Solving Word Problems Systematically

Unit: Mathematics

MA Curriculum Frameworks (2016): SP1, SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Assign (declare) variables in a word problem according to the conventions used in physics.
- Substitute values for variables in an equation.

Success Criteria:

- Variables match the quantities given and match the units.
- Quantities are substituted for the correct variables in the equation.

Tier 2 Vocabulary: equation, variable

Language Objectives:

- Describe the quantities used in physics, list their variables, and explain why that particular variable might have been chosen for the quantity.

Tier 2 Vocabulary: unit

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Use this space for summary and/or additional notes:

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word(s)	symbol	word(s)	symbol
of	×	percent (“per” + “cent”)	÷ 100
per, out of	÷		
is	=	change in *x*, difference in *x*	Δx

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable	quantity	S.I. unit	variable
mass	kg	m	power	W	P^*
distance, length	m	d , ℓ	pressure	Pa	P^*
height	m	h	momentum	N·s	p^*
area	m^2	A	density	kg/m^3	ρ^*
acceleration	m/s^2	a	moles	mol	n
volume	m^3	V	temperature	K	T
velocity (speed)	m/s	v	heat	J	Q
time	s	t	electric charge	C	q , Q

*Note the subtle differences between uppercase “*P*”, lowercase “*p*”, and the Greek letter ρ (“rho”).

Any time you see a number in a word problem that has a unit that you recognize (such as one listed in this table), notice which quantity the unit is measuring and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes:

Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$\rho^{*} = \frac{m}{V}$ and you're given: $m = 12.3$ g and $V = 2.8$ cm^3

simply substitute 12.3 g for m, and 2.8 cm^3 for V, giving:

$$\rho = \frac{12.3\,\text{g}}{2.8\,\text{cm}^3} = 4.4\,\frac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often easier to leave the units out when you substitute numbers for variables and then add them back in at the end:

$$\rho = \frac{12.3}{2.8} = 4.4\,\frac{\text{g}}{\text{cm}^3}$$

Many physics teachers disagree with this approach and insist on having students include the units with the number throughout the calculation. However, this can lead to confusion about which symbols are variables and which are units. For example, if a device applies a power of 150 W for a duration of 30 s and we wanted to find out the amount of work done, we would have:

$$P = \frac{W}{t}$$
$$150\,\text{W} = \frac{W}{30\,\text{s}}$$

The student would need to realize that the W on the left side of the equation is the unit "watts", and the *W* on the right side of the equation is the variable *W*, which stands for "work".

* Physicists use the Greek letter ρ ("rho") for density. Note that the Greek letter ρ is different from the Roman letter "p".

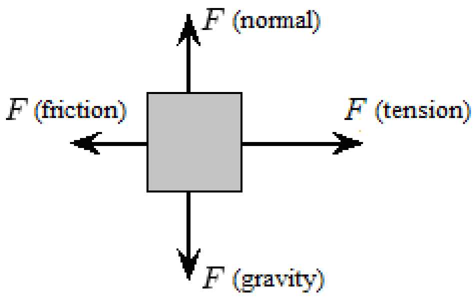
Use this space for summary and/or additional notes:

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

1. The force of gravity, pulling downward.
2. The "normal" force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

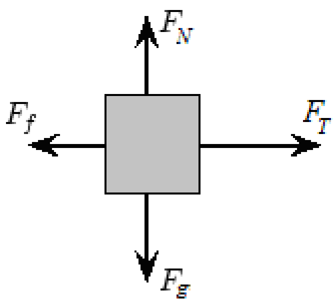
The variable for force is "*F*". There are four types of forces, which means "*F*" means four different things in this problem:



To make the diagram easier to read, we add subscripts to the variable "*F*". Note that in most cases, the subscript is the first letter of the word that describes the particular instance of the variable:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.

This results in the following free-body diagram:



We use these same subscripts in the equations that relate to the problem. For example:

$$F_g = mg \quad \text{and} \quad F_f = \mu F_N$$

Use this space for summary and/or additional notes:

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—***it needs to be smaller than the other letters and lowered slightly***. For example, when we write F_g, the variable is F (force) and the subscript $_g$ attached to it tells which kind of force it is (gravity). This might occur in the following equation:

$$F_g = mg$$ ← right ☺

It is important that the subscript $_g$ on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$Fg = mg$$
$$F\cancel{g} = m\cancel{g}$$ ← wrong! ☹
$$F = m$$

Another common use of subscripts is the subscript "o" to mean "initial". (Imagine that the "story problem" is shown as a video. When the slider is at the beginning of the video, the time is shown as 0, and the values of all of the variables at that time are shown with a subscript of o.)

