Graphical Solutions (Linearization)

Unit: Laboratory & Measurement

MA Curriculum Frameworks (2016): SP4, SP5

AP® Physics 2 Learning Objectives: SP2, SP5

Mastery Objective(s): (Students will be able to…)

- Use a graph to calculate the relationship between two variables.

Success Criteria:

- Graph has the independent variable on the *x*-axis and the dependent variable on the *y*-axis.
- Graph includes best-fit line that appears to minimize the total accumulated distance between the points and the line.
- Axes and best-fit line drawn with straightedge and divisions on axes are evenly spaced.
- Slope of line determined correctly (rise/run) and used correctly in calculation of desired result.

Tier 2 Vocabulary: plot, axes

Language Objectives:

- Explain why a best-fit line gives a better answer than calculating an average.
- Explain how the slope of the line relates to the desired quantity.

Tier 2 Vocabulary: best-fit, independent variable, dependent variable

Notes:

Most experiments in a high-school physics class involve finding a mathematical relationship between two quantities. While it is possible to simply measure each quantity once and calculate, or to measure several times and calculate the average, an approach that measures the relationship across a range of values will provide a better result.

Use this space for summary and/or additional notes:

A common way to achieve this is to manipulate equations and plot data such that the expected result is a straight line, and the experimental quantity is calculated by plotting a best-fit line and determining the slope. This means that you need to plot graphs ***accurately***, either on graph paper or using a computer or calculator. If you use graph paper:

- The data points need to be as close to their actual locations as you are capable of drawing.
- The best-fit line needs to be as close as you can practically get to its mathematically correct location.
- The best-fit line must be drawn with a straightedge.
- The slope needs to be calculated using the actual rise and run of points on the best-fit line.

Common mistakes:

- Axes are labeled unevenly. (The "skip" between divisions is not consistent.)
- There is a break in either or both axes, but the best-fit line is drawn through zero anyway.
- Points are not plotted exactly. (In this case, "close enough" usually isn't!)

As mentioned in the previous section, a good rule of thumb for quantitative experiments is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Once you have your data points, arrange the equation into $y = mx + b$ form, such that the slope (or $^{1}/_{\text{slope}}$) is the quantity of interest. Then accurately plot your data and draw a best-fit line. The slope of this line will be the quantity of interest (or its reciprocal).

Use this space for summary and/or additional notes:

For example, suppose you wanted to calculate the spring constant of a spring by measuring the displacement caused by an applied force. You are given the following data:

Applied Force (N)	0.0	1.0	2.0	3.0	5.0
Displacement (m)	−0.01	0.05	0.16	0.20	0.34
Uncertainty (m)	± 0.06	± 0.06	± 0.06	± 0.06	± 0.06

The equation is $F_s = kx$, which is already in $y = mx + b$ form. However, we varied the force and measured the displacement, which means force is the independent variable (x-axis), and displacement is the dependent variable (y-axis). Therefore, we need to rearrange the equation to:

$$\begin{array}{ccccc} y & = m & x & +b \\ \downarrow & \downarrow & \downarrow & \downarrow \\ x & = \left(\frac{1}{k}\right) & F_s & +0 \end{array}$$

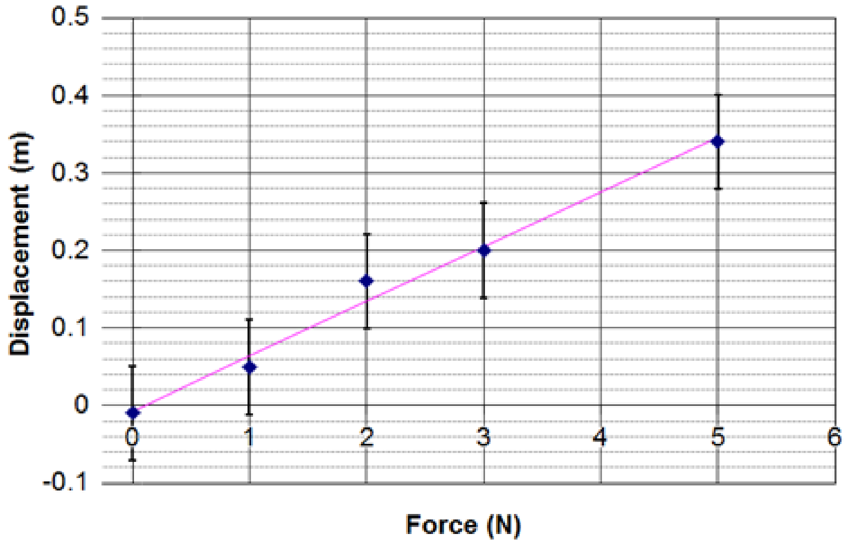
This means that if we plot a graph of all of our data points, a graph of F_s *vs.* x will have a slope of $\frac{1}{k}$.

You therefore need to:

1. Plot the data points, expressing the uncertainties as error bars.
2. Draw a best-fit line that passes through each error bar and minimizes the total accumulated distance away from each data point. (You can use linear regression, provided that the regression line actually passes through each error bar. If the line cannot pass through all of the error bars, you need to determine what the problem was with the outlier(s).) You may disregard a data point in your determination of the best-fit line *only* if you know *and can explain* the problem that caused it to be an outlier.

Use this space for summary and/or additional notes:

The plot looks like the following:



We calculate the slope using the actual rise (Δy) and run (Δx) from the graph. The best-fit line goes through the points (0, 0) and (3.0, 0.21). From these points, we would calculate the slope as:

$$m = \frac{\Delta y}{\Delta x} = \frac{0.21 - 0}{3.0 - 0} = 0.07$$

Because the slope is $\frac{1}{k}$, the spring constant is the reciprocal of the slope of the above graph. $\frac{1}{0.07} = 14\,\frac{\text{N}}{\text{m}}$ (rounded to two significant figures).

Use this space for summary and/or additional notes: