# Solving Equations Symbolically

**Unit:** Mathematics

## MA Curriculum Frameworks (2016): SP5

**AP® Physics 2 Learning Objectives:** SP 2.2

**Mastery Objective(s):** (Students will be able to…)

- Rearrange algebraic expressions to solve for any variable in the expression.

**Success Criteria:**

- Rearrangements are algebraically correct.

**Tier 2 Vocabulary: equation, variable**

## Language Objectives:

- Describe how the rules of algebra are applied to expressions that contain only variables.

**Tier 2 Vocabulary:** N/A

## Notes:

In solving physics problems, we are more often interested in the relationship between the quantities in the problem than we are in the numerical answer.

For example, suppose we are given a problem in which a person with a mass of 65 kg accelerates on a bicycle from rest $(0\,\frac{\text{m}}{\text{s}})$ to a velocity of $10\,\frac{\text{m}}{\text{s}}$ over a duration of 12 s and we wanted to know the force that was applied.

We could calculate acceleration as follows:

$$v - v_o = at$$
$$10 - 0 = a(12)$$
$$a = \frac{10}{12} = 0.8\overline{3}\,\frac{\text{m}}{\text{s}^2}$$

Then we could use Newton's second law:

$$F = ma$$
$$F = (65)(0.8\overline{3}) = 54.2\,\text{N}$$

We have succeeded in answering the question. However, the question and the answer are of no consequence. Obtaining the correct answer shows that we can manipulate two related equations and come out with the correct number.

Use this space for summary and/or additional notes:

However, if instead we decided that we wanted to come up with an expression for force in terms of the quantities given (mass, initial and final velocities and time), we would need to rearrange the relevant equations to give an expression for force in terms of those quantities.

Just like algebra with numbers, rearranging an equation to solve for a variable is simply "undoing PEMDAS:"

1. "Undo" addition and subtraction by doing the opposing operation. If a variable is added, subtract it from both sides; if the variable is subtracted, then add it to both sides.

$$\begin{aligned} a + c &= b \\ -c &= -c \\ a &= b - c \end{aligned}$$

2. "Undo" multiplication and division by doing the opposing operation. If a variable is multiplied, divide both sides by it; if the variable is in the denominator, multiply both sides by it. *Note: whenever you have variables in the denominator that are on the same side of the equation as the variable you are solving for, always multiply both sides by it to clear the fraction.*

$$\frac{xy}{y} = \frac{z}{y} \qquad x = \frac{z}{y}$$

$$\frac{n}{r} = s \qquad r \cdot \frac{n}{r} = s \cdot r \qquad n = sr \qquad \frac{n}{s} = \frac{sr}{s} \qquad \frac{n}{s} = r$$

3. "Undo" exponents by taking the appropriate root of both sides. (Most often, the exponent will be 2, which means take the square root.) Similarly, you can "undo" roots by raising both sides to the appropriate power.

$$\begin{aligned} t^2 &= 4ab \\ \sqrt{t^2} &= \sqrt{4ab} \\ t &= \sqrt{4} \cdot \sqrt{ab} = 2\sqrt{ab} \end{aligned}$$

4. When you are left with only parentheses and nothing outside of them, you can drop the parentheses, and then repeat steps 1–3 above until you have nothing left but the variable of interest.

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Returning to the previous problem:

We know that $F = ma$. We are given *m*, but not *a*, which means we need to replace *a* with an expression that includes only the quantities given.

First, we find an expression that contains *a*:

$$v - v_o = at$$

We recognize that $v_o = 0$, and we use algebra to rearrange the rest of the equation so that *a* is on one side, and everything else is on the other side.

$$v - v_o = \underline{a}t$$

$$v - 0 = \underline{a}t$$

$$v = \underline{a}t$$

$$\underline{a} = \frac{v}{t}$$

Finally, we replace *a* in the first equation with $\frac{v}{t}$ from the second:

$$F = ma$$

$$F = (m)\left(\frac{v}{t}\right)$$

$$F = \frac{mv}{t}$$

If the only thing we want to know is the value of *F* in one specific situation, we can substitute numbers at this point. However, we can also see from our final equation that increasing the mass or velocity will increase the numerator, which will increase the value of the fraction, which means the force would increase. We can also see that increasing the time would increase the denominator, which would decrease the value of the fraction, which means the force would decrease.

Solving the problem symbolically gives a relationship that holds true for all problems of this type in the natural world, instead of merely giving a number that answers a single pointless question. This is why the College Board and many college professors insist on symbolic solutions to equations.

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# Homework Problems

1. Given $a = 2bc$ and $e = c^2 d$, write an expression for *e* in terms of *a*, *b*, and *d*.

2. Given $w = \frac{3}{2}xy^2$ and $z = \frac{q}{y}$:

    a. Write an expression for *z* in terms of *q*, *w*, and *x*.

    b. If you wanted to maximize the value of the variable *z* in question #2 above, what adjustments could you make to the values of *q*, *w*, and *x*?

    c. Changing which of the variables *q*, *w*, or *x* would give the largest change in the value of *z*?

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