Vectors

Unit: Mathematics

MA Curriculum Frameworks (2016): SP5

AP® Physics 2 Learning Objectives: SP 2.2

Mastery Objective(s): (Students will be able to…)

- Identify the magnitude and direction of a vector.
- Combine vectors graphically and calculate the magnitude and direction.

Success Criteria:

- Magnitude is calculated correctly (Pythagorean theorem).
- Direction is correct: angle (using trigonometry) or direction (*e.g.,* "south", "to the right", "in the negative direction", *etc.*)

Tier 2 Vocabulary: magnitude, direction

Language Objectives:

- Explain what a vector is and what its parts are.

Tier 2 Vocabulary: vector, sign, direction

Notes:

vector: a quantity that has both a magnitude (value) and a direction.

E.g., if you are walking $1\frac{m}{s}$ to the north, the magnitude is $1\frac{m}{s}$ and the direction is north.

scalar: a quantity that has a value but does not have a direction. (A scalar is what you think of as a "regular" number, including its unit.)

magnitude: the scalar part of a vector (*i.e.,* the number and its units, but without the direction). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if $\vec{\boldsymbol{F}}$ is 25 N to the east, then $\|\vec{\boldsymbol{F}}\| = 25\,\text{N}$. However, to make typesetting easier, it is common to use regular absolute value bars instead, *e.g.,* $|\vec{F}| = 25\,\text{N}$.

resultant: a vector that is the result of a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes:

Variables that represent vectors are traditionally typeset in ***bold Italics***. Vector variables may also optionally have an arrow above the letter:

$$\boldsymbol{J}, \vec{\boldsymbol{F}}, \boldsymbol{v}$$

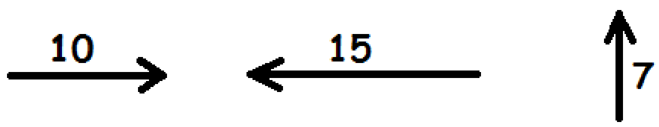
Variables that represent scalars are traditionally typeset in *plain Italics:*

$$V, t, \lambda$$

Note that a variable that represents only the magnitude of a vector quantity is generally typeset as if it were a scalar:

> For example, suppose $\vec{\boldsymbol{F}}$ is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount ***and*** the direction.)
>
> The magnitude would be 25 N, and would be represented by the variable *F*.

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:

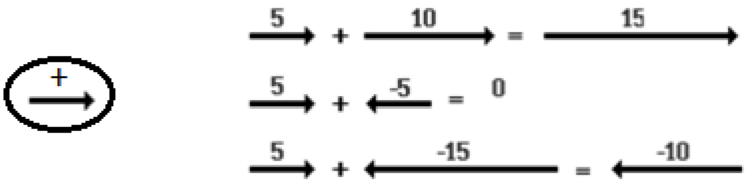


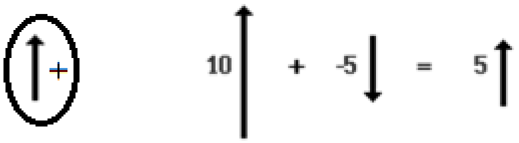
magnitude 10	magnitude 15	magnitude 7
direction: "to the right", 0°	direction: "to the left", +180°	direction: "up", +90°

Use this space for summary and/or additional notes:

Adding & Subtracting Vectors

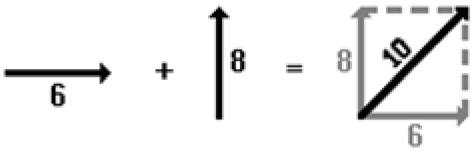
If the vectors have the same direction or opposite directions, the resultant is easy to envision:





If the vectors are not in the same direction, we move them so they start from the same place and complete the parallelogram. If they are perpendicular, we can add them by doing the following:

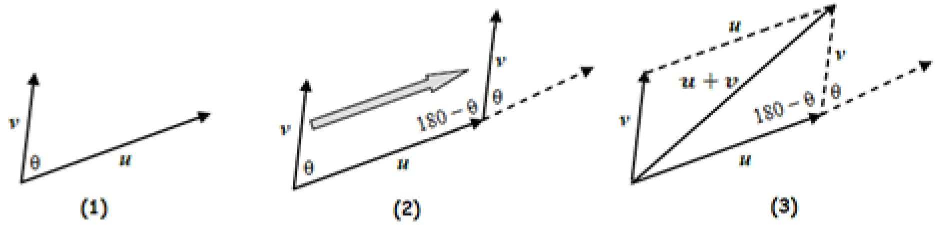
1. Translate (slide) the vectors so that they are either tip-to-tail or tail-to-tail.*
2. Calculate the length of the resultant by completing the rectangle and using the Pythagorean theorem:



$(6^2 + 8^2 = 10^2)$

Note that the sum of these two vectors has a magnitude (length) of 10, not 14.