For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity "v_o*" where the subscript "o" means "at time zero", *i.e.,* at the beginning of the problem. The final velocity is simply "v" without the zero.

* pronounced either "v-zero" or "v-naught"

Use this space for summary and/or additional notes:

The Problem-Solving Process

1. Identify the quantities in the problem, based on the units and any other information in the problem.
2. Assign the appropriate variables to those quantities.
3. Find an equation that relates all of the variables.
4. Use algebra to rearrange the equation to solve for the variable you're looking for. ("Undo PEMDAS.")
5. Substitute the values of the variables into the equation.
    a. If you have only one variable left, it should be the one you're looking for.
    b. If you have more than one variable left, repeat this sequence, finding another equation that uses one of the variables you have left, plus other quantities that you know.
6. Solve the equation(s), using algebra.
7. Apply the appropriate unit(s) to the result.

Use this space for summary and/or additional notes:

Sample Problem

A force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object? (*mechanics/forces*)

We have units of N and kg, and we're looking for acceleration. We need to look these up in the *Appendix: Physics* Reference Tables in the appendix at the end of these notes.

From *Table C. Quantities, Variables and Units* on page 611 of the reference tables, we find:

Symbol	Unit	Quantity	Variable
N	newton	force	$\vec{F}$
kg	kilogram	mass	m
		acceleration	$\vec{a}$

We can label these quantities on the problem itself:

$\vec{F}$ m $\vec{a}$

A force of <u>30 N</u> acts on an object with a mass of <u>1.5 kg</u>. What is the <u>acceleration</u> of the object?

Now we know that we need an equation that relates the variables $\vec{F}$, *m*, and . ($\vec{F}$ and $\vec{a}$ *are* in boldface with an arrow above them because they are vector quantities. We will discuss vectors a little later in the course.)

Now that we have the variables, we find a formula that relates them. From the second formula box ("Forces") in *Table D. Mechanics Formulas and Equations* on page 612 of the reference tables, we find that:

$$\Delta V = \frac{W}{q} = \vec{E} \bullet \vec{d}$$

$$\vec{F}_{net} = m\vec{a}$$

Rearranging to solve for $\vec{a}$ gives:

$$R = \frac{\rho L}{A}$$

$$\vec{F}_{net} = m\vec{a}$$

$$\frac{\vec{F}_{net}}{m} = \vec{a}$$

$$\frac{30}{1.5} = \vec{a} = 20$$

Again from *Table C. Quantities, Variables and Units*, we find that acceleration has units of meters per second squared, so our final answer is $20\ \frac{\text{m}}{\text{s}^2}$.

Use this space for summary and/or additional notes:

Homework Problems

To solve these problems, refer to your Appendix: Physics Reference Tables starting on page 609. To make the equations easier to find, the table and section of the table in your Physics Reference Tables where the equation can be found is given in parentheses.

1. What is the velocity of a car that travels 90. m in 4.5 s? (*mechanics/kinematics*)

   Answer: $20.\,\frac{m}{s}$

2. If a force of 100. N acts on a mass of 5.0 kg, what is its acceleration? (*mechanics/forces*)

   Answer: $20.\,\frac{m}{s^2}$

3. If the momentum of a block is *p* and its velocity is *v*, derive an expression for the mass, *m*, of the block.

   (*If you are not sure how to do this problem, do #4 below and use the steps to guide your algebra.*)

   (*mechanics/momentum*)

   Answer: $m = \frac{p}{v}$

Use this space for summary and/or additional notes:

4. If the momentum of a block is 18 N·s and its velocity is $3\,\frac{\text{m}}{\text{s}}$, what is the mass of the block?
   (You must start with the equations in your Physics Reference Tables. *You may only use the answer to question #3 above as a starting point if you have already solved that problem.*)
   (*mechanics/momentum*)

   Answer: 6 kg

5. What is the potential energy due to gravity of a 95 kg anvil that is about to fall off a 150 m cliff onto Wile E. Coyote's head?
   (*mechanics/energy, work & power*)

