" an electron from the innermost shell (the K-shell) and incorporates it into the nucleus. This process is exactly the reverse of *β−* decay; during electron capture, a quark changes flavor from up to down, which changes a proton (uud) into a neutron (udd):

$${}^{23}_{12}\text{Mg} + {}^{0}_{-1}e \rightarrow {}^{23}_{11}\text{Na}$$

Note that *β+* decay and electron capture produce the same products. Electron capture can sometimes (but not often) occur without *β+* decay. However, *β+* decay is ***<u>always</u>*** accompanied by electron capture.

Atoms are likely to undergo electron capture (and usually also *β+* decay) if they have too many protons and not enough neutrons to achieve a stable neutron/proton ratio. Almost all isotopes that are lighter than the isotopes of the same element that fall within the band of stability undergo electron capture, and usually also *β+* decay.

<u>Net effects of electron capture:</u>

- An electron is absorbed by the nucleus.
- Atom loses 1 proton and gains 1 neutron (atomic number goes down by 1; mass number does not change)

gamma (γ) rays: most radioactive decay produces energy. Some of that energy is emitted in the form of gamma rays, which are very high energy photons of electromagnetic radiation. (Radio waves, visible light, and X-rays are other types of electromagnetic radiation.) Because gamma rays are waves (which have no mass), they can penetrate far into substances and can do a lot of damage. Because gamma rays are not particles, emission of gamma rays does not change the composition of the nucleus.

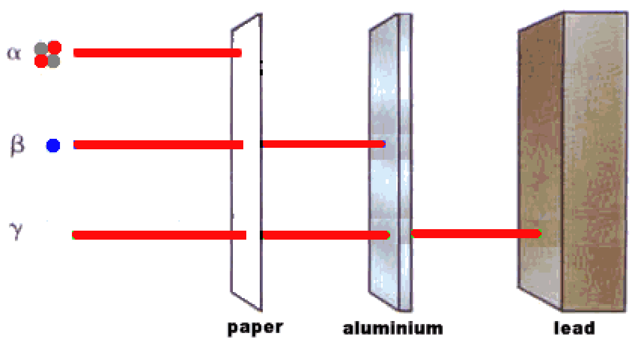
All of the types of radioactive decay mentioned in these notes also produce γ rays. This means to be complete, we would add gamma radiation to each of the radioactive decay equations described above:

$${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} + {}^{0}_{0}\gamma \qquad {}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}e + {}^{0}_{0}\gamma$$

$${}^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}e + {}^{0}_{0}\gamma \qquad {}^{23}_{12}\text{Mg} + {}^{0}_{-1}e \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{0}\gamma$$

penetrating power: the distance that radioactive particles can penetrate into/through another substance is directly related to the velocity of the emission (faster = more penetrating) and inversely related to the mass of the emission (heaver = less penetrating):

gamma rays	**beta particles**	**alpha particles**
lightest	⟷	heaviest
fastest	⟷	slowest
most penetrating power	⟷	least penetrating power



Note also that denser substances (such as lead) do a better job of blocking and absorbing radioactive emissions. This is why lead is commonly used as shielding for experiments involving radioactive substances.

Nuclear Equations

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 5.C.1.1, 7.C.3.1

Mastery Objective(s): (Students will be able to…)

- Determine the products of α, $\beta-$, and $\beta+$ decay and electron capture.

Success Criteria:

- Equations give the correct starting material and products.

Language Objectives:

- Describe the changes to the nucleus during radioactive decay.

Tier 2 Vocabulary: decay, capture

Notes:

nuclear equation: a chemical equation describing the process of an isotope undergoing radioactive decay. For example:

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

In a nuclear equation, the number of protons (atomic number) and the total mass (mass number) are conserved on both sides of the arrow. If you look at the bottom (atomic) numbers, and replace the arrow with an = sign, you would have the following:

$$92 = 90 + 2$$

Similarly, if you look at the top (mass) numbers, and replace the arrow with an = sign, you would have:

$$238 = 234 + 4$$

Sample problems:

Q: What are the products of beta-minus ($\beta-$) decay of ^{131}I?

A: A $\beta-$ particle is an electron, which we write as ${}^{0}_{-1}e$ in a nuclear equation. This means ^{131}I decays into some unknown particle plus ${}^{0}_{-1}e$. The equation is:

$$^{131}_{53}\text{I} \rightarrow {}^{m}_{p}X + {}^{0}_{-1}e$$

We can write the following equations for the atomic and mass numbers:

Atomic #s: $53 = p + -1 \;\rightarrow\; p = 54$; therefore X is Xe

Mass #s: $131 = m + 0 \;\rightarrow\; m = 131$

Therefore, particle X is $^{131}_{54}\text{Xe}$ So our final answer is:

The two products of decay in this reaction are $^{131}_{54}\text{Xe}$ and ${}^{0}_{-1}e$.

Q: Which particle was produced in the following radioactive decay reaction:

$$^{212}_{86}\text{Rn} \rightarrow {}^{208}_{84}\text{Po} + {}^{m}_{p}X$$

A: The two equations are:

Atomic #s: $86 = 84 + p \;\rightarrow\; p = 2$; therefore X is He

Mass #s: $212 = 208 + m \;\rightarrow\; m = 4$

Therefore, particle X is $^{4}_{2}\text{He}$, which means it is an α particle.

Homework Problems

For these problems, you will need to use a Figure CC. Periodic Table of the Elements (on page 481 of your Physics Reference Tables) and radioactive decay information from *Table EE. Selected Radioisotopes* on page 482 of your Physics Reference Tables.

Give the nuclear equation(s) for radioactive decay of the following:

1. **(M)** ^{222}Rn
2. **(M)** ^{85}Kr
3. **(S)** ^{220}Fr
4. **(S)** ^{37}K
5. **(S)** ^{3}H

Give the starting material for the following materials produced by radioactive decay:

6. **(M)** Alpha (α) decay resulting in $^{267}_{108}Hs$
7. **(M)** Beta-minus ($\beta-$) decay resulting in $^{185}_{75}Re$

Mass Defect & Binding Energy

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 1.4.C.1, 5.B.11.1

Mastery Objective(s): (Students will be able to…)

- Calculate the binding energy of an atom.
- Calculate the energy given off by a radioactive decay based on the binding energies before and after.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain where the energy behind the strong force (which holds the nucleus together) comes from.

Tier 2 Vocabulary: defect

Notes:

mass defect: the difference between the actual mass of an atom, and the sum of the masses of the protons, neutrons, and electrons that it contains. The mass defect is the amount of "missing" mass that was turned into binding energy.

- A proton has a mass of 1.6726×10^{-27} kg = 1.0073 amu
- A neutron has a mass of 1.6749×10^{-27} kg = 1.0087 amu
- An electron has a mass of 9.1094×10^{-31} kg = 0.0005486 amu

To calculate the mass defect, total up the masses of each of the protons, neutrons, and electrons in an atom. The actual (observed) atomic mass of the atom is always *less* than this number. The "missing mass" is called the mass defect.

binding energy: the energy that holds the nucleus of an atom together through the strong nuclear force

The binding energy comes from the small amount of mass (the mass defect) that was released as energy when the nucleus was formed, given by the equation:

$$E = mc^2$$

where E is the binding energy, m is the mass defect, and c is the speed of light ($3\times10^8\,\frac{\text{m}}{\text{s}}$), which means c^2 is $9\times10^{16}\,\frac{\text{m}^2}{\text{s}^2}$ (a very large number)!

You can figure out how much energy is produced by spontaneous radioactive decay by calculating the difference in the sum of the binding energies of the atoms before and after the decay.

Sample problem:

Q: Calculate the mass defect of 1 mole of uranium-238.

A: $^{238}_{92}\text{U}$ has 92 protons, 146 neutrons, and 92 electrons. This means the total mass of one atom of $^{238}_{92}\text{U}$ should theoretically be:

92 protons × 1.0073 amu = 92.6704 amu

146 neutrons × 1.0087 amu = 147.2661 amu

92 electrons × 0.000 5486 amu = 0.0505 amu

92.6704 + 147.2661 + 0.0505 = 239.9870 amu

The actual observed mass of one atom of $^{238}_{92}\text{U}$ is 238.0003 amu.

The mass defect of one atom of $^{238}_{92}\text{U}$ is therefore
239.9870 − 238.0003 = 1.9867 amu.

One mole of $^{238}_{92}\text{U}$ would have a mass of 238.0003 g, and therefore a total mass defect of 1.9867 g, or 0.001 9867 kg.

Because $E = mc^2$, that means the binding energy of one mole of $^{238}_{92}\text{U}$ is:

$$0.001\,9867\,\text{kg}\times(3.00\times10^8)^2 = 1.79\times10^{14}\text{ J}$$

In case you don't realize just how large that number is, the binding energy of just 238 g (1 mole) of $^{238}_{92}\text{U}$ would be enough energy to heat every house on Earth for an entire winter!

Half-Life

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP Physics 2 Learning Objectives/Essential Knowledge (2024): 7.C.3.1

Mastery Objective(s): (Students will be able to…)

- Calculate the amount of material remaining after an amount of time.
- Calculate the elapsed time based on the amount of material remaining.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain why the *mass* of material that decays keeps decreasing.

Tier 2 Vocabulary: life, decay

Labs, Activities & Demonstrations:

- half-life of dice or M & M candies

Notes:

The atoms of radioactive elements are unstable, and they spontaneously decay (change) into atoms of other elements.

For any given atom, there is a certain probability, *P*, that it will undergo radioactive decay in a given amount of time. The half-life, τ, is how much time it would take to have a 50% probability of the atom decaying. If you start with *n* atoms, after one half-life, half of them (0.5*n*) will have decayed.

If we start with 32 g of ^{53}Fe, which has a half-life (τ) of 8.5 minutes, we would observe the following:

# minutes	0	8.5	17	25.5	34
# half lives	0	1	2	3	4
amount left	32 g	16 g	8 g	4 g	2 g

Amount of Material Remaining

Most half-life problems in a first-year high school physics course involve a whole number of half-lives and can be solved by making a table like the one above. However, on the AP® exam you can expect problems that do not involve a whole number of half-lives, and you need to use the exponential decay equation.

Because *n* is decreasing, the number of atoms (and consequently also the mass) remaining after any specific period of time follows the exponential decay function:

$$A = A_o(\tfrac{1}{2})^n$$

where *A* is the amount you have now, A_o is the amount you started with, and *n* is the number of half-lives that have elapsed.

Because the number of half-lives equals the total time elapsed (t) divided by the half-life (τ), we can replace $n = {}^{t}\!/_{\tau}$ and rewrite the equation as:

$$A = A_o\left(\frac{1}{2}\right)^{t/\tau} \text{ or } \frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/\tau}$$

If you want to find either *A* or A_o, you can plug the values for *t* and τ into the above equation.

Sample Problem:

Q: If you start with 228 g of ^{90}Sr, how much would remain after 112.4 years?

A: $A_0 = 228$ g

A = A

τ = 28.1 years (from the "Selected Radioisotopes" table in your reference tables)

t = 112.4 years

$$A = A_0\left(\frac{1}{2}\right)^{t/\tau}$$

$$A = (228)\left(\frac{1}{2}\right)^{112.4/28.1} = (228)\left(\frac{1}{2}\right)^4 = (228)\left(\frac{1}{16}\right) = 14.25\text{ g}$$

Or, if the decay happens to occur over an integer number of half-lives (as in this example), you can use a chart:

# years	0	28.1	56.2	84.3	112.4
# half lives	0	1	2	3	4
amount left	228 g	114 g	57 g	28.5 g	14.25 g

Finding the Time that has Passed

Integer Number of Half-Lives

If the amount you started with divided by the amount left is an exact power of two, you have an integer number of half-lives and you can just make a table.

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 4 g remaining? The half-life (τ) of ^{131}I is 8.07 days.

A: $\frac{64}{4}=16$ which is a power of 2, so we can simply make a table:

# half lives	0	1	2	3	4
amount remaining	64 g	32 g	16 g	8 g	4 g

From the table, after 4 half-lives, we have 4 g remaining.

The half-life (τ) of ^{131}I is 8.07 days.

$$8.07 \times 4 = 32.3 \text{ days}$$

Non-Integer Number of Half-Lives

If you need to find the elapsed time and it is not an exact half-life, you need to use logarithms.

In mathematics, *the only reason you ever need to use logarithms is when you need to solve for a variable that's in an exponent*. For example, suppose we have the expression of the form $a^b = c$.

If *b* is a constant, we can solve for either *a* or *c*, as in the expressions:

$$a^3 = 21 \quad (\sqrt[3]{a^3} = \sqrt[3]{21} = 2.76)$$
$$6^2 = c \quad (6^2 = 36)$$

However, we can't do this if *a* and *c* are constants and we need to solve for *b*, as in the expression:

$$3^b = 17$$

To solve for *b*, we need to get *b* out of the exponent. We do this by taking the logarithm of both sides:

$$b\log(3) = \log(17)$$
$$b = \frac{\log(17)}{\log(3)} = \frac{1.23}{0.477} = 2.58$$

It doesn't matter which base you use. For example, using ln instead of log gives the same result:

$$b\ln(3) = \ln(17)$$
$$b = \frac{ln(17)}{ln(3)} = \frac{2.83}{1.10} = 2.58$$

We can apply this same logic to the half-life equation:

$$\frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/\tau}$$
$$\log A - \log A_o = \frac{t}{\tau}\log\left(\frac{1}{2}\right)$$

Sample problem:

Q: If you started with 64 g of ^{131}I, how long would it take until there was only 5.75 g remaining? The half-life (τ) of ^{131}I is 8.07 days.

A: We have 5.75 g remaining. However, $\frac{64}{5.75} = 11.13$, which is not a power of two.

This means we don't have an integer number of half-lives, so we need to use logarithms:

$$\frac{A}{A_o} = \left(\frac{1}{2}\right)^{t/\tau}$$

$$\log A - \log A_o = \frac{t}{\tau} \log\left(\frac{1}{2}\right)$$

$$\log 5.75 - \log 64 = \frac{t}{8.07} \log\left(\frac{1}{2}\right)$$

$$0.7597 - 1.8062 = \frac{t}{8.07}(-0.3010)$$

$$-1.0465 = -0.03730\,t$$

$$28.1\text{ days} = t$$

Homework Problems

For these problems, you will need to use half-life information from *Table EE. Selected Radioisotopes* on page 482 of your physics reference tables.

1. **(M)** If a lab had 128 g of ^{3}H waste 49 years ago, how much of it would be left today? (*Note: you may round off to a whole number of half-lives.*)

   Answer: 8 g

2. **(S)** Suppose you set aside a 20. g sample of ^{42}K at 5:00pm on a Friday for an experiment, but you are not able to perform the experiment until 9:00am on Monday (64 hours later). How much of the ^{42}K will be left?

   Answer: 0.56 g

3. **(M)** If a school wants to dispose of small amounts of radioactive waste, they can store the materials for ten half-lives, and then dispose of the materials as regular trash.

   a. If we had a sample of ^{32}P, how long would we need to store it before disposing of it?

      Answer: 143 days

   b. If we had started with 64 g of ^{32}P, how much ^{32}P would be left after ten half-lives? Approximately what fraction of the original amount would be left?

      Answer: 0.063 g; approximately $\frac{1}{1000}$ of the original amount.

4. **(M)** If the carbon in a sample of human bone contained 30. % of the expected amount of ^{14}C, approximately how old is the sample?

   Answer: 9 950 years

honors (not AP®)

Nuclear Fission & Fusion

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-8

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Identify nuclear processes as "fission" or "fusion".
- Describe the basic construction and operation of fission-based and fusion-based nuclear reactors.

Success Criteria:

- Descriptions account for how the energy is produced and how the radiation is contained.

Language Objectives:

- Explain how fission-based and fusion-based nuclear reactors work.

Tier 2 Vocabulary: fusion, nuclear

Notes:

Fission

fission: splitting of the nucleus of an atom, usually by bombarding it with a high-speed neutron.

When atoms are split by bombardment with neutrons, they can divide in hundreds of ways. For example, when ^{235}U is hit by a neutron, it can split more than 200 ways. Three examples that have been observed are:

$$^{1}_{0}n + ^{235}_{92}\text{U} \rightarrow ^{90}_{37}\text{Rb} + ^{144}_{55}\text{Cs} + 2\,^{1}_{0}n$$
$$^{1}_{0}n + ^{235}_{92}\text{U} \rightarrow ^{87}_{35}\text{Br} + ^{146}_{57}\text{La} + 3\,^{1}_{0}n$$
$$^{1}_{0}n + ^{235}_{92}\text{U} \rightarrow ^{72}_{30}\text{Br} + ^{160}_{62}\text{Sm} + 4\,^{1}_{0}n$$

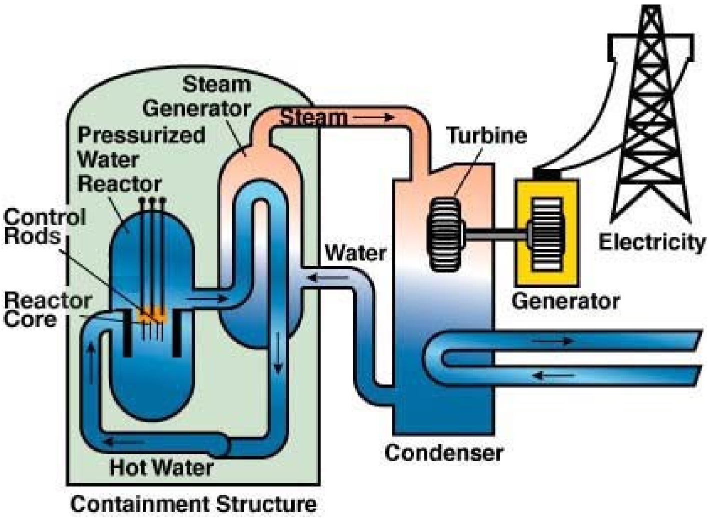
Note that each of these bombardments produces more neutrons. A reaction that produces more fuel (in this case, neutrons) than it consumes will accelerate. This self-propagation is called a chain reaction.

Note also that the neutron/proton ratio of ^{235}U is about 1.5. The stable neutron/proton ratio of each of the products would be approximately 1.2. This means that almost all of the products of fission reactions have too many neutrons to be stable, which means they will themselves undergo $\beta-$ decay.

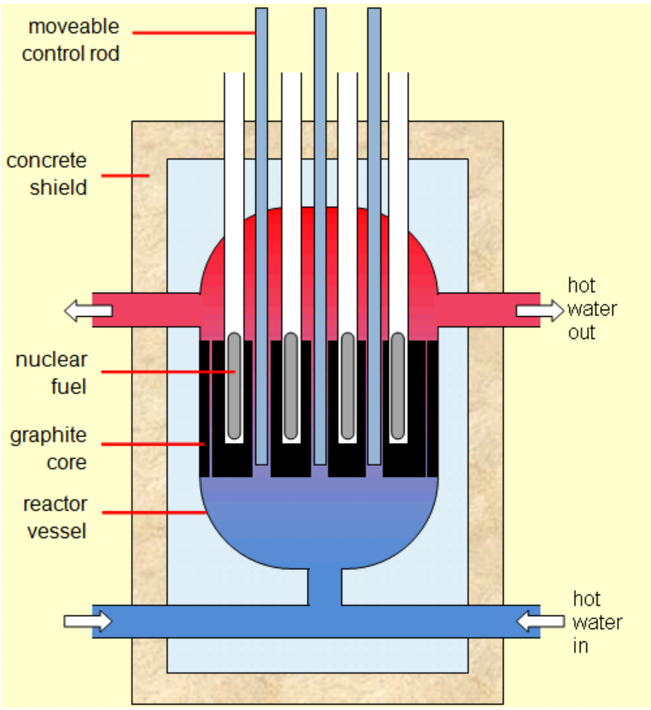
honors (not AP®)

Nuclear Fission Reactors

In a nuclear reactor, the heat from a fission reaction is used to heat water. The radioactive hot water from the reactor (under pressure, so it can be heated well above 100 °C without boiling) is used to boil clean (non-radioactive) water. The clean steam is used to turn a turbine, which generates electricity.



The inside of the reactor looks like this:



The fuel is the radioactive material (such as ^{235}U) that is undergoing fission. The graphite in the core of the reactor is used to absorb some of the neutrons. The moveable control rods are adjusted so they can absorb some or all of the remaining neutrons as desired. If the control rods are all the way down, all of the neutrons are absorbed and no heating occurs. When the reactor is in operation, the control rods are raised just enough to make the reaction proceed at the desired rate.

honors (not AP®)

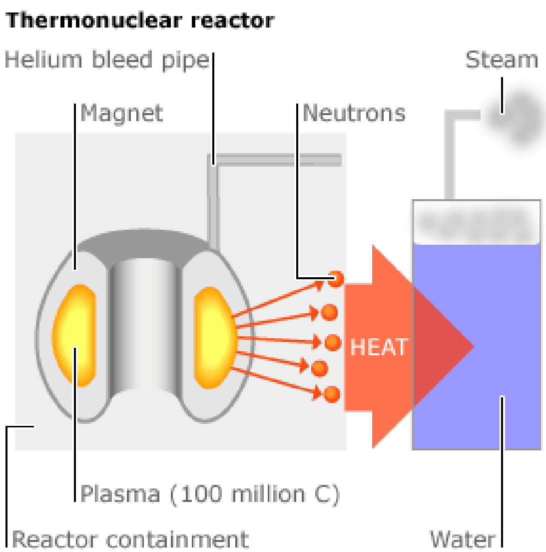
Fusion

<u>fusion</u>: the joining together of the nuclei of two atoms, accomplished by colliding them at high speeds.

Nuclear fusion reactions occur naturally on stars (such as the sun), and are the source of the heat and energy that stars produce.

On the sun, fusion occurs between atoms of deuterium (^{2}H) to produce helium:

$$^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He}$$



The major challenge in building nuclear fusion reactors is the high temperatures produced—on the order of $10^6 – 10^9$ °C. In a tokamak fusion reactor, the starting materials are heated until they become plasma—a sea of highly charged ions and electrons. The highly charged plasma is kept away from the sides by powerful electromagnets.

At the left is a schematic of the ITER tokamak reactor currently under construction in southern France.

MIT has a smaller tokamak reactor at its Plasma Science & Fusion Center. The MIT reactor is able to conduct fusion reactions lasting for only a few seconds; if the reaction continued beyond this point, the current in the electromagnets that is necessary to generate the high magnetic fields required to confine the reaction would become hot enough to melt the copper wire and fuse the coils of the electromagnet together.

After each "burst" (short fusion reaction), the electromagnets in the MIT reactor need to be cooled in a liquid nitrogen bath (−196 °C) for fifteen minutes before the reactor is ready for the next burst.

honors (not AP®)

Practical Uses for Nuclear Radiation

Unit: Atomic and Nuclear Physics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS1-8

NGSS Standards/MA Curriculum Frameworks (2006): N/A

Mastery Objective(s): (Students will be able to…)

- Identify & describe practical (peaceful) uses for nuclear radiation.

Success Criteria:

- Descriptions give examples and explain how radiation is essential to the particular use.

Language Objectives:

- Explain how radiation makes certain scientific procedures possible.

Tier 2 Vocabulary: radiation

Notes:

While most people think of the dangers and destructive power of nuclear radiation, there are a lot of other uses of radioactive materials:

Power Plants: nuclear reactors can generate electricity in a manner that does not produce CO_2 and other greenhouse gases.

Cancer Therapy: nuclear radiation can be focused in order to kill cancer cells in patients with certain forms of cancer. Radioprotective drugs are now available that can help shield non-cancerous cells from the high-energy gamma rays.

Radioactive Tracers: chemicals made with radioactive isotopes can be easily detected in complex mixtures or even in humans. This enables doctors to give a patient a chemical with a small amount of radioactive material and track the progress of the material through the body and determine where it ends up. It also enables biologists to grow bacteria with radioactive isotopes and follow where those isotopes end up in subsequent experiments.

Irradiation of Food: food can be exposed to high-energy gamma rays in order to kill germs. These gamma rays kill all of the bacteria in the food, but do not make the food itself radioactive. (Gamma rays cannot build up inside a substance.) This provides a way to create food that will not spoil for months on a shelf in a store. There is a lot of irrational fear of irradiated food in the United States, but irradiation is commonly used in Europe. For example, irradiated milk will keep for months on a shelf at room temperature without spoiling.

honors (not AP®)

Carbon Dating: Because ^{14}C is a long-lived isotope (with a half-life of 5 700 years), the amount of ^{14}C in archeological samples can give an accurate estimate of their age. One famous use of carbon dating was its use to prove that the Shroud of Turin (the supposed burial shroud of Jesus Christ) was fake, because it was actually made between 1260 C.E. and 1390 C.E.

Smoke Detectors: In a smoke detector, ^{241}Am emits positively-charged alpha particles, which are directed towards a metal plate. This steady flow of positive charges completes an electrical circuit. If there is a fire, smoke particles neutralize positive charges. This makes the flow of charges through the electrical circuit stop, which is used to trigger the alarm.