The same process applies to adding vectors that are not perpendicular:

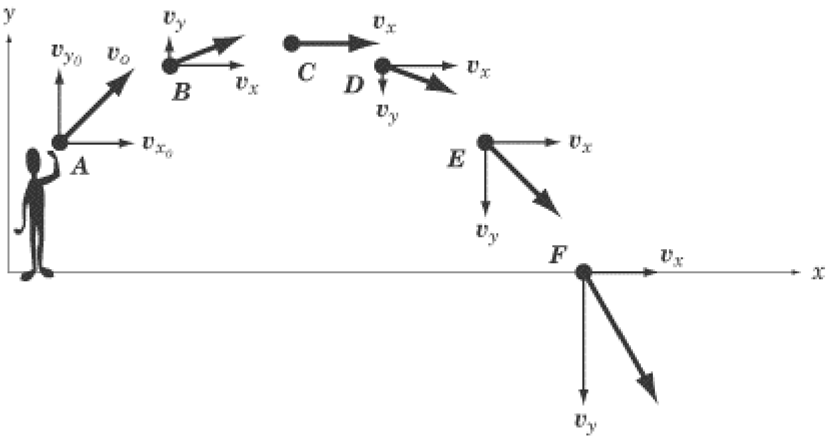


However, the trigonometry needed for these calculations is beyond the scope of this course.

* In this section, examples are shown translating vectors tail-to-tail and completing the parallelogram. While this does not always result in the best representation of the physics involved, it is less confusing for students to keep the procedure consistent when they are first learning.

Use this space for summary and/or additional notes:

One type of physics problem that commonly uses vectors is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, $\vec{\boldsymbol{v}}$, at angle θ, the vector can be represented as the sum of a horizontal vector $\vec{\boldsymbol{v}}_x$ and a vertical vector $\vec{\boldsymbol{v}}_y$. This is useful because the horizontal vector $\vec{\boldsymbol{v}}_x$ gives us the component (portion) of the vector in the *x*-direction, and the vertical vector $\vec{\boldsymbol{v}}_y$ gives us the component of the vector in the *y*-direction.



Notice that $\vec{\boldsymbol{v}}_x$ remains constant, but $\vec{\boldsymbol{v}}_y$ changes (because of the effects of gravity).

Because perpendicular vectors do not affect each other, we can apply equations to the two directions separately.

As you saw in projectile motion (which you learned about in physics 1), we use the equation $\vec{\boldsymbol{d}} = \vec{\boldsymbol{v}}_o t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$, applying it separately in the *x*- and *y*-directions. This gives us two equations.

In the horizontal (*x*)-direction:

$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_{o,x} t + \frac{1}{2}\cancelto{0}{\vec{\boldsymbol{a}}_x} t^2$$

$$\vec{\boldsymbol{d}}_x = \vec{\boldsymbol{v}}_x t$$

In the vertical (*y*)-direction:

$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y} t + \frac{1}{2}\vec{\boldsymbol{a}}_y t^2$$

$$\vec{\boldsymbol{d}}_y = \vec{\boldsymbol{v}}_{o,y} t + \frac{1}{2}\vec{\boldsymbol{g}} t^2$$

Use this space for summary and/or additional notes:

Note that each of the vector quantities ($\vec{d}$, $\vec{v}_o$ and $\vec{a}$) has independent *x*- and *y*-components. For example, $\vec{v}_{o,x}$ (the component of the initial velocity in the *x*-direction) is independent of $\vec{v}_{o,y}$ (the component of the initial velocity in the *x*-direction). This means *we treat them as completely separate variables*, and we can solve for one without affecting the other.

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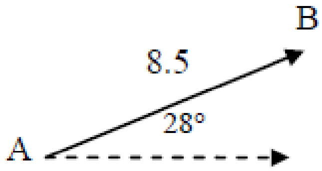
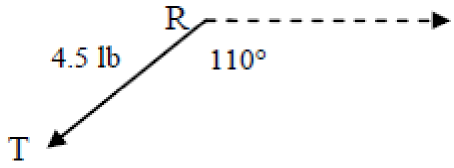
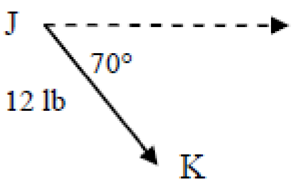
Homework Problems

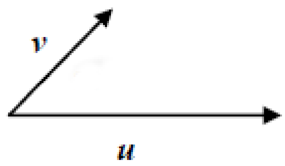
Label the magnitude and direction of each of the following:

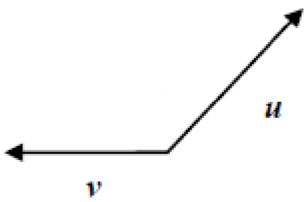
1. 

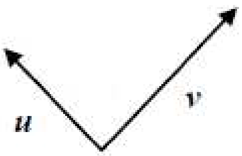
2. 

3. 

Sketch the resultant of each of the following.

4. 

5. 

6. 

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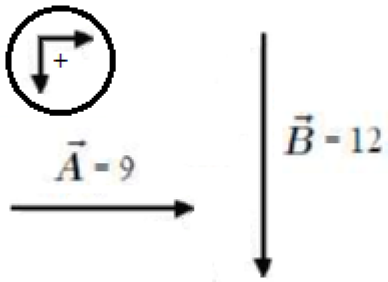
Consider the following vectors $\vec{A} \& \vec{B}$.

Vector $\vec{A}$ has a magnitude of 9 and its direction is the positive horizontal direction (to the right).



7. $\vec{A}-\vec{B}$ ..Sketch the resultant of $\vec{A}+\vec{B}$, and determine its magnitude and direction.

8. Sketch the resultant of $\vec{A}-\vec{B}$ (which is the same as $\vec{A}+(-\vec{B})$, and determine its magnitude and direction.

Use this space for summary and/or additional notes: