Heat Transfer

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS3-4a

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.3.A, 9.3.A.1, 9.3.A.1.i, 9.3.A.1.ii, 9.3.A.2, 9.3.A.3, 15.4.A, 15.4.A.1, 15.4.A.2, 15.4.A.3, 15.4.A.3.i, 15.4.A.3.ii, 15.4.A.3.iii

Mastery Objective(s): (Students will be able to…)

- Explain heat transfer by conduction, convection and radiation.
- Calculate heat transfer using Fourier's Law of Heat Conduction.

Success Criteria:

- Descriptions & explanations account for observed behavior.
- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain the mechanisms by which heat is transferred.

Tier 2 Vocabulary: conduction, radiation

Labs, Activities & Demonstrations:

- Radiometer & heat lamp.
- Almond & cheese stick.
- Flammable soap bubbles.
- Drop of food coloring in water *vs.* ice water

Notes:

Heat transfer is the flow of heat energy from one object to another. Heat transfer usually occurs through three distinct mechanisms: conduction, radiation, and convection.

conduction: transfer of heat through collisions of particles by objects that are ***in direct contact*** with each other. Conduction occurs when there is a net transfer of momentum from the molecules of an object with a higher temperature transfer to the molecules of an object with a lower temperature.

thermal conductivity (k): a measure of the amount of heat that a given length of a substance can conduct in a specific amount of time. Thermal conductivity is measured in units of $\frac{\text{J}}{\text{m·s·°C}}$ or $\frac{\text{W}}{\text{m·°C}}$.

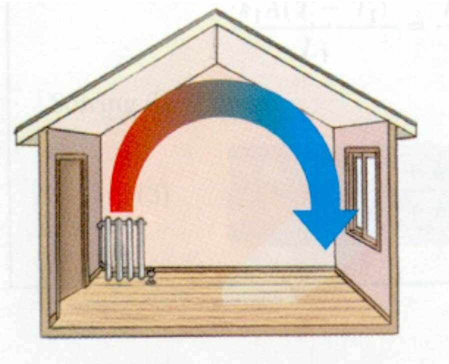
conductor: an object that allows heat to pass through itself easily; an object with high thermal conductivity.

insulator: an object that does not allow heat to pass through itself easily; a poor conductor of heat; an object with low thermal conductivity.

radiation: transfer of heat *through space* via electromagnetic waves (light, microwaves, *etc.*)

convection: transfer of heat *by motion of particles* that have a higher temperature exchanging places with particles that have a lower temperature. Convection usually occurs when air moves around a room.

Natural convection occurs when particles move because of differences in density. In a heated room, because cool air is more dense than warm air, the force of gravity is stronger on the cool air, and it is pulled harder toward the ground than the warm air. The cool air displaces the warm air, pushing it upwards out of the way.



In a room with a radiator, the radiator heats the air, which causes it to expand and be displaced upward by the cool air nearby. When the (less dense) warm air reaches the ceiling, it spreads out, and it continues to cool as it spreads. When the air reaches the opposite wall, it is forced downward toward the floor, across the floor, and back to the radiator.

Forced convection can be achieved by moving heated or cooled air using a fan. Examples of this include ceiling fans and convection ovens. If your radiator does not warm your room enough in winter, you can use a fan to speed up the process of convection. (Make sure the fan is moving the air in the same direction that would happen from natural convection. Otherwise, the fan will be fighting against physics!)

Calculating Heat Transfer by Conduction

Heat transfer by conduction can be calculated using Fourier's Law of Heat Conduction:

$$P = \frac{Q}{t} = \pm kA\frac{\Delta T}{L}$$

where:

P = power (W)

Q = heat transferred (J)

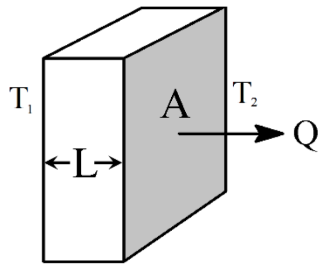
t = time (s)

k = coëfficient of thermal conductivity $\left(\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)$

A = cross-sectional area (m^2)

ΔT = temperature difference (K or °C)

L = length (m)



The ± sign means that the value can be positive or negative, because the sign for Q is chosen based on whether the heat transfers into (+) or out of (−) the system.

