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Thermal Expansion

Unit: Thermal Physics (Heat)

NGSS Standards/MA Curriculum Frameworks (2016): N/A

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate changes in length & volume for solids, liquids and gases that are undergoing thermal expansion or contraction.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct equations.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain what the heat is used for in each step of a heating curve.

Tier 2 Vocabulary: expand, contract

Labs, Activities & Demonstrations:

- Balloon with string & heat gun.
- Brass ball & ring.
- Bi-metal strip.

Notes:

expand: to become larger

contract: to become smaller

thermal expansion: an increase in the length and/or volume of an object caused by a change in temperature.

When a substance is heated, the particles it is made of move farther and faster. This causes the particles to move farther apart, which causes the substance to expand.

Solids tend to keep their shape when they expand. (Liquids and gases do not have a definite shape to begin with.)

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0 °C to 4 °C. (Water expands as the temperature increases above 4 °C.)

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Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coëfficient of thermal expansion. The changes in length and volume are given by the equation:

$$\text{Length: } \Delta L = \alpha L_i \Delta T$$

$$\text{Volume: } \Delta V = \beta V_i \Delta T$$

where:

ΔL = change in length (m)
L_i = initial length (m)
α = linear coëfficient of thermal expansion (°C^{-1} or K^{-1})

ΔV = change in volume (m^3)
V_i = initial volume (m^3)
β = volumetric coëfficient of thermal expansion (°C^{-1} or K^{-1})

ΔT = temperature change (°C or K)

Values of α and β at 20°C for some solids and liquids:

Substance	α (°C^{-1})	β (°C^{-1})	Substance	α (°C^{-1})	β (°C^{-1})
aluminum	2.3×10^{-5}	6.9×10^{-5}	gold	1.4×10^{-5}	4.2×10^{-5}
copper	1.7×10^{-5}	5.1×10^{-5}	iron	1.18×10^{-5}	3.33×10^{-5}
brass	1.9×10^{-5}	5.6×10^{-5}	lead	2.9×10^{-5}	8.7×10^{-5}
diamond	1×10^{-6}	3×10^{-6}	mercury	6.1×10^{-5}	1.82×10^{-4}
ethanol		7.5×10^{-4}	silver	1.8×10^{-5}	5.4×10^{-5}
glass	8.5×10^{-6}	2.55×10^{-6}	water (liq.)	6.9×10^{-5}	2.07×10^{-4}

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<ins>expansion joint</ins>: a space deliberately placed between two objects to allow room for the objects to expand without coming into contact with each other.

Bridges often have expansion joints in order to leave room for sections of the bridge to expand or contract without damaging the bridge or the roadway.

Railroad rails are sometimes welded together in order to create a smoother ride, which enables high-speed trains to use them. Unfortunately, if expansion joints are not placed at frequent enough intervals, thermal expansion can cause the rails to bend and buckle, resulting in derailments:

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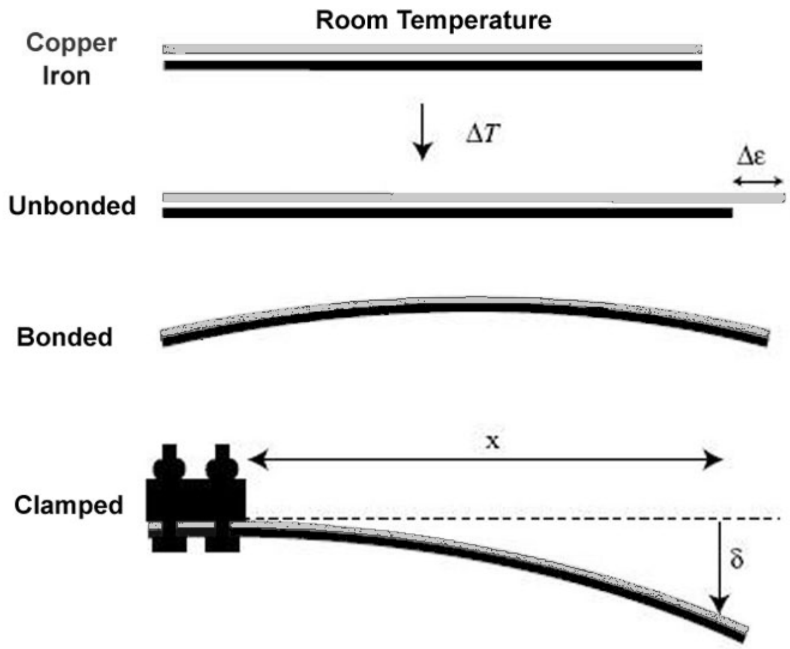
bimetal strip: a strip made from two metals with different coëfficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.