Appendix: AP® Physics 2 Equation Tables

ADVANCED PLACEMENT PHYSICS PHYSICS 2 IN PLAIN ENGLISH, EFFECTIVE 2017

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67\times10^{-27}$ kg	Electron charge magnitude,	$e = 1.60\times10^{-19}$ C
Neutron mass,	$m_n = 1.67\times10^{-27}$ kg	1 electron volt,	$1\text{ eV} = 1.60\times10^{-19}$ J
Electron mass,	$m_e = 9.11\times10^{-31}$ kg	Speed of light,	$c = 3.00\times10^{8}$ m/s
Avogadro's number,	$N_o = 6.02\times10^{23}\ \text{mol}^{-1}$	Universal gravitational constant,	$G = 6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$
Universal gas constant,	$R = 8.31\ \frac{\text{J}}{\text{mol}\cdot\text{K}}$	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$
Boltzmann's constant,	$k_B = 1.38\times10^{-23}\ \frac{\text{J}}{\text{K}}$		

1 unified atomic mass unit,	$1\text{ u} = 1.66\times10^{-27}\text{ kg} = 931\ \frac{\text{MeV}}{c^2}$
Planck's constant,	$h = 6.63\times10^{-34}\text{ J}\cdot\text{s} = 4.14\times10^{-15}\text{eV}\cdot\text{s}$
	$hc = 1.99\times10^{-25}\text{J}\cdot\text{m} = 1.24\times10^{-3}\text{eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_o = 8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$
Coulomb's law constant,	$k = 1/4\pi\varepsilon_o = 9.0\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
Vacuum permeability,	$\mu_o = 4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$
Magnetic constant,	$k' = \mu_o/4\pi = 1\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}$
1 atmosphere pressure,	$1\text{ atm} = 1.0\times10^{5}\ \frac{\text{N}}{\text{m}^2} = 1.0\times10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole	mol	watt,	W	farad,	F
	kilogram,	k	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
sin θ	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
cos θ	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
tan θ	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. In all situations, positive work is defined as work done **on** a system.
III. The direction of current is conventional current: the direction in which positive charge would drift.
IV. Assume all batteries and meters are ideal unless otherwise stated.
V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.
VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

MECHANICS

$v_x = v_{xo} + a_x t$

$x = x_o + v_{xo}t + \frac{1}{2}a_x t^2$

$v_x^2 = v_{xo}^2 + 2a_x(x - x_o)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$|\vec{F}_f| \le \mu |\vec{F}_n|$

$a_c = \frac{v^2}{r}$

$\vec{p} = m\vec{v}$

$\Delta\vec{p} = \vec{F}\Delta t$

$K = \frac{1}{2}mv^2$

$\Delta E = W = F_{\parallel}d = Fd\cos\theta$

$P = \frac{\Delta E}{\Delta t}$

$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$\omega = \omega_o + \alpha t$

$x = A\cos(\omega t) = A\cos(2\pi f t)$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$\tau = r_{\perp}F = rF\sin\theta$

$L = I\omega$

$\Delta L = \tau\Delta t$

$K = \frac{1}{2}I\omega^2$

$|\vec{F}_s| = k|\vec{x}|$

$U_s = \frac{1}{2}kx^2$

$\Delta U_g = mg\Delta y$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$

$\vec{g} = \frac{\vec{F}_g}{m}$

$U_g = G\frac{m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_o}\frac{|q_1 q_2|}{r^2}$

$\vec{E} = \frac{\vec{F}_E}{q}$

$|\vec{E}| = \frac{1}{4\pi\varepsilon_o}\frac{|q|}{r^2}$

$\Delta U_E = q\Delta V$

$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$

$|\vec{E}| = \left|\frac{\Delta V}{\Delta r}\right|$

$\Delta V = \frac{Q}{C}$

$C = \kappa\varepsilon_o\frac{A}{d}$

$E = \frac{Q}{\varepsilon_o A}$

$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$

$I = \frac{\Delta Q}{\Delta t}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$R = \frac{\rho\ell}{A}$

$P = I\Delta V$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$B = \frac{\mu_o}{2\pi}\frac{I}{R}$

$\vec{F}_M = q\vec{v} \times \vec{B}$

$|\vec{F}_M| = |q\vec{v}||\sin\theta||\vec{B}|$

$\vec{F}_M = \vec{I}\ell \times \vec{B}$

$|\vec{F}_M| = |\vec{I}\ell||\sin\theta||\vec{B}|$

$\Phi_B = \vec{B} \bullet \vec{A}$

$\Phi_B = |\vec{B}|\cos\theta|\vec{A}|$

$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$

$\mathcal{E} = B\ell v$

FLUID MECHANICS AND THERMAL PHYSICS

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P = P_o + \rho g h$

$F_b = \rho V g$

$A_1 v_1 = A_2 v_2$

$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$

$PV = nRT = Nk_B T$

$K = \frac{3}{2}k_B T$

$W = -P\Delta V$

$\Delta U = Q + W$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$E = hf$

$K_{max} = hf - \phi$

$\lambda = \frac{h}{p}$

$E = mc^2$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$\lambda = \frac{v}{f}$

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$

$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$

$\Delta L = m\lambda$

$d \sin\theta = m\lambda$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = bh$

Circle
$A = \frac{1}{2}bh$

Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

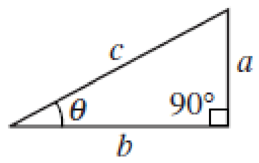
A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



Appendix: Physics Reference Tables

Contents:

Table A. Metric Prefixes

Factor		Prefix	Symbol
1 000 000 000 000 000 000 000 000 000 000	10^{30}	quetta	Q
1 000 000 000 000 000 000 000 000 000	10^{27}	ronna	R
1 000 000 000 000 000 000 000 000	10^{24}	yotta	Y
1 000 000 000 000 000 000 000	10^{21}	zeta	Z
1 000 000 000 000 000 000	10^{18}	exa	E
1 000 000 000 000 000	10^{15}	peta	P
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto	h
10	10^{1}	deca	da
1	10^{0}	—	—
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n
0.000 000 000 001	10^{-12}	pico	p
0.000 000 000 000 001	10^{-15}	femto	f
0.000 000 000 000 000 001	10^{-18}	atto	a
0.000 000 000 000 000 000 001	10^{-21}	zepto	z
0.000 000 000 000 000 000 000 001	10^{-24}	yocto	y
0.000 000 000 000 000 000 000 000 001	10^{-27}	ronto	r
0.000 000 000 000 000 000 000 000 000 001	10^{-30}	quecto	q

↓ Move Decimal Point to the Right ↓

↑ Move Decimal Point to the Left ↑

Table B. Physical Constants

Description	Symbol	Precise Value	Common Approximation
acceleration due to gravity on Earth strength of gravity field on Earth	g	$9.7639\,\frac{m}{s^2}$ to $9.8337\,\frac{m}{s^2}$ average value at sea level is $9.806\,65\,\frac{m}{s^2}$	$9.8\,\frac{m}{s^2} \equiv 9.8\,\frac{N}{kg}$ or $10\,\frac{m}{s^2} \equiv 10\,\frac{N}{kg}$
universal gravitational constant	G	$6.673\,84(80)\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$	$6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2}$
speed of light in a vacuum	c	$299\,792\,458\,\frac{m}{s}$ *	$3.00\times10^{8}\,\frac{m}{s}$
elementary charge (proton or electron)	e	$\pm1.602\,176\,634\times10^{-19}$ C*	$\pm1.60\times10^{-19}$ C
1 coulomb (C)		$6.241\,509\,074\times10^{18}$ elementary charges	6.24×10^{18} elementary charges
(electric) permittivity of a vacuum	ε_o	$8.854\,187\,82\times10^{-12}\,\frac{A^2\cdot s^4}{kg\cdot m^3}$	$8.85\times10^{-12}\,\frac{A^2\cdot s^4}{kg\cdot m^3}$
(magnetic) permeability of a vacuum	μ_o	$4\pi\times10^{-7} = 1.256\,637\,06\times10^{-6}\,\frac{T\cdot m}{A}$	$1.26\times10^{-6}\,\frac{T\cdot m}{A}$
electrostatic constant	k	$\frac{1}{4\pi\varepsilon_o} = 8.987\,551\,787\,368\,176\,4\times10^{9}\,\frac{N\cdot m^2}{C^2}$ *	$8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$
1 electron volt (eV)		$1.602\,176\,565(35)\times10^{-19}$ J	1.60×10^{-19} J
Planck's constant	h	$6.626\,070\,15\times10^{-34}$ J·s*	6.63×10^{-34} J·s
1 universal (atomic) mass unit (u)		$931.494\,061(21)$ MeV/ c^2 $1.660\,538\,921(73)\times10^{-27}$ kg	931 MeV/c^2 1.66×10^{-27} kg
Avogadro's constant	N_A	$6.022\,140\,76\times10^{23}$ mol^{-1} *	6.02×10^{23} mol^{-1}
Boltzmann constant	k_B	$1.380\,649\times10^{-23}\,\frac{J}{K}$ *	$1.38\times10^{-23}\,\frac{J}{K}$
universal gas constant	R	$8.314\,4621(75)\,\frac{J}{mol\cdot K}$	$8.31\,\frac{J}{mol\cdot K}$
Rydberg constant	R_H	$\frac{m_e e^4}{8\varepsilon_o^2 h^3 c} = 10\,973\,731.6\,\frac{1}{m}$	1.10×10^{7} m^{-1}
Stefan-Boltzmann constant	σ	$\frac{2\pi^5 R^4}{15h^3c^2} = 5.670\,374\,419\times10^{-8}\,\frac{J}{m^2\cdot s\cdot K^4}$	$5.67\times10^{-8}\,\frac{J}{m^2\cdot s\cdot K^4}$
standard atmospheric pressure at sea level		101 325 Pa ≡ 1.01325 bar*	100 000 Pa ≡ 1.0 bar
rest mass of an electron	m_e	$9.109\,382\,15(45)\times10^{-31}$ kg	9.11×10^{-31} kg
mass of a proton	m_p	$1.672\,621\,777(74)\times10^{-27}$ kg	1.67×10^{-27} kg
mass of a neutron	m_n	$1.674\,927\,351(74)\times10^{-27}$ kg	1.67×10^{-27} kg

*denotes an exact value (by definition)

Table C. Quantities, Variables and Units

Quantity	Variable	MKS Unit Name	MKS Unit Symbol	S.I. Base Unit
position	$\vec{\boldsymbol{x}}$	meter*	m	m
distance/displacement, (length, height)	$d, \vec{\boldsymbol{d}}, (L, h)$	meter*	m	m
angle	θ	radian, degree	—, °	—
area	A	square meter	m^2	m^2
volume	V	cubic meter, liter	m^3	m^3
time	t	second*	s	s
velocity	$\vec{\boldsymbol{v}}$	meter/second	$\frac{m}{s}$	$\frac{m}{s}$
speed of light	c			
angular velocity	$\vec{\boldsymbol{\omega}}$	radians/second	$\frac{1}{s^2}$, s^{-1}	$\frac{1}{s^2}$, s^{-1}
acceleration	$\vec{\boldsymbol{a}}$	meter/second²	$\frac{m}{s^2}$	$\frac{m}{s^2}$
acceleration due to gravity	$\vec{\boldsymbol{g}}$			
angular acceleration	$\vec{\boldsymbol{\alpha}}$	radians/second²	$\frac{1}{s^2}$, s^{-2}	$\frac{1}{s^2}$, s^{-2}
mass	m	kilogram*	kg	kg
force	$\vec{\boldsymbol{F}}$	newton	N	$\frac{kg \cdot m}{s^2}$
gravitational field	$\vec{\boldsymbol{g}}$	newton/kilogram	$\frac{N}{kg}$	$\frac{m}{s^2}$
pressure	P	pascal	Pa	$\frac{kg}{m \cdot s^2}$
energy (generic)	E	joule	J	$\frac{kg \cdot m^2}{s^2}$
potential energy	U			
kinetic energy	K, E_k			
heat	Q			
work	W	joule , newton-meter	J , N·m	$\frac{kg \cdot m^2}{s^2}$
torque	$\vec{\boldsymbol{\tau}}$	newton-meter	N·m	$\frac{kg \cdot m^2}{s^2}$
power	P	watt	W	$\frac{kg \cdot m^2}{s^3}$
momentum	$\vec{\boldsymbol{p}}$	newton-second	N·s	$\frac{kg \cdot m}{s}$
impulse	$\vec{\boldsymbol{J}}$			
moment of inertia	I	kilogram-meter²	$kg \cdot m^2$	$kg \cdot m^2$
angular momentum	$\vec{\boldsymbol{L}}$	newton-meter-second	N·m·s	$\frac{kg \cdot m^2}{s}$
frequency	f	hertz	Hz	s^{-1}
wavelength	λ	meter	m	m
period	T	second	s	s
index of refraction	n	—	—	—
electric current	$\vec{\boldsymbol{I}}$	ampere*	A	A
electric charge	q	coulomb	C	A·s
electric potential potential difference (voltage) electromotive force (emf)	V ΔV ε	volt	V	$\frac{kg \cdot m^2}{A \cdot s^3}$
electrical resistance	R	ohm	Ω	$\frac{kg \cdot m^2}{A^2 \cdot s^3}$
capacitance	C	farad	F	$\frac{A^2 \cdot s^4}{m^2 \cdot kg}$
electric field	$\vec{\boldsymbol{E}}$	netwon/coulomb volt/meter	$\frac{N}{C}$, $\frac{V}{m}$	$\frac{kg \cdot m}{A \cdot s^3}$
magnetic field	$\vec{\boldsymbol{B}}$	tesla	T	$\frac{kg}{A \cdot s^2}$
temperature	T	kelvin*	K	K
amount of substance	n	mole*	mol	mol
luminous intensity	I_v	candela*	cd	cd

Variables representing vector quantities are typeset in ***bold italics*** with ***arrows***. * = S.I. base unit

Table D. Mechanics Formulas and Equations

Topic	Formulas
Kinematics (Distance, Velocity & Acceleration)	$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$ $\frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2} (= \vec{v}_{ave.})$ $\vec{v} - \vec{v}_o = \vec{a}t$ $\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ $\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}$
Forces & Dynamics	$\sum\vec{F} = \vec{F}_{net} = m\vec{a}$ $F_f \le \mu_s F_N \qquad F_f = \mu_k F_N$ $\vec{F}_g = m\vec{g} = \frac{Gm_1m_2}{r^2}$
Circular/ Centripetal Motion & Force	$a_c = \frac{v^2}{r}$ $F_c = ma_c$
Simple Harmonic Motion	$T = \frac{1}{f}$ $T_s = 2\pi\sqrt{\frac{m}{k}} \qquad T_p = 2\pi\sqrt{\frac{L}{g}}$ $\vec{F}_s = -k\vec{x}$ $U_s = \frac{1}{2}kx^2$
Energy, Work & Power	$U_g = mgh = \frac{Gm_1m_2}{r}$ $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $W = \Delta E = \Delta(U_g + K)$ $W = F_{\parallel}d = \vec{F}_{net} \bullet \vec{d} = Fd\cos\theta$ $TME = U_g + K$ $TME_i + W = TME_f$ $P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv\cos\theta$
Momentum	$\vec{p} = \sum m\vec{v}$ $\Sigma m_i\vec{v}_i + \vec{J} = \Sigma m_f\vec{v}_f$ $\vec{J} = \Delta\vec{p} = \vec{F}_{net}t$

var. = name of quantity (unit)

- Δ = change in something (*E.g.,* Δx means change in x)
- Σ = sum
- d = distance (m)
- $\vec{d}$ = displacement (m)
- $\vec{x}$ = position (m)
- t = time (s)
- $\vec{v}$ = velocity $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{v}_{ave.}$ = average velocity $\left(\frac{\text{m}}{\text{s}}\right)$
- $\vec{a}$ = acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
- f = frequency $\left(\text{Hz} = \frac{1}{\text{s}}\right)$
- $\vec{F}$ = force (N)
- $\vec{F}_{net}$ = net force (N)
- F_f = force due to friction (N)
- $\vec{F}_g$ = force due to gravity (N)
- $\vec{F}_N$ = normal force (N)
- m = mass (kg)
- $\vec{g}$ = strength of gravity field = aceleration due to gravity $= 10\,\frac{\text{N}}{\text{kg}} = 10\,\frac{\text{m}}{\text{s}^2}$ on Earth
- G = gravitational constant $= 6.67\times10^{-11}\,\frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}$
- r = radius (m)
- μ = coëfficient of friction* (*dimensionless*)
- θ = angle (°, radians)
- k = spring constant $\left(\frac{\text{N}}{\text{m}}\right)$
- $\vec{x}$ = displacement of spring (m)
- L = length of pendulum (m)
- E = energy (J)
- $K = E_k$ = kinetic energy (J)
- U = potential energy (J)
- TME = total mechanical energy (J)
- h = height (m)
- Q = heat (J)
- P = power (W)
- W = work (J, N·m)
- T = (time) period (Hz)
- $\vec{p}$ = momentum (N·s)
- $\vec{J}$ = impulse (N·s)
- π = pi (mathematical constant) = 3.14159 26535 89793...

*characteristic property of a substance (to be looked up)

Table E. Approximate Coëfficients of Friction

Substance	Static (μ_s)	Kinetic (μ_k)	Substance	Static (μ_s)	Kinetic (μ_k)
rubber on concrete (dry)	0.90	0.68	wood on wood (dry)	0.42	0.30
rubber on concrete (wet)		0.58	wood on wood (wet)	0.2	
rubber on asphalt (dry)	0.85	0.67	wood on metal	0.3	
rubber on asphalt (wet)		0.53	wood on brick	0.6	
rubber on ice		0.15	wood on concrete	0.62	
steel on ice	0.03	0.01	Teflon on Teflon	0.04	0.04
waxed ski on snow	0.14	0.05	Teflon on steel	0.04	0.04
aluminum on aluminum	1.2	1.4	graphite on steel	0.1	
cast iron on cast iron	1.1	0.15	leather on wood	0.3–0.4	
steel on steel	0.74	0.57	leather on metal (dry)	0.6	
copper on steel	0.53	0.36	leather on metal (wet)	0.4	
diamond on diamond	0.1		glass on glass	0.9–1.0	0.4
diamond on metal	0.1–0.15		metal on glass	0.5–0.7	

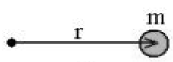
Table F. Angular/Rotational Mechanics Formulas and Equations

Angular Kinematics (Distance, Velocity & Acceleration)	$\Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\theta}} - \vec{\boldsymbol{\theta}}_o$ $\frac{\Delta\vec{\boldsymbol{\theta}}}{t} = \frac{\vec{\boldsymbol{\omega}}_o + \vec{\boldsymbol{\omega}}}{2} (= \vec{\boldsymbol{\omega}}_{ave.})$ $\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\omega}}_o = \vec{\boldsymbol{\alpha}}t$ $\Delta\vec{\boldsymbol{\theta}} = \vec{\boldsymbol{\omega}}_o t + \frac{1}{2}\vec{\boldsymbol{\alpha}}t^2$ $\vec{\boldsymbol{\omega}}^2 - \vec{\boldsymbol{\omega}}_o^2 = 2\vec{\boldsymbol{\alpha}}(\Delta\vec{\boldsymbol{\theta}})$
Circular/ Centripetal Motion	$s = r\Delta\theta \quad v_T = r\omega \quad a_T = r\alpha$ $a_c = \frac{v^2}{r} = \omega^2 r$
Rotational Dynamics	$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$ $I = \_\_mr^2 = \int_0^m r^2 dm$ $F_c = ma_c = mr\omega^2$ $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} \qquad \tau = rF\sin\theta = r_\perp F$ $\sum \vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\tau}}_{net} = I\vec{\boldsymbol{\alpha}}$
Simple Harmonic Motion	$T = \frac{1}{f} = \frac{2\pi}{\omega}$ $x = A\cos(2\pi ft) + \phi$
Angular Momentum	$\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}} = I\vec{\boldsymbol{\omega}} \qquad L = rp\sin\theta = I\omega$ $\Delta\vec{\boldsymbol{L}} = \vec{\boldsymbol{\tau}}\Delta t$
Angular/ Rotational Energy, Work & Power	$K_r = \frac{1}{2}I\omega^2$ $K = K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $W_r = \tau\Delta\theta$ $P = \frac{W}{t} = \tau\omega$

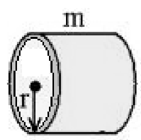
var. = name of quantity (unit)

- Δ = change in something (*E.g.*, Δx = change in x)
- Σ = sum
- s = arc length (m)
- t = time (s)
- a_c = centripetal acceleration $\left(\frac{\text{m}}{\text{s}^2}\right)$
- F_c = centripetal force (N)
- m = mass (kg)
- r = radius (m)
- $\vec{\boldsymbol{r}}$ = radius (vector)
- θ = angle (°, radians)
- $\vec{\boldsymbol{\omega}}$ = angular velocity $\left(\frac{\text{rad}}{\text{s}}\right)$
- $\vec{\boldsymbol{\alpha}}$ = angular velocity $\left(\frac{\text{rad}}{\text{s}^2}\right)$
- $\vec{\boldsymbol{\tau}}$ = torque (N·m)
- x = position (m)
- f = frequency (Hz)
- A = amplitude (m)
- ϕ = phase offset (°, rad)
- E = energy (J)
- $K = E_k$ = kinetic energy (J)
- K_t = translational kinetic energy (J)
- K_r = rotational kinetic energy (J)
- P = power (W)
- W = work (J, N·m)
- $\vec{\boldsymbol{p}}$ = momentum (N·s)
- $\vec{\boldsymbol{L}}$ = angular momentum (N·m·s)

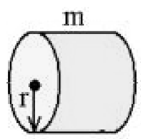
Table G. Moments of Inertia



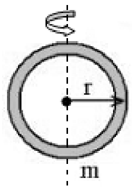
Point Mass: $I = mr^2$



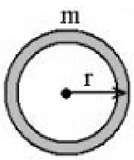
Hollow Cylinder: $I = mr^2$



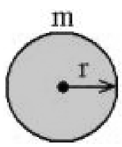
Solid Cylinder: $I = \frac{1}{2}mr^2$



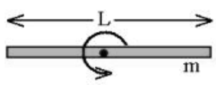
Hoop About Diameter: $I = \frac{1}{2}mr^2$



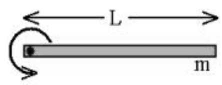
Hollow Sphere: $I = \frac{2}{3}mr^2$



Solid Sphere: $I = \frac{2}{5}mr^2$



Rod About the Middle: $I = \frac{1}{12}mL^2$



Rod About the End: $I = \frac{1}{3}mL^2$

Table H. Heat and Thermal Physics Formulas and Equations

Temperature	$T_{°F} = 1.8(T_{°C}) + 32$ $T_K = T_{°C} + 273.15$
Heat	$Q = mC\Delta T$ $Q_{melt} = m\Delta H_{fus}$ $Q_{boil} = m\Delta H_{vap}$ $C_p - C_v = R$ $\Delta L = \alpha L_i \Delta T$ $\Delta V = \beta V_i \Delta T$ $P = \frac{Q}{t} = (\pm) kA\frac{\Delta T}{L}$ $P = \frac{Q}{t} = \varepsilon\sigma AT^4$ (*in this section, P = power*)
Thermo-dynamics	$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ $PV = nRT = Nk_BT$ $P\Delta V = nR\Delta T = Nk_B\Delta T$ $\Delta U = Q + W$ $U = \frac{3}{2}nRT \quad \Delta U = \frac{3}{2}nR\Delta T$ $W = -P\Delta V = -\int_{V_1}^{V_2} P\,dV$ $K_{(molecular)} = \frac{3}{2}RT$ $U = \frac{3}{2}nRT = \frac{3}{2}Nk_BT$ $\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$ $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{\mu}}$ $\Delta S = \frac{Q_{rev}}{T} \geq \frac{Q}{T}$ $A = U - TS$ $\Delta A = \Delta U - T\Delta S$ (*in this section, P = pressure*)

var. = name of quantity (unit)

- Δ = change in something (*E.g.*, Δx = change in x)
- $T = T_K$ = Kelvin temperature (K)
- $T_{°F}$ = Fahrenheit temperature (°F)
- $T_{°C}$ = Celsius temperature (°C)
- Q = heat (J, kJ)
- m = mass (kg)
- C = specific heat capacity* $\left(\frac{kJ}{kg\cdot°C}, \frac{J}{g\cdot°C}\right)$
- t = time (s)
- L = length (m)
- k = coëfficient of thermal conductivity* $\left(\frac{J}{m\cdot s\cdot°C}, \frac{W}{m\cdot°C}\right)$
- ε = emissivity* (*dimensionless*)
- H_{fus} = latent heat of fusion $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$
- H_{vap} = heat of vaporization $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$
- σ = Stefan-Boltzmann constant $= 5.67\times10^{-8}\ \frac{J}{m^2\cdot s\cdot K^4}$
- V = volume (m^3)
- α = linear coëfficient of thermal expansion* ($°C^{-1}$)
- β = volumetric coëfficient of thermal expansion* ($°C^{-1}$)
- P = power (W)
- P = pressure (Pa)
- n = number of moles (mol)
- N = number of molecules
- R = gas constant $= 8.31\ \frac{J}{mol\cdot K}$
- k_B = Boltzmann constant $= 1.38\times10^{-23}\ \frac{J}{K}$
- U = internal energy (J)
- W = work (J, N·m)
- v_{rms} = root mean square speed $\left(\frac{m}{s}\right)$
- μ = molecular mass* (kg)
- M = molar mass* $\left(\frac{kg}{mol}\right)$
- K = kinetic energy (J)
- Q_{rev} = "reversible" heat (J)
- S = entropy $\left(\frac{J}{K}\right)$
- A = Helmholtz free energy (J)

*characteristic property of a substance (to be looked up)

Table I. Thermodynamics Equation Map

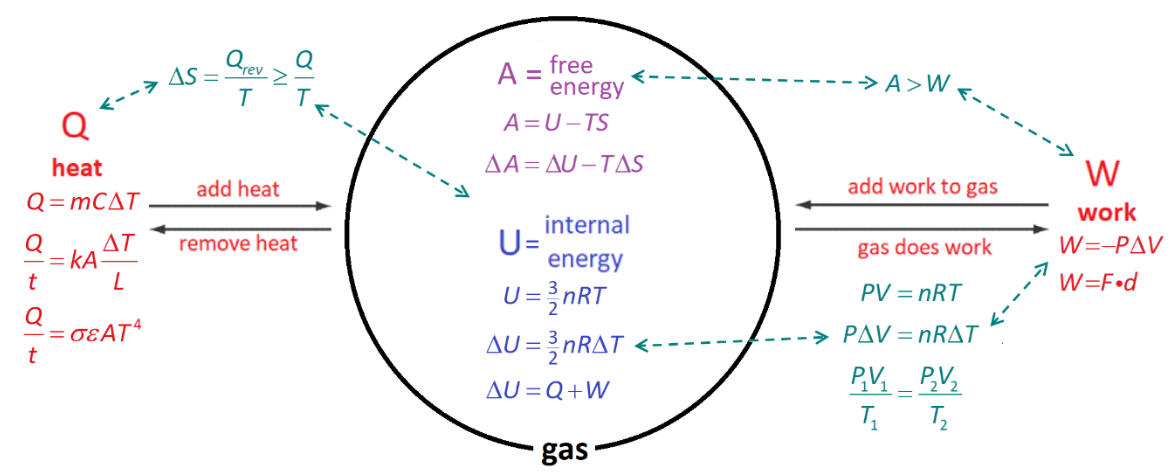


Table J. Pressure-Volume Curves and Equations

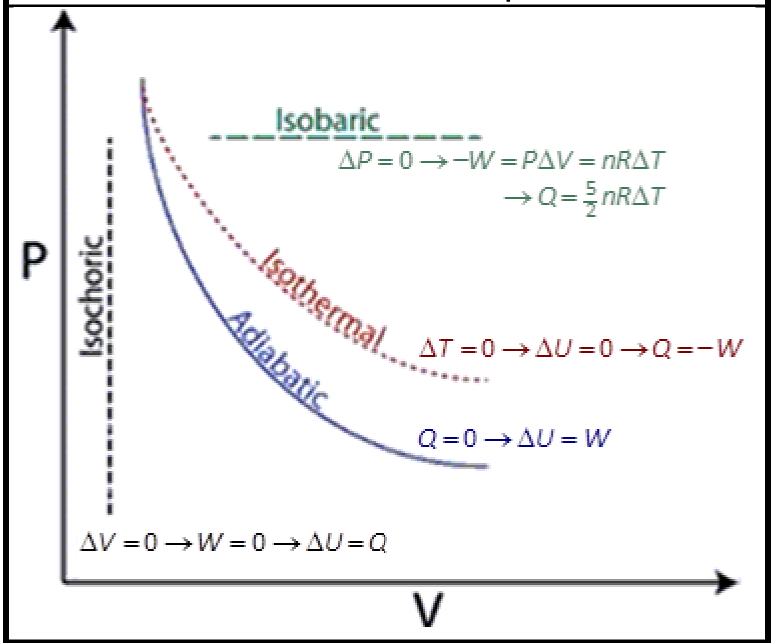


Table K. Thermal Properties of Selected Materials

Substance	Melting Point (°C)	Boiling Point (°C)	Heat of Fusion ΔH_{fus} $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$	Heat of Vaporization ΔH_{vap} $\left(\frac{kJ}{kg}, \frac{J}{g}\right)$	Specific Heat Capacity C $\left(\frac{kJ}{kg\cdot °C}\right)$ at 25°C	Thermal Conductivity k $\left(\frac{J}{m\cdot s\cdot °C}\right)$ at 25°C	Emissivity ε black body = 1	Coefficients of Expansion at 20°C: Linear α (°C^{-1})	Volumetric β (°C^{-1})
air (gas)	—	—	—	—	1.012	0.024	—	—	—
aluminum (solid)	659	2467	395	10460	0.897	250	0.09*	2.3×10^{-5}	6.9×10^{-5}
ammonia (gas)	−75	−33.3	339	1369	4.7	0.024	—	—	—
argon (gas)	−189	−186	29.5	161	0.520	0.016	—	—	—
carbon dioxide (gas)	−78		574		0.839	0.0146	—	—	—
copper (solid)	1086	1187	134	5063	0.385	401	0.03*	1.7×10^{-5}	5.1×10^{-5}
brass (solid)	—	—	—	—	0.380	120	0.03*	1.9×10^{-5}	5.6×10^{-5}
diamond (solid)	3550	4827	10 000	30 000	0.509	2200	—	1×10^{-6}	3×10^{-6}
ethanol (liquid)	−117	78	104	858	2.44	0.171	—	2.5×10^{-4}	7.5×10^{-4}
glass (solid)	—	—	—	—	0.84	0.96–1.05	0.92	8.5×10^{-6}	2.55×10^{-5}
gold (solid)	1063	2660	64.4	1577	0.129	310	0.025*	1.4×10^{-5}	4.2×10^{-5}
granite (solid)	1240	—	—	—	0.790	1.7–4.0	0.96	—	—
helium (gas)	—	−269	—	21	5.193	0.142	—	—	—
hydrogen (gas)	−259	−253	58.6	452	14.30	0.168	—	—	—
iron (solid)	1535	2750	289	6360	0.450	80	0.31	1.18×10^{-5}	3.33×10^{-5}
lead (solid)	327	1750	24.7	870	0.160	35	0.06	2.9×10^{-5}	8.7×10^{-5}
mercury (liquid)	−39	357	11.3	293	0.140	8	—	6.1×10^{-5}	1.82×10^{-4}
paraffin wax (solid)	46–68	~300	~210	—	2.5	0.25	—	—	—
silver (solid)	962	2212	111	2360	0.233	429	0.025*	1.8×10^{-5}	5.4×10^{-5}
zinc (solid)	420	906	112	1760	0.387	120	0.05*	$\sim 3\times10^{-5}$	8.9×10^{-5}
steam (gas) @ 100°C			—	2260	2.080	0.016	—	—	—
water (liq.) @ 25°C	0	100	334		4.181	0.58	0.95	6.9×10^{-5}	2.07×10^{-4}
ice (solid) @ −10°C				—	2.11	2.18	0.97	—	—

*polished surface

Table L. Electricity Formulas & Equations

Electrostatic Charges & Electric Fields	$\boldsymbol{F}_e = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}$ $\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_e}{q} = \frac{Q}{\varepsilon_o A}$ $\quad \vec{\boldsymbol{E}} = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2} = \frac{\Delta V}{\Delta r}$ $W = q\vec{\boldsymbol{E}}\bullet\vec{\boldsymbol{d}} = qEd_{\parallel} = qEd\cos\theta$ $\Delta V = \frac{W}{q} = \vec{\boldsymbol{E}}\bullet\vec{\boldsymbol{d}} = Ed_{\parallel} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ $\Delta U_E = q\Delta V \qquad U_E = \frac{kq_1q_2}{r}$
Circuits and Electrical Components	$\Delta V = IR \qquad I = \frac{\Delta Q}{\Delta t} = \frac{\Delta V}{R}$ $\mathcal{E} = IR$ $P = I\Delta V = I^2R = \frac{(\Delta V)^2}{R}$ $W = Pt = I\Delta Vt$ $R = \frac{\rho L}{A}$ $C = \kappa\varepsilon_o\frac{A}{d}$ $Q = C\Delta V$ $U_{capacitor} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$ $P_{total} = P_1 + P_2 + P_3 + \ldots = \sum P_i$ $U_{total} = U_1 + U_2 + U_3 + \ldots = \sum U_i$
Series Circuits (or Series Sections of Circuits)	$I_{total} = I_1 = I_2 = I_3 = \ldots$ $\Delta V_{total} = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = \sum \Delta V_i$ $R_{equiv.} = R_1 + R_2 + R_3 + \ldots = \sum R_i$ $Q_{total} = Q_1 = Q_2 = Q_3 = \ldots$ $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots = \sum \frac{1}{C_i}$
Parallel Circuits (or Parallel Sections of Circuits)	$I_{total} = I_1 + I_2 + I_3 + \ldots = \sum I_i$ $\Delta V_{total} = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots$ $\frac{1}{R_{equiv.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \sum \frac{1}{R_i}$ $Q_{total} = Q_1 + Q_2 + Q_3 + \ldots = \sum Q_i$ $C_{total} = C_1 + C_2 + C_3 + \ldots = \sum C_i$
Resistor-Capacitor (RC) Circuits	charging: $\frac{I}{I_o} = e^{-t/RC}$ charging: $\frac{Q}{Q_{max}} = 1 - e^{-t/RC}$ discharging: $\frac{I}{I_o} = \frac{V}{V_o} = \frac{Q}{Q_{max}} = e^{-t/RC}$

var. = name of quantity (unit)

Δ = change in something. (*E.g.,* Δx = change in x)
$\vec{\boldsymbol{F}}_e$ = force due to electric field (N)
ε_o = electric permittivity of a vacuum
$= 8.85\times10^{-12}\ \frac{A^2\bullet s^4}{kg\bullet m^3}$
k = electrostatic constant
$= \frac{1}{4\pi\varepsilon_o} = 9.0\times10^{9}\ \frac{N\bullet m^2}{C^2}$
q = point charge (C)
Q = charge (C)
$\vec{\boldsymbol{E}}$ = electric field $\left(\frac{N}{C}, \frac{V}{m}\right)$
V = electric potential (V)
ΔV = voltage = electric potential difference (V)
$\mathcal{E}$ = emf = electromotive force (V)
W = work (J, N·m)
$\kappa = \varepsilon_r$ = relative permittivity* (*dimensionless*)
d = distance (m)
r = radius (m)
$\vec{\boldsymbol{I}}$ = current (A)
t = time (s)
R = resistance (Ω)
P = power (W)
ρ = resistivity (Ω·m)
L = length (m)
A = cross-sectional area (m^2)
C = capacitance (F)
U = potential energy (J)
π = pi (mathematical constant)
= 3.14159 26535 89793...
e = Euler's number (mathematical constant)
= 2.78182 81812 84590...