Note that for insulation (the kind you have in the walls and attic of your home), you want the lowest possible thermal conductivity—you don't want the insulation to conduct the heat from the inside of your house to the outside! Because most people think that bigger numbers are better, the industry has created a measure of the effectiveness of insulation called the "R value". It is essentially the reciprocal of $\frac{k}{L}$, which means lower conductivity and more thickness gives better insulation.

Sample Problem:

Q: A piece of brass is 5.0 mm (0.0050 m) thick and has a cross-sectional area of 0.010 m^2. If the temperature on one side of the metal is 65°C and the temperature on the other side is 25°C, how much heat will be conducted through the metal in 30. s? The coefficient of thermal conductivity for brass is $120\,\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$.

A: $$\frac{Q}{t} = kA\frac{\Delta T}{L}$$

$$\frac{Q}{30} = (120)(0.010)\left(\frac{65-25}{0.0050}\right) = 9600$$

$$Q = 288\,000\ \text{J} = 288\ \text{kJ}$$

(Note that because the quantities of heat that we usually measure are large, values are often given in kilojoules or megajoules instead of joules.)

Calculating Heat Transfer by Radiation

Heat transfer by radiation is based on the temperature of a substance and its ability emit heat (emissivity). The equation is:

$$P = \frac{Q}{t} = \varepsilon \sigma A T^4$$

where:

- P = power (W)
- Q = heat (J)
- t = time (s)
- ε = emissivity (dimensionless; "blackbody" ≡ 1)
- σ = Stefan-Boltzmann constant $\left(\sigma = 5.67\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)$
- A = area (m^2)
- T = temperature (K)

Note that because the equation contains T (rather than ΔT), the temperature needs to be in Kelvin.

emissivity (ε): a ratio of the amount of heat radiated by a substance to the amount of heat that would be radiated by a perfect "blackbody" of the same dimensions.

Emissivity is a dimensionless number (meaning that it has no units, because the units cancel), and is specific to the substance.

blackbody: an object that absorbs all of the heat energy that comes in contact with it (and reflects none of it).

Stefan-Boltzmann constant (σ): the constant that makes the above equation come out in watts. Note that the Stefan-Boltzmann constant is defined from other constants:

$\sigma = \frac{2\pi^5 k_B^4}{15h^3c^2}$, where k_B is the Boltzmann constant, h is Planck's constant, and c is the speed of light in a vacuum.

Wien's Displacement Law

In 1893, German physicist Wilhelm Wien discovered that radiation from a blackbody occurs in the form of electromagnetic radiation, over a range of wavelengths. The wavelength at which the maximum energy is radiated decreases as the temperature increases:

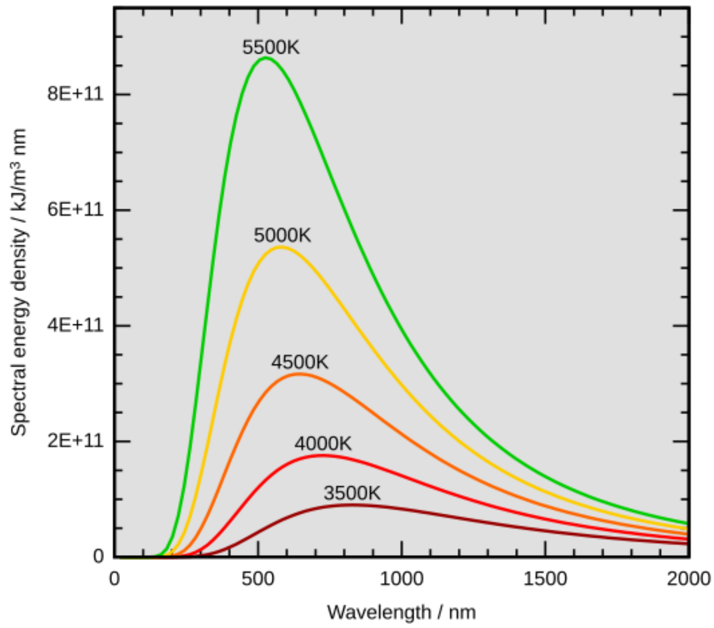


Image by 4C. Used with permission.

The wavelength of maximum radiation is described by the equation:

$$\lambda_{max} = \frac{b}{T}$$

where:

λ_{max} = wavelength of maximum radiated energy

b = Wien's displacement constant $= 2.897\,771\,955 \times 10^{-3}\,\text{m}\cdot\text{K}$

T = temperature (K)

honors (not AP®)

Wien's displacement law was superseded in 1900, when German physicist Max Planck derived a more general equation, now called Planck's radiation law. This law gives an equation for the spectral energy density (energy per unit volume per unit frequency):

$$u_\nu(\nu,T) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{\left(h\nu/k_B T\right)} - 1}$$

where:

$u_\nu(\nu,T) =$ spectral energy density (a function of wavelength & temperature)
$h =$ Planck's constant $= 6.626\,070\,15 \times 10^{-34}\ \text{J}\cdot\text{s}$
$c =$ speed of light in a vacuum $= 2.997\,924\,58\ \frac{\text{m}}{\text{s}}$
$k_B =$ Boltzmann constant $= 1.380\,649 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$
$T =$ temperature (K)

Homework Problems

You will need to look up coefficients of thermal conductivity in *Table K. Thermal Properties of Selected Materials* on page 473 of your reference tables.

1. **(S)** The surface of a hot plate is made of 12.0 mm (0.012 m) thick aluminum and has an area of 64 cm^2 (which equals 0.0064 m^2). If the heating coils maintain a temperature of 80.°C underneath the surface and the air temperature is 22°C, how much heat can be transferred through the plate in 60. s?

   Answer: 464 000 J* or 464 kJ

2. **(S)** A cast iron frying pan is 5.0 mm thick. If it contains boiling water (100°C), how much heat will be transferred into your hand if you place your hand against the bottom for two seconds?
   (Assume your hand has an area of 0.0040 m^2, and that body temperature is 37°C.)

   Answer: +8 064 J or +8.064 kJ
   (positive because the direction is stated as "*<u>into</u>* your hand")

3. **(M)** A plate of metal has thermal conductivity k and thickness L. One side has a temperature of T_h and the other side has a temperature of T_c, derive an expression for the cross-sectional area A that would be needed in order to transfer a certain amount of heat, Q, through the plate in time t.

   Answer: $A = \dfrac{QL}{kt\,(T_h - T_c)}$

---

* Note: Questions #1 and #3 do not specify the direction of heat transfer, so the answer could be either positive or negative.

4. **(M)** A glass window in a house has an area of 0.67 m^2 and a thickness of 2.4 mm $(2.4\times10^{-3}\text{ m})$. The temperature inside the house is 21 °C, and the outside temperature is 0 °C.

   a. **(M)** How much heat is lost through the window in 1 hour (3600 s) due to conduction?

      Use $1.0\frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}$ for the thermal conductivity of the glass.

      Answer: −21 100 000 J = −21 100 kJ
      (negative because heat is *lost* through the window)

   b. **(M – honors; A – AP®)** How much heat is lost through the window in 1 hour (3600 s) due to radiation? (Assume the temperature of the entire glass is 21 °C for this problem.)

      Hint: Remember to convert the temperature to Kelvin.

      Answer: −940 000 J = −940 kJ
      (negative because heat is *lost* through the window)

   c. **(M – honors; A – AP®)** Which mode of heat transfer (conduction *vs.* radiation) accounts for the greater amount of heat loss?