   Answer: 142 500 J

6. A 25 Ω resistor is placed in an electrical circuit with a voltage of 110 V. How much current flows through the resistor?
   (*electricity/circuits*)

   Answer: 4.4 A

Use this space for summary and/or additional notes:

7. What is the frequency of a wave that is traveling at a velocity of $300.\,\frac{m}{s}$ and has a wavelength of 10. m?
(*waves/waves*)

Answer: 30. Hz

8. What is the energy of a photon that has a frequency of 6×10^{15} Hz ?
(*atomic, particle & nuclear physics/energy*)

Answer: 3.96×10^{-18} J

9. A piston with an area of $2.0\,m^2$ is compressed by a force of 10 000 N. What is the pressure applied by the piston?
(*fluid mechanics/pressure*)

Answer: 5 000 Pa

10. Derive an expression for the acceleration, *a*, of a car whose velocity changes from v_o to *v* in time *t*.
(*If you are not sure how to do this problem, do #11 below and use the steps to guide your algebra.*)
(*mechanics/kinematics*)

Answer: $a=\frac{v-v_o}{t}$

Use this space for summary and/or additional notes:

11. What is the acceleration of a car whose velocity changes from $60.\,\frac{\text{m}}{\text{s}}$ to $80.\,\frac{\text{m}}{\text{s}}$ over a period of 5.0 s?

    Hint: v_o is the initial velocity and v is the final velocity.

    (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #10 above as a starting point if you have already solved that problem.*)

    (*mechanics/kinematics*)

    Answer: $4.0\,\frac{\text{m}}{\text{s}^2}$

12. If the normal force on an object is 100. N and the coëfficient of kinetic friction between the object and the surface it is sliding on is 0.35, what is the force of friction on the object as it slides along the surface? (*mechanics/forces*)

    Answer: 35 N

13. A 1200 W hair dryer is plugged into a electrical circuit with a voltage of 110 V. How much electric current flows through the hair dryer? (*electricity/circuits*)

    Answer: 10.9 A

14. A car has mass *m* and kinetic energy *K*. Derive an expression for its velocity, *v*. You may use your work from problem #15 below to guide your algebra.

    (*If you are not sure how to do this problem, do #11 above and use the steps to guide your algebra.*)

    (*mechanics/energy*)

Use this space for summary and/or additional notes:

Answer: $v = \sqrt{\frac{2K}{m}}$

15. A car has a mass of 1 200 kg and kinetic energy of 240 000 J. What is its velocity?

    (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #14 above as a starting point if you have already solved that problem.*)

    (*mechanics/energy*)

    Answer: $20.\ \frac{m}{s}$

16. What is the velocity of a photon (wave of light) through a block of clear plastic that has an index of refraction of 1.40?

    Hint: You will need to look up the index of refraction in your Physcis Reference Tables.

    (*waves/reflection & refraction*)

    Answer: $2.14 \times 10^{8}\ \frac{m}{s}$

17. If a pressure of 100 000 Pa is applied to a gas and the volume decreases by $0.05\ m^3$, how much work was done on the gas?

    (Note: represent the *change* in volume as ΔV.)

    (*fluid mechanics/work*)

    Answer: 5 000 J

Use this space for summary and/or additional notes:

18. If the distance from a mirror to an object is s_o and the distance from the mirror to the image is s_i, derive an expression for the distance from the lens to the focus, *f*. You may use your work from problem #17 to guide your algebra.

    (*If you are not sure how to do this problem, do #19 below and use the steps to guide your algebra.*)

    (*waves/mirrors & lenses*)

    Answer: $f = \dfrac{s_i + s_o}{s_i\, s_o}$

19. If the distance from a mirror to an object is 0.8 m and the distance from the mirror to the image is 0.6 m, what is the distance from the mirror to the focus?

    (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #18 above as a starting point if you have already solved that problem.*)

    (*waves/mirrors & lenses*)

    Answer: 0.343 m

20. What is the momentum of a photon that has a wavelength of 400 nm?
    (*Hint: you need to convert nanometers to meters.*)
    (*atomic, particle & nuclear physics/energy*)

    Answer: 1.65×10^{-27} N·s

Use this space for summary and/or additional notes: