Gas Laws

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-8(MA)

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.2.A, 9.2.A.1, 9.2.A.2, 9.2.A.3, 9.2.A.4

Mastery Objective(s): (Students will be able to…)

- Qualitatively describe the relationship between any two of the quantities: *number of particles*, *temperature*, *pressure*, and *volume* in terms of Kinetic Molecular Theory (KMT).
- Quantitatively determine the *number of particles*, *temperature*, *pressure*, or *volume* in a before & after problem in which one or more of these quantities is changing.

Success Criteria:

- Descriptions relate behavior at the molecular level to behavior at the macroscopic level.
- Solutions have the correct quantities substituted for the correct variables.
- Chosen value of the gas constant has the same units as the other quantities in the problem.
- Algebra and rounding to appropriate number of significant figures is correct.

Language Objectives:

- Identify each quantity based on its units and assign the correct variable to it.
- Understand and correctly use the terms “pressure,” “volume,” and “temperature,” and “ideal gas.”
- Explain the placement of each quantity in the ideal gas law.

Tier 2 Vocabulary: ideal, law

Labs, Activities & Demonstrations:

- Vacuum pump (pressure & volume) with:
  - balloon (air vs. water)
  - shaving cream
- Absolute zero apparatus (pressure & temperature)
- Balloon with tape (temperature & volume)
- Can crush (pressure, volume & temperature)

Notes:

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost. (See the section on *Kinetic-Molecular Theory*, starting on page 92.)

Avogadro's Principle

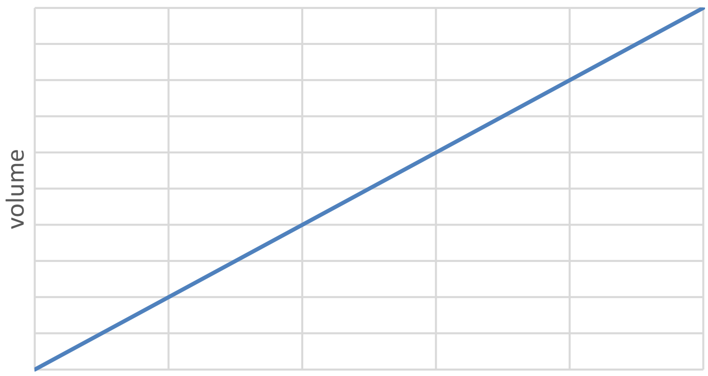
In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of particles.

Demonstration	Outcome	What the molecules are doing	Conclusion
put more (moles of) air into a balloon $n\uparrow$	the volume of the balloon got larger $V\uparrow$	crowding each other → pushing each other farther away	*n* and *V* are directly proportional. $\frac{V}{n} = \text{constant}$

If the pressure and temperature are constant, then for an ideal gas:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \text{and} \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}\,^*$$

(particles) (moles)



Although Avogadro's principle was originally stated in terms of particles (*N*) of gas, it is almost always more convenient to work with moles (*n*).

* Avogadro's principle is usually stated $\frac{n_1}{V_1} = \frac{n_2}{V_2}$. I have inverted it in these notes so that the quantities in the numerator and denominator are the same as the quantities in the numerator and denominator of the combined gas law.

(Note that by convention, gas laws use subscripts "1" and "2" instead of "o" for initial no subscript for final.)

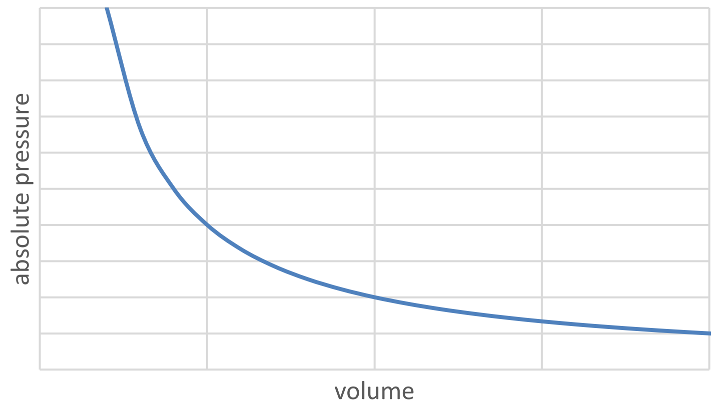
Boyle's Law

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional.

Demonstration	Outcome	What the molecules are doing	Conclusion
decrease pressure by putting a balloon in a vacuum chamber $P\downarrow$	the volume of the air inside the balloon increased $V\uparrow$	expanding the space = more surface area ⟶ less force per unit area (less pressure)	*P* and *V* are inversely proportional. PV = constant

Therefore, if the temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1V_1 = P_2V_2$$



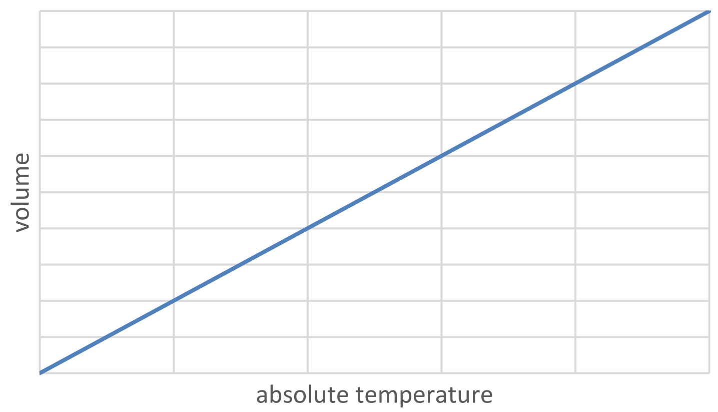
Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional.

Demonstration	Outcome	What the molecules are doing	Conclusion
place masking tape around balloon and heat with hot air gun $T\uparrow$	the volume of the air got larger and expanded the balloon except where the tape pinched it $V\uparrow$	moving more slowly → pushing each other less far away	*V* and *T* are directly proportional. $\frac{V}{T} = \text{constant}$

If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



Note that if a plot of temperature *vs.* volume is extrapolated to a volume of zero, the *x*-intercept will be absolute zero.

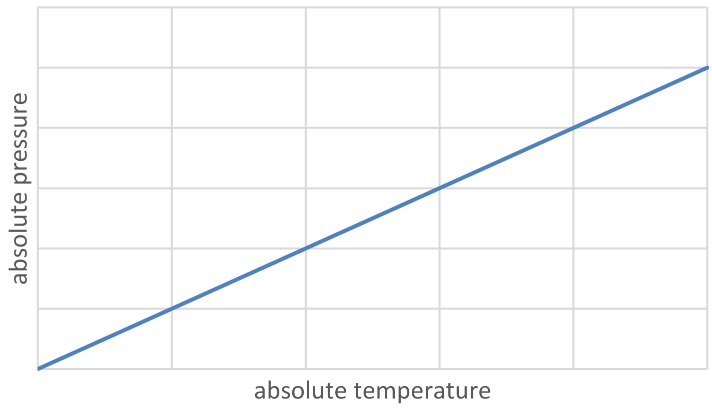
Gay-Lussac's Law

In 1702, French physicist Guillaume Amontons discovered that there is a relationship between the pressure and temperature of a gas. However, precise thermometers were not invented until after Amontons' discovery so it wasn't until 1808, more than a century later, that French chemist Joseph Louis Gay-Lussac confirmed this law mathematically. The pressure law is most often attributed to Gay-Lussac, though some texts refer to it as Amontons' Law.

Demonstration	Outcome	What the molecules are doing	Conclusion
increase temperature by heating a metal sphere full of air $T\uparrow$	the pressure of the air increased $P\uparrow$	moving faster → colliding with more force	*P* and *T* are directly proportional. $\frac{P}{T}=\text{constant}$

If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}$$



Note that at absolute zero, gas molecules do not exert any pressure on the walls of the container.

The Combined Gas Law

We can combine each of the above principles. When we do this (keeping *P* and *V* in the numerator and *n* (or *N*) and *T* in the denominator for consistency), we get following relationship for an ideal gas:

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} = \text{constant} \qquad \frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2} = \text{constant}$$

using moles using particles

Note, however, that in most situations where we want to calculate properties of a gas, the number of moles or particles remains constant. This means $n_1 = n_2$ or $N_1 = N_2$, and we can cancel it from the equation. This gives:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

The above equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

When using the combined gas law, any quantity that is not changing may be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.)

For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel *T* from both sides of the combined gas law, you get:

$$\frac{P_1V_1}{\cancel{T_1}} = \frac{P_2V_2}{\cancel{T_2}}$$ which simplifies to $P_1V_1 = P_2V_2$ (Boyle's Law)

Solving Problems Using the Combined Gas Law

You can use this method to solve any "before/after" gas law problem:

1. Determine which variables you have
2. Determine which values are ***initial*** (#1) *vs.* ***final*** (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don't forget that temperatures ***must*** be in Kelvin!)

Note: because quantities appear on both sides of the equation, it is not necessary to use S.I. units when solving problems using the combined gas law. It is, however, important to ***use the same units for the same quantity on both sides of the equation***.

Sample problem:

Q: A gas has a temperature of 25 °C and a pressure of 1.5 bar. If the gas is heated to 35 °C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25 °C and 35 °C), and two pressures (1.5 bar and the new pressure that we're looking for).

2. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):	Time 2 ("after"):
P_1 = 1.5 bar	$P_2 = P_2$
T_1 = 25 °C + 273 = 298 K	T_2 = 35 °C + 273 = 308 K

3. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Gay-Lussac's Law)}$$

4. Plug in our values and solve:

$$\frac{1.5\,\text{bar}}{298\,\text{K}} = \frac{P_2}{308\,\text{K}} \quad \rightarrow \quad \boxed{P_2 = 1.55\text{ bar}}$$

Homework Problems

Solve these problems using one of the gas laws in this section. Remember to convert temperatures to Kelvin!

1. **(M)** A sample of oxygen gas occupies a volume of 250. mL at a pressure of 740. torr. What volume will it occupy at 800. torr?

   Answer: 231.25 mL

2. **(M)** A sample of O_2 is at a temperature of 40.0 °C and occupies a volume of 2.30 L. To what temperature should it be raised to occupy a volume of 6.50 L?

   Answer: 612 °C

3. **(S)** H_2 gas was cooled from 150. °C to 50. °C. Its new pressure is 750 torr. What was its original pressure?

   Answer: 980 torr

4. **(S)** A 2.00 L container of N_2 had a pressure of 3.20 atm. What volume would be necessary to decrease the pressure to 98.0 kPa?

   (*Hint: notice that the pressures are in different units. You will need to convert one of them so that both pressures are in either* atm *or* kPa.)

   Answer: 6.62 L

5. **(S)** A sample of air has a volume of 60.0 mL at S.T.P. What volume will the sample have at 55.0 °C and 745 torr?

   Answer: 73.5 mL

6. **(M)** N_2 gas is enclosed in a tightly stoppered 500. mL flask at 20.0 °C and 1 atm. The flask, which is rated for a maximum pressure of 3.00 atm, is heated to 680. °C. Will the flask explode?

   Answer: P_2 = 3.25 atm. Yes, the flask will explode.

7. A scuba diver's 10. L air tank is filled to a pressure of 210 bar at a dockside temperature of 32.0 °C. When the diver is breathing the air underwater, the water temperature is 8.0 °C, and the pressure is 2.1 bar.

   a. **(M)** What volume of air does the diver use?

      Answer: 921 L

   b. **(S)** If the diver uses air at the rate of 8.0 L/min, how long will the diver's air last?

      Answer: 115 min