*characteristic property of a substance (to be looked up)

Table M. Electricity & Magnetism Formulas & Equations

Magnetism and Electro-magnetism	$\vec{F}_M = q(\vec{v} \times \vec{B})$ $F_M = qvB\sin\theta$ $\vec{F}_M = \ell(\vec{I} \times \vec{B})$ $F_M = \ell I B\sin\theta$ $\Delta V = \ell(\vec{v} \times \vec{B})$ $\Delta V = \ell vB\sin\theta$ $B = \frac{\mu_o}{2\pi}\frac{I}{r}$ $\Phi_B = \vec{B} \bullet \vec{A} = BA\cos\theta$ $\mathcal{E} = \frac{\Delta\Phi_B}{\Delta t} = BLv$
Electro-magnetic Induction	$\frac{\#turns_{in}}{\#turns_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$ $P_{in} = P_{out}$

var. = name of quantity (unit)

- Δ = change in something. (*E.g.,* Δx = change in *x*)
- $\vec{F}_e$ = force due to electric field (N)
- $\vec{v}$ = velocity (of moving charge or wire) $\left(\frac{\text{m}}{\text{s}}\right)$
- q = point charge (C)
- ΔV = voltage = electric potential difference (V)
- $\mathcal{E}$ = emf = electromotive force (V)
- r = radius (m) = distance from wire
- $\vec{I}$ = current (A)
- L = length (m)
- t = time (s)
- A = cross-sectional area (m^2)
- $\vec{B}$ = magnetic field (T)
- μ_o = magnetic permeability of a vacuum = $4\pi \times 10^{-7}\ \frac{\text{T}\bullet\text{m}}{\text{A}}$
- Φ_B = magnetic flux ($\text{T}\cdot\text{m}^2$)

Table N. Resistor Color Code

Color	Digit	Multiplier
black	0	$\times 10^0$
brown	1	$\times 10^1$
red	2	$\times 10^2$
orange	3	$\times 10^3$
yellow	4	$\times 10^4$
green	5	$\times 10^5$
blue	6	$\times 10^6$
violet	7	$\times 10^7$
gray	8	$\times 10^8$
white	9	$\times 10^9$
gold		± 5%
silver		± 10%

Table O. Symbols Used in Electrical Circuit Diagrams

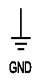
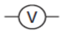
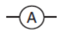
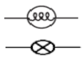
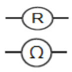
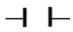
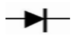
Component	Symbol	Component	Symbol
wire		battery	
switch		ground	
fuse		resistor	
voltmeter		variable resistor (rheostat, potentiometer, dimmer)	
ammeter		lamp (light bulb)	
ohmmeter		capacitor	
		diode	

Table P. Resistivities at 20°C

Conductors		Semiconductors		Insulators	
Substance	**Resistivity** $(\Omega\cdot\text{m})$	**Substance**	**Resistivity** $(\Omega\cdot\text{m})$	**Substance**	**Resistivity** $(\Omega\cdot\text{m})$
silver	1.59×10^{-8}	germanium	0.001 to 0.5	deionized water	1.8×10^{5}
copper	1.72×10^{-8}	silicon	0.1 to 60	glass	1×10^{9} to 1×10^{13}
gold	2.44×10^{-8}	sea water	0.2	rubber, hard	1×10^{13} to 1×10^{13}
aluminum	2.82×10^{-8}	drinking water	20 to 2 000	paraffin (wax)	1×10^{13} to 1×10^{17}
tungsten	5.60×10^{-8}			air	1.3×10^{16} to 3.3×10^{16}
iron	9.71×10^{-8}			quartz, fused	7.5×10^{17}
nichrome	1.50×10^{-6}				
graphite	3×10^{-5} to 6×10^{-4}				

Table Q. Waves & Optics Formulas & Equations

Waves	$v=\lambda f$ $f=\frac{1}{T}$ $v_{\text{wave on a string}}=\sqrt{\frac{F_T}{\mu}}$ $f_{\text{doppler shifted}}=f\left(\frac{\vec{v}_{\text{wave}}+\vec{v}_{\text{detector}}}{\vec{v}_{\text{wave}}+\vec{v}_{\text{source}}}\right)$ $x=A\cos(2\pi ft+\phi)$	*var. = name of quantity (unit)* Δ = change in something (*E.g.,* Δx = change in *x*) v = velocity of wave $\left(\frac{\text{m}}{\text{s}}\right)$ $\vec{v}$ = velocity of source or detector $\left(\frac{\text{m}}{\text{s}}\right)$ f = frequency (Hz) λ = wavelength (m) A = amplitude (m) x = position (m) T = period (of time) (s) F_T = tension (force) on string (N) μ = elastic modulus of string $\left(\frac{\text{kg}}{\text{m}}\right)$ θ = angle (°, rad) ϕ = phase offset (°, rad) θ_i = angle of incidence (°, rad) θ_r = angle of reflection (°, rad) θ_c = critical angle (°, rad) n = index of refraction* (*dimensionless*) c = speed of light in a vacuum $=3.00\times10^8\ \frac{\text{m}}{\text{s}}$ $f=s_f=d_f$ = distance to focus of mirror/lens (m) r_c = radius of curvature of spherical mirror (m) $s_i=d_i$ = distance from mirror/lens to image (m) $s_o=d_o$ = distance from mirror/lens to object (m) h_i = height of image (m) h_o = height of object (m) M = magnification (*dimensionless*) d = separation (m) L = distance from the opening (m) m = an integer ______________ *characteristic property of a substance (to be looked up)
Reflection, Refraction & Diffraction	$\theta_i=\theta_r$ $n=\frac{c}{v}$ $n_1\sin\theta_1=n_2\sin\theta_2$ $\theta_c=\sin^{-1}\left(\frac{n_2}{n_1}\right)$ $\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}$ $\Delta L=m\lambda=d\sin\theta$	
Mirrors & Lenses	$f=\frac{r_c}{2}$ $\frac{1}{s_i}+\frac{1}{s_o}=\frac{1}{f}$ $M=\frac{h_i}{h_o}=-\frac{s_i}{s_o}$	

Table R. Absolute Indices of Refraction

Measured using $f=5.89\times10^{14}$ Hz (yellow light) at 20 °C unless otherwise specified

Substance	Index of Refraction	Substance	Index of Refraction
air (0 °C and 1 atm)	1.000293	silica (quartz), fused	1.459
ice (0 °C)	1.309	Plexiglas	1.488
water	1.3330	Lucite	1.495
ethyl alcohol	1.36	glass, borosilicate (Pyrex)	1.474
human eye, cornea	1.38	glass, crown	1.50–1.54
human eye, lens	1.41	glass, flint	1.569–1.805
safflower oil	1.466	sodium chloride, solid	1.516
corn oil	1.47	PET (#1 plastic)	1.575
glycerol	1.473	zircon	1.777–1.987
honey	1.484–1.504	cubic zirconia	2.173–2.21
silicone oil	1.52	diamond	2.417
carbon disulfide	1.628	silicon	3.96

Figure S. The Electromagnetic Spectrum

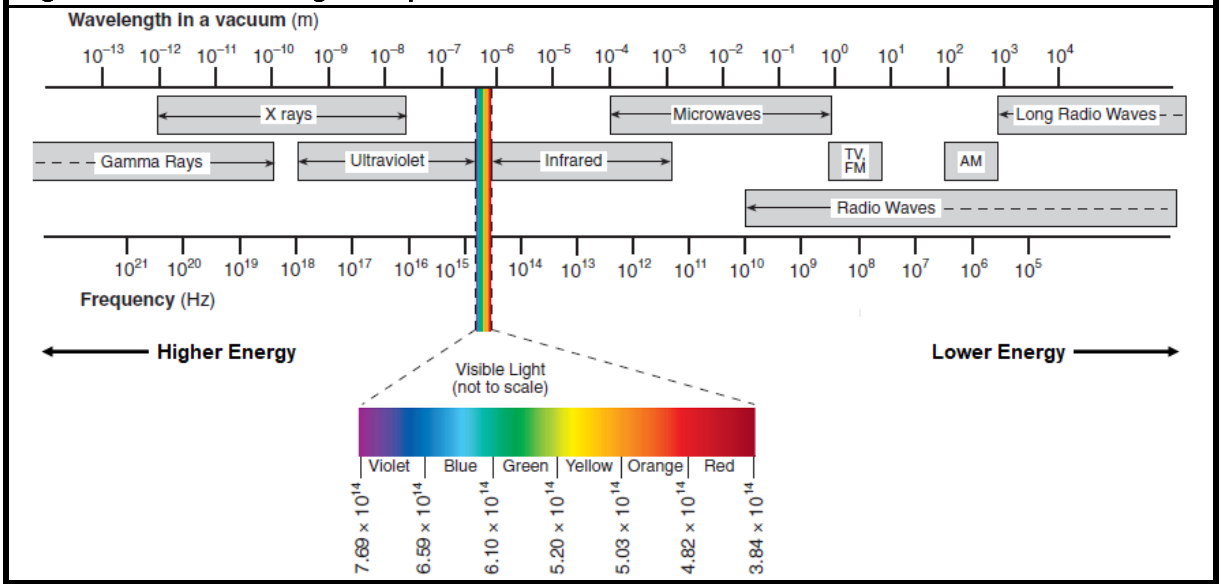


Table T. Planetary Data

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Distance from Sun (m)	5.79×10^{10}	1.08×10^{11}	1.50×10^{11}	2.28×10^{11}	7.79×10^{11}	1.43×10^{12}	2.87×10^{12}	4.52×10^{12}	5.91×10^{12}
Radius (m)	2.44×10^{6}	6.05×10^{6}	6.38×10^{6}	3.40×10^{6}	7.15×10^{7}	6.03×10^{7}	2.56×10^{7}	2.48×10^{7}	1.19×10^{6}
Mass (kg)	3.30×10^{23}	4.87×10^{24}	5.97×10^{24}	6.42×10^{23}	1.90×10^{27}	5.68×10^{26}	8.68×10^{25}	1.02×10^{26}	1.30×10^{22}
Density $\left(\frac{kg}{m^3}\right)$	5429	5243	5514	3934	1326	687	1270	1638	1850
Orbit (years)	0.24	0.61	1.00	1.88	11.8	29	84	164	248
Rotation Period (hours)	1408	−5833	23.9	24.6	9.9	10.7	−17.2	16.1	−153.3
Tilt of axis	0.034°	177.4°	23.4°	25.2°	3.1°	26.7°	97.8°	28.3°	122.5°
# of observed satellites	0	0	1	2	92	83	27	14	5
Mean temp. (°C)	167	464	15	−65	−110	−140	−195	−200	−225
Global magnetic field	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes

Data from NASA Planetary Fact Sheet, https://nssdc.gsfc.nasa.gov/planetary/factsheet/ last updated 11 February 2023.

Table U. Sun & Moon Data

Radius of the sun (m)	6.96×10^{8}
Mass of the sun (kg)	1.99×10^{30}
Radius of the moon (m)	1.74×10^{6}
Mass of the moon (kg)	7.35×10^{22}
Distance of moon from Earth (m)	3.84×10^{8}

Table V. Fluids Formulas and Equations

Fluids	$\rho = \frac{m}{V}$ $P = \frac{F}{A}$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ $P_{\text{hydrostatic}} = P_H = \rho g h$ $F_B = \rho V_d g$ $P_{\text{dynamic}} = P_D = \frac{1}{2}\rho v^2$ $A_1 v_1 = A_2 v_2$ $P_{total} = P_{ext.} + P_H + P_D$ $P_1 + P_{H,1} + P_{D,1} = P_2 + P_{H,2} + P_{D,2}$ $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$	$var.$ = name of quantity (unit) Δ = change in something. (*E.g.*, Δx = change in x) ρ = density $\left(\frac{\text{kg}}{\text{m}^3}\right)$ m = mass (kg) V = volume (m^3) P = presure (Pa) g = gravitational field = $9.8\,\frac{\text{N}}{\text{kg}} \approx 10\,\frac{\text{N}}{\text{kg}}$ h = height or depth (m) A = area (m^2) v = velocity (of fluid) $\left(\frac{\text{m}}{\text{s}}\right)$ F = force (N) ______________ *characteristic property of a substance (to be looked up)

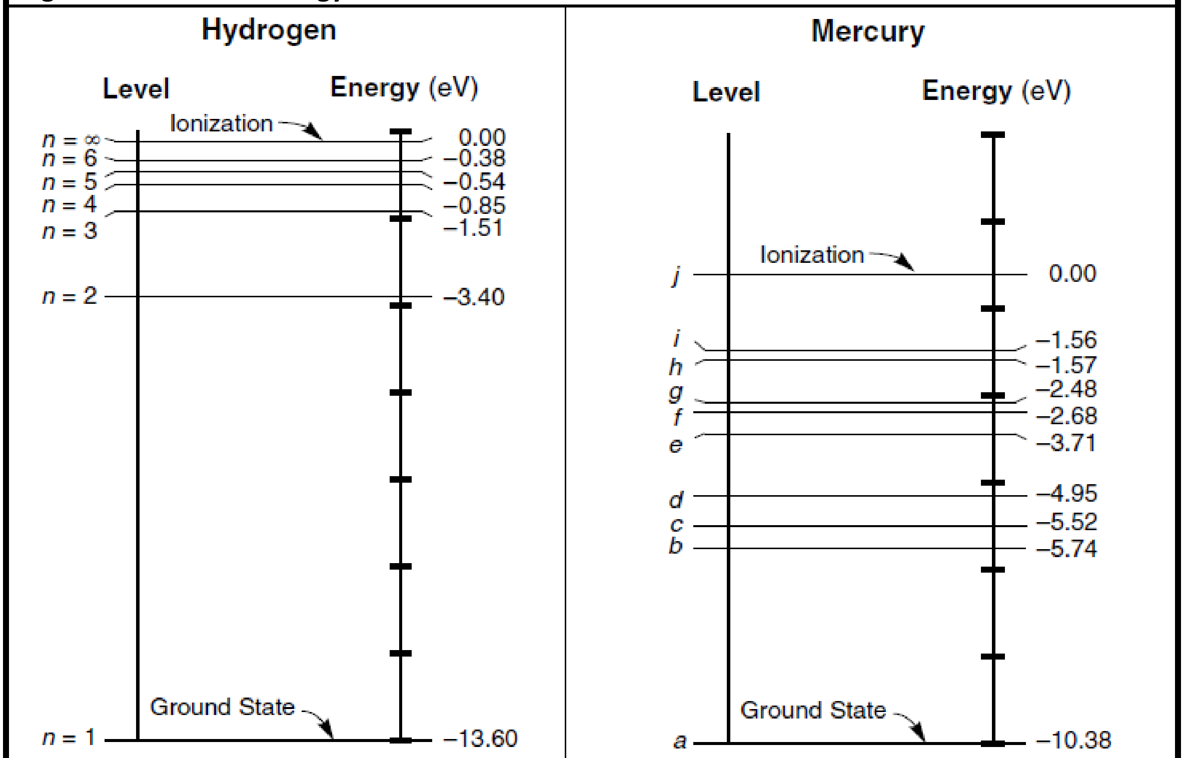
Table W. Properties of Water and Air

Temp. (°C)	Water: Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Water: Speed of Sound $\left(\frac{\text{m}}{\text{s}}\right)$	Water: Vapor Pressure (Pa)	Air: Density $\left(\frac{\text{kg}}{\text{m}^3}\right)$	Air: Speed of Sound $\left(\frac{\text{m}}{\text{s}}\right)$
0	999.78	1 403	611.73	1.288	331.30
5	999.94	1 427	872.60	1.265	334.32
10	999.69	1 447	1 228.1	1.243	337.31
20	998.19	1 481	2 338.8	1.200	343.22
25	997.02	1 496	3 169.1	1.180	346.13
30	995.61	1 507	4 245.5	1.161	349.02
40	992.17	1 526	7 381.4	1.124	354.73
50	990.17	1 541	9 589.8	1.089	360.35
60	983.16	1 552	19 932	1.056	365.88
70	980.53	1 555	25 022	1.025	371.33
80	971.79	1 555	47 373	0.996	376.71
90	965.33	1 550	70 117	0.969	382.00
100	954.75	1 543	101 325	0.943	387.23

Table X. Atomic & Particle Physics (Modern Physics)

Energy	$E_{photon} = hf = \frac{hc}{\lambda} = pc = \hbar\omega$ $E_{k,\max} = hf - \phi$ $\lambda = \frac{h}{p}$ $E_{photon} = E_i - E_f$ $E^2 = (pc)^2 + (mc^2)^2$ $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$	*var.* = name of quantity (unit) Δ = change in something. (*E.g.*, Δx = change in *x*) E = energy (J) h = Planck's constant = 6.63×10^{-34} J·s $\hbar$ = reduced Planck's constant = $\frac{h}{2\pi} = 1.05\times10^{-34}$ J·s f = frequency (Hz) v = velocity $\left(\frac{\text{m}}{\text{s}}\right)$ c = speed of light = $3.00\times10^{8}\ \frac{\text{m}}{\text{s}}$ λ = wavelength (m) p = momentum (N·s) m = mass (kg) K = kinetic energy (J) ϕ = work function* (J) R_H = Rydberg constant = $1.10\times10^{7}\ \text{m}^{-1}$
Special Relativity	$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $\gamma = \frac{L_o}{L} = \frac{\Delta t'}{\Delta t} = \frac{m_{rel}}{m_o}$	γ = Lorentz factor (dimensionless) L = length in moving reference frame (m) L_o = length in stationary reference frame (m) $\Delta t'$ = time in stationary reference frame (s) Δt = time in moving reference frame (s) m_o = mass in stationary reference frame (kg) m_{rel} = apparent mass in moving reference frame (kg) __________ *characteristic property of a substance (to be looked up)

Figure Y. Quantum Energy Levels



Energy Levels for the Hydrogen Atom

A Few Energy Levels for the Mercury Atom

Figure Z. Particle Sizes

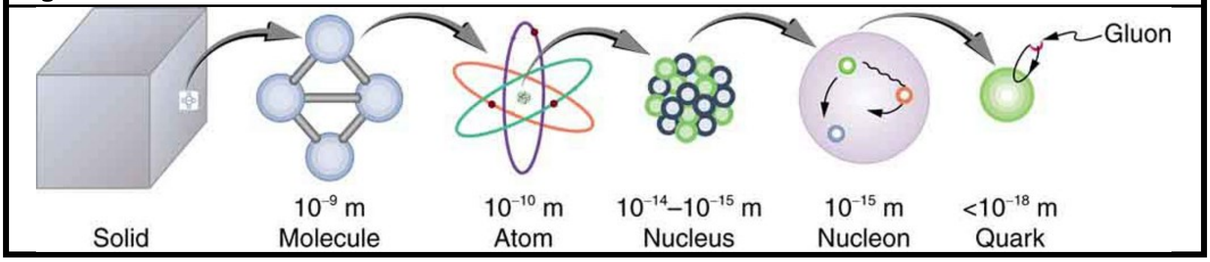


Figure AA. Classification of Matter

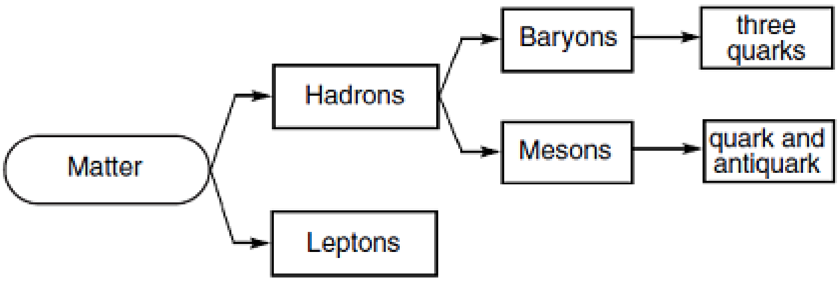


Table BB. The Standard Model of Elementary Particles

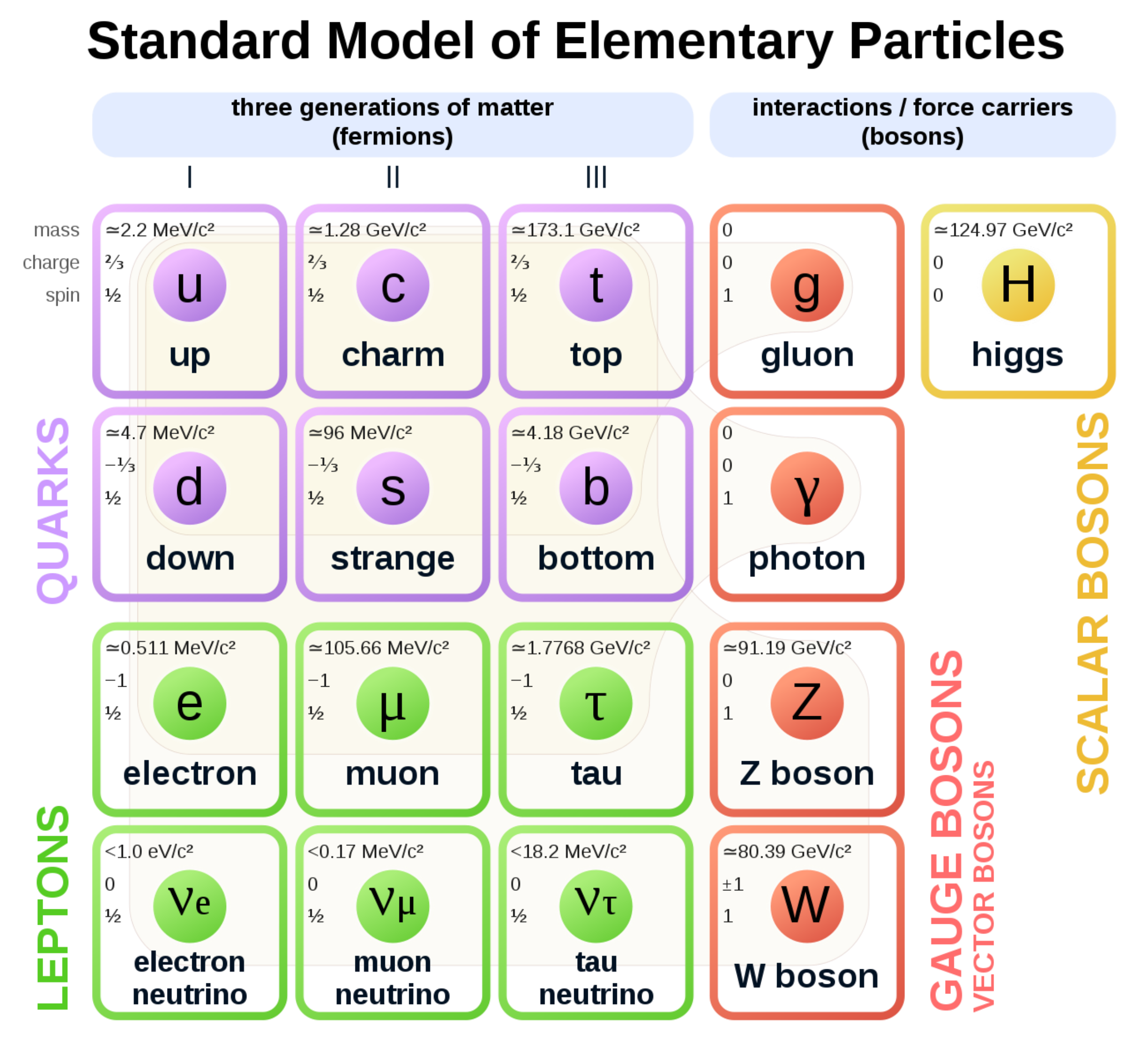


Figure CC. Periodic Table of the Elements

Period	1 I A	2 II A	3 III B	4 IV B	5 V B	6 VI B	7 VII B	8 VIII B	9 VIII B	10 VIII B	11 I B	12 II B	13 III A	14 IV A	15 V A	16 VI A	17 VII A	18 VIII A
1	1 H hydrogen 1.008																	2 He helium 4.003
2	3 Li lithium 6.968	4 Be beryllium 9.012											5 B boron 10.81	6 C carbon 12.01	7 N nitrogen 14.01	8 O oxygen 16.00	9 F fluorine 19.00	10 Ne neon 20.18
3	11 Na sodium 22.99	12 Mg magnesium 24.31											13 Al aluminum 26.98	14 Si silicon 28.09	15 P phosphorus 30.97	16 S sulfur 32.07	17 Cl chlorine 35.45	18 Ar argon 39.95
4	19 K potassium 39.10	20 Ca calcium 40.08	21 Sc scandium 44.96	22 Ti titanium 47.87	23 V vanadium 50.94	24 Cr chromium 52.00	25 Mn manganese 54.94	26 Fe iron 55.85	27 Co cobalt 58.93	28 Ni nickel 58.69	29 Cu copper 63.55	30 Zn zinc 65.38	31 Ga gallium 69.72	32 Ge germanium 72.63	33 As arsenic 74.92	34 Se selenium 78.97	35 Br bromine 79.90	36 Kr krypton 83.80
5	37 Rb rubidium 85.47	38 Sr strontium 87.62	39 Y yttrium 88.91	40 Zr zirconium 91.22	41 Nb niobium 92.91	42 Mo molybdenum 95.95	43 Tc technetium 98	44 Ru ruthenium 101.1	45 Rh rhodium 102.9	46 Pd palladium 106.4	47 Ag silver 107.9	48 Cd cadmium 112.4	49 In indium 114.8	50 Sn tin 118.7	51 Sb antimony 121.8	52 Te tellurium 127.6	53 I iodine 126.9	54 Xe xenon 131.3
6	55 Cs cesium 132.9	56 Ba barium 137.3	71 Lu lutetium 175.0	72 Hf hafnium 178.5	73 Ta tantalum 180.9	74 W tungsten 183.8	75 Re rhenium 186.2	76 Os osmium 190.2	77 Ir iridium 192.2	78 Pt platinum 195.1	79 Au gold 197.0	80 Hg mercury 200.6	81 Tl thallium 204.4	82 Pb lead 207.2	83 Bi bismuth 209.0	84 Po polonium 209	85 At astatine 210	86 Rn radon 222
7	87 Fr francium 223	88 Ra radium 226	103 Lr lawrencium 262	104 Rf rutherfordium 267	105 Db dubnium 268	106 Sg seaborgium 271	107 Bh bohrium 272	108 Hs hassium 270	109 Mt meitnerium 276	110 Ds darmstadtium 281	111 Rg roentgentium 280	112 Cn copernicum 285	113 Nh nihonium 284	114 Fl flerovium 289	115 Mc moscovium 288	116 Lv livermorium 293	117 Ts tennessine 292	118 Og oganesson 294

lanthanides (rare earth metals)	57 La lanthanum 138.9	58 Ce cerium 140.1	59 Pr praseodymium 140.9	60 Nd neodymium 144.2	61 Pm promethium 145	62 Sm samarium 150.4	63 Eu europium 152.0	64 Gd gadolinium 157.3	65 Tb terbium 158.9	66 Dy dysprosium 162.5	67 Ho holmium 164.9	68 Er erbium 167.3	69 Tm thulium 168.9	70 Yb ytterbium 173.1
actinides	89 Ac actinium 227	90 Th thorium 232.0	91 Pa protactinium 231.0	92 U uranium 238.0	93 Np neptunium 237	94 Pu plutonium 244	95 Am americium 243	96 Cm curium 247	97 Bk berkelium 247	98 Cf californium 251	99 Es einsteinium 252	100 Fm fermium 257	101 Md mendelevium 258	102 No nobelium 259