Sample Problems:

Q: Find the change in length of an 0.40 m brass rod that is heated from 25 °C to 980 °C.

A: For brass, $\alpha = 1.9\times10^{-5}\ {}^{\circ}\mathrm{C}^{-1}$.

$$\Delta L = \alpha L_i \Delta T$$
$$\Delta L = (1.9\times10^{-5})(0.40)(955)$$
$$\Delta L = 0.0073\text{ m}$$

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Q: A typical mercury thermometer contains about 0.22 cm^3 (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25 °C to 50. °C.

A: For mercury, $\beta = 1.82\times10^{-4}\ {}^{\circ}\mathrm{C}^{-1}$.

$$\Delta V = \beta V_i \Delta T$$
$$\Delta V = (1.82\times10^{-4})(0.22)(25)$$
$$\Delta V = 0.000\,91\ \mathrm{cm}^3$$

If the distance from the 25 °C to the 50 °C mark is about 3.0 cm, we could use this information to figure out the bore (diameter of the column of mercury) of the thermometer:

$$V = \pi r^2 h$$
$$0.000\,91 = (3.14)r^2(3.0)$$
$$r^2 = \frac{0.000\,91}{(3.14)(3.0)} = 9.66\times10^{-5}$$
$$r = \sqrt{9.66\times10^{-5}} = 0.0098\ \mathrm{cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is (2)(0.0098) = 0.0197 cm, which is about 0.20 mm.

Homework Problems

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You will need to look up coëfficients of thermal expansion in *Table K. Thermal Properties of Selected Materials* on page 473 of your Physics Reference Tables.

1. **(S)** A brass rod is 27.50 cm long at 25 °C. How long would the rod be if it were heated to 750. °C in a flame?

   Answer: 27.88 cm

2. **(M)** A steel bridge is 625 m long when the temperature is 0 °C.

   a. If the bridge did not have any expansion joints, how much longer would the bridge be on a hot summer day when the temperature is 35 °C?
   (Use the linear coëfficient of expansion for iron.)

   Answer: 0.258 m

   b. Why do bridges need expansion joints?

3. **(M)** A 15.00 cm long bimetal strip is aluminum on one side and copper on the other. If the two metals are the same length at 20.0 °C, how long will each be at 800. °C?

   Answers: aluminum: 15.269 cm; copper: 15.199 cm

4. **(S)** A glass volumetric flask is filled with water exactly to the 250.00 mL line at 50. °C. What volume will the water occupy after it cools down to 20. °C?

   Answer: 248.45 mL

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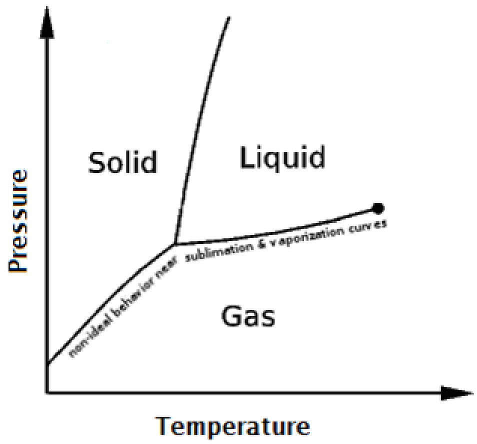
Thermal Expansion of Gases



ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.,* gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)

For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles' Law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1 and T_1 are the initial volume and temperature, and V_2 and T_2 are the final volume and temperature, respectively. Volume can be any volume unit (as long as it is the same on both sides), but temperature *must be* in Kelvin.

Sample Problem:

Q: If a 250 mL container of air is heated from 25 °C to 95 °C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

T_1 = 25 °C + 273 = 298 K T_2 = 95 °C + 273 = 368 K

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{250}{298} = \frac{V_2}{368}$$

$$V_f = 308.7 \approx 310 \text{ mL}$$

Because we used mL for V_1, the value of V_2 is therefore also in mL.

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Homework Problems

1. **(S)** A sample of argon gas was cooled, and its volume went from 380. mL to 250. mL. If its final temperature was −45.0 °C, what was its original temperature?

   Answer: 347 K or 74 °C

2. **(M)** A balloon contains 250. mL of air at 50 °C. If the air in the balloon is cooled to 20.0 °C, what will be the new volume of the air?

   Answer: 226.8mL