Table DD. Symbols Used in Nuclear Physics

Name	Notation	Symbol
alpha particle	${}^{4}_{2}\text{He}$ or ${}^{4}_{2}\alpha$	□
beta particle (electron)	${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta$	□ −
gamma radiation	${}^{0}_{0}\gamma$	□
neutron	${}^{1}_{0}n$	n
proton	${}^{1}_{1}\text{H}$ or ${}^{1}_{1}p$	p
positron	${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta$	□ +

Table FF. Constants Used in Nuclear Physics

Constant	Value
mass of an electron (m_e)	0.00055 amu
mass of a proton (m_p)	1.00728 amu
mass of a neutron (m_n)	1.00867 amu
Bequerel (Bq)	1 disintegration/second
Curie (Ci)	3.7 x 10^{10} Bq

Figure GG. Neutron/Proton Stability Band

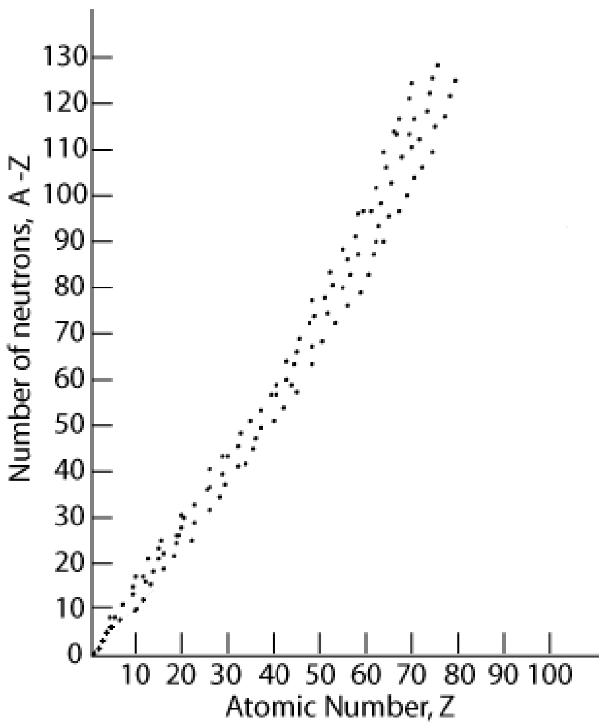


Table EE. Selected Radioisotopes

Nuclide	Half-Life	Decay Mode
^{3}H	12.26 y	□ −
^{14}C	5730 y	□ −
^{16}N	7.2 s	□ −
^{19}Ne	17.2 s	□ +
^{24}Na	15 h	□ −
^{27}Mg	9.5 min	□ −
^{32}P	14.3 d	□ −
^{36}Cl	3.01 × 10^{5} y	□ −
^{37}K	1.23 s	□ +
^{40}K	1.26 × 10^{9} y	□ +
^{42}K	12.4 h	□ −
^{37}Ca	0.175 s	□ −
^{51}Cr	27.7 d	□
^{53}Fe	8.51 min	□ −
^{59}Fe	46.3 d	□ −
^{60}Co	5.26 y	□ −
^{85}Kr	10.76 y	□ −
^{87}Rb	4.8 × 10^{10} y	□ −
^{90}Sr	28.1 y	□ −
^{99}Tc	2.13 x 10^{5} y	□ −
^{131}I	8.07 d	□ −
^{137}Cs	30.23 y	□ −
^{153}Sm	1.93 d	□ −
^{198}Au	2.69 d	□ −
^{222}Rn	3.82 d	□
^{220}Fr	27.5 s	□
^{226}Ra	1600 y	□
^{232}Th	1.4 x 10^{10} y	□
^{233}U	1.62 x 10^{5} y	□
^{235}U	7.1 x 10^{8} y	□
^{238}U	4.51 x 10^{9} y	□
^{239}Pu	2.44 x 10^{4} y	□
^{241}Am	432 y	□

Table HH. Mathematics Formulas

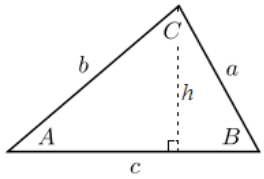
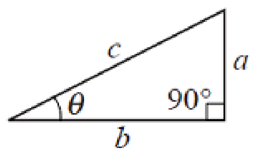
Scientific Notation	$3\times10^4 = 3\times10\,000 = 30\,000 \qquad 2\times10^{-3} = 2\times0.001 = 0.002$ $(3\times10^4)(2\times10^{-3}) = (3\cdot2)(10^4\cdot10^{-3}) = 6\times10^{4+(-3)} = 6\times10^1 = 60$	
Rounding **(to underlined place)**	$15\,\underline{3}54 \rightarrow 15\,\underline{4}00 \qquad 27\,\underline{2}49.99 \rightarrow 27\,\underline{2}00$ $0.037\,\underline{5}00 \rightarrow 0.037\underline{5}$	
Algebra with Fractions	$\frac{a}{b}+\frac{c}{d} = \frac{a}{b}\cdot\frac{d}{d}+\frac{c}{d}\cdot\frac{b}{b} = \frac{ad+cb}{bd}$ $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd} \qquad \frac{a}{b/c}=a\cdot\frac{c}{b}$ $\frac{a}{x}=b \rightarrow \cancel{x}\cdot\frac{a}{\cancel{x}}=b\cdot x \rightarrow a=bx \rightarrow \frac{a}{b}=\frac{bx}{\cancel{b}} \rightarrow \frac{a}{b}=x$	
Quadratic Equation	$ax^2+bx+c=0 \quad \rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	
All Triangles	$A=\frac{1}{2}bh$ $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $c^2=a^2+b^2-2ab\cos C$	
Right Triangles	$c^2=a^2+b^2$ $\sin\theta=\frac{a}{c}=\frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta=\frac{b}{c}=\frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{a}{b}=\frac{\text{opposite}}{\text{adjacent}}$ $b=c\cos\theta$ $a=c\sin\theta$	
Rectangles, Parallelograms and Trapezoids	$A=\bar{b}h$	$a,b,c=$ length of a side of a triangle $\theta=$ angle $A=$ area $C=$ circumference $S=$ surface area $V=$ volume $b=$ base $\bar{b}=$ average base $=\frac{b_1+b_2}{2}$ $h=$ height $L=$ length $w=$ width $r=$ radius
Rectangular Solids	$V=Lwh$	
Circles	$C=2\pi r$ $A=\pi r^2$	
Cylinders	$S=2\pi rL+2\pi r^2=2\pi r(L+r)$ $V=\pi r^2 L$	
Spheres	$S=4\pi r^2$ $V=\frac{4}{3}\pi r^3$	

Table II. Values of Trigonometric Functions

degree	radian	sine	cosine	tangent	degree	radian	sine	cosine	tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.747
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.331
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.430
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.301
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.081
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.636
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.290
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	∞

Table JJ. Some Exact and Approximate Conversions

Length	1 cm	≈	width of a small paper clip		
	1 inch (in.)	≡	2.54 cm		
	length of a US dollar bill	=	6.14 in.	=	15.6 cm
	12 in.	≡	1 foot (ft.)	≈	30 cm
	3 ft.	≡	1 yard (yd.)	≈	1 m
	1 m	≡	0.3048 ft.	=	39.37 in.
	1 km	≈	0.6 mi.		
	5,280 ft.	≡	1 mile (mi.)	≈	1.6 km
Mass / Weight	1 small paper clip	≈	0.5 g		
	US 1¢ coin (1983–present)	=	2.5 g		
	US 5¢ coin	=	5 g		
	1 oz.	≈	30 g		
	one medium-sized apple	≈	1 N	≈	3.6 oz.
	1 pound (lb.)	≡	16 oz.	≈	454 g
	1 pound (lb.)	≈	4.45 N		
	1 ton	≡	2000 lb.	≈	0.9 tonne
	1 tonne	≡	1000 kg	≈	1.1 ton
Volume	1 pinch	≈	$^{1}/_{16}$ teaspoon (tsp.)		
	1 dash	≈	$^{1}/_{8}$ teaspoon (tsp.)		
	1 mL	≈	10 drops		
	1 tsp.	≈	5 mL	≈	60 drops
	3 tsp.	≡	1 tablespoon (Tbsp.)	≈	15 mL
	2 Tbsp.	≡	1 fluid ounce (fl. oz.)	≈	30 mL
	8 fl. oz.	≡	1 cup (C)	≈	250 mL
	16 fl. oz.	≡	1 U.S. pint (pt.)	≈	500 mL
	20 fl. oz.	≡	1 Imperial pint (UK)	≈	600 mL
	2 pt. (U.S.)	≡	1 U.S. quart (qt.)	≈	1 L
	4 qt. (U.S.)	≡	1 U.S. gallon (gal.)	≈	3.8 L
	4 qt. (UK) ≡ 5 qt. (U.S.)	≡	1 Imperial gal. (UK)	≈	4.7 L
Speed / Velocity	1 $^{\text{m}}/_{\text{s}}$	=	3.6 $^{\text{km}}/_{\text{h}}$	≈	2.24 $^{\text{mi.}}/_{\text{h}}$
	60 $^{\text{mi.}}/_{\text{h}}$	≈	100 $^{\text{km}}/_{\text{h}}$	≈	27 $^{\text{m}}/_{\text{s}}$
Energy	1 cal	≈	4.18 J		
	1 Calorie (food)	≡	1 kcal	≈	4.18 kJ
	1 BTU	≈	1.06 kJ		
Power	1 hp	≈	746 W		
	1 kW	≈	1.34 hp		
Temperature	0 K	≡	−273.15 °C	=	absolute zero
	0 °R	≡	−459.67 °F	=	absolute zero
	0 °F	≈	−18 °C ≡ 459.67 °R		
	32 °F	=	0 °C ≡ 273.15 K	=	water freezes
	70 °F	≈	21 °C	≈	room temperature
	212 °F	=	100 °C	=	water boils
Speed of light	300 000 000 $^{\text{m}}/_{\text{s}}$	≈	186 000 $^{\text{mi.}}/_{\text{s}}$	≈	1 $^{\text{ft.}}/_{\text{ns}}$

Table KK. Greek Alphabet

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Table LL. Decimal Equivalents

$\frac{1}{2}=0.5$	$\frac{1}{5}=0.2$
$\frac{1}{3}=0.33\overline{3}$	$\frac{2}{5}=0.4$
$\frac{2}{3}=0.66\overline{6}$	$\frac{3}{5}=0.6$
$\frac{1}{4}=0.25$	$\frac{4}{5}=0.8$
$\frac{3}{4}=0.75$	$\frac{1}{8}=0.125$
$\frac{1}{6}=0.16\overline{6}$	$\frac{3}{8}=0.375$
$\frac{5}{6}=0.83\overline{3}$	$\frac{5}{8}=0.625$
$\frac{1}{7}=0.\overline{142857}$	$\frac{7}{8}=0.875$
$\frac{2}{7}=0.\overline{285714}$	$\frac{1}{9}=0.11\overline{1}$
$\frac{3}{7}=0.\overline{428571}$	$\frac{2}{9}=0.22\overline{2}$
$\frac{4}{7}=0.\overline{571428}$	$\frac{4}{9}=0.44\overline{4}$
$\frac{5}{7}=0.\overline{714285}$	$\frac{5}{9}=0.55\overline{5}$
$\frac{6}{7}=0.\overline{857142}$	$\frac{7}{9}=0.77\overline{7}$
$\frac{1}{11}=0.09\overline{09}$	$\frac{8}{9}=0.88\overline{8}$
$\frac{2}{11}=0.18\overline{18}$	$\frac{1}{16}=0.0625$
$\frac{3}{11}=0.27\overline{27}$	$\frac{3}{16}=0.1875$
$\frac{4}{11}=0.36\overline{36}$	$\frac{5}{16}=0.3125$
$\frac{5}{11}=0.45\overline{45}$	$\frac{7}{16}=0.4375$
$\frac{6}{11}=0.54\overline{54}$	$\frac{9}{16}=0.5625$
$\frac{7}{11}=0.63\overline{63}$	$\frac{11}{16}=0.6875$
$\frac{8}{11}=0.72\overline{72}$	$\frac{13}{16}=0.8125$
$\frac{9}{11}=0.81\overline{81}$	$\frac{15}{16}=0.9375$
$\frac{10}{11}=0.90\overline{90}$	

Index