# Fluid Motion & Bernoulli's Law

**Unit:** Fluids & Pressure

**MA Curriculum Frameworks (2016):** HS-PS2-10(MA), HS-PS2-1

**AP® Physics 2 Learning Objectives:** 5.B.10.1, 5.B.10.2, 5.B.10.3, 5.F.1.1

**Mastery Objective(s):** (Students will be able to…)

- Solve problems involving fluid flow using Bernoulli's Equation.

**Success Criteria:**

- Problems are set up & solved correctly with the correct units.

**Language Objectives:**

- Explain why a fluid has less pressure when the flow rate is faster.

**Tier 2 Vocabulary:** fluid, velocity

## Labs, Activities & Demonstrations:

- Blow across paper (unfolded & folded)
- Blow between two empty cans.
- Ping-pong ball and air blower (without & with funnel)
- Venturi tube
- Leaf blower & large ball

## Notes:

flow: the net movement of a fluid

velocity of a fluid: the average velocity of a particle of fluid as the fluid flows past a reference point. (unit = $\frac{m}{s}$)

volumetric flow rate: the volume of a fluid that passes through a section of pipe in a given amount of time. (unit = $\frac{m^3}{s}$)

mass flow rate: the mass of fluid that passes through a section of pipe in a given amount of time. (unit = $\frac{kg}{s}$)

Use this space for summary and/or additional notes:

Big Ideas | Details

## Continuity

If a pipe has only one inlet and one outlet, all of the fluid that flows in must flow out, which means the volumetric flow rate through the pipe $\frac{V}{t}$ must be constant.

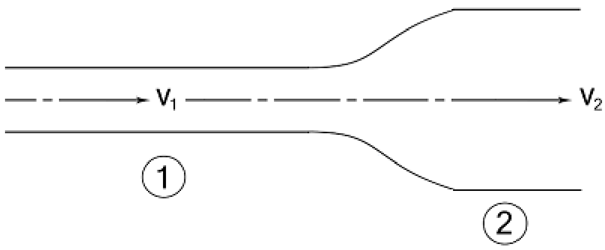
Because volume is area times length, we can write the volumetric flow rate as:

$$\frac{V}{t} = \frac{Ad}{t}$$

Assuming the velocity is constant through a section of the pipe as long as the size and elevation are not changing, we can substitute $v = \frac{d}{t}$, giving:

$$\frac{V}{t} = \frac{Ad}{t} = A \cdot \frac{d}{t} = Av = \text{constant}$$

If the volumetric flow rate remains constant but the diameter of the pipe changes:




In order to squeeze the same volume of fluid through a narrower opening, the fluid needs to flow faster. Because *Av* must be constant, the cross-sectional area times the velocity in one section of the pipe must be the same as the cross-sectional velocity in the other section.

$$Av = \text{constant}$$
$$A_1 v_1 = A_2 v_2$$

This equation is called the *continuity equation*, and it is one of the important tools that you will use to solve these problems.

Note that ***the continuity equation applies only in situations in which the flow rate is constant***, such as inside of a pipe.

Use this space for summary and/or additional notes:

## Dynamic Pressure

When a fluid is flowing, the fluid must have kinetic energy, which equals the work that it takes to move that fluid.

Recall the equations for work and kinetic energy:

$$K = \tfrac{1}{2}mv^2$$
$$W = \Delta K = F_{||}d$$

Combining these (the work-energy theorem) gives $\tfrac{1}{2}mv^2 = F_{||}d$ .

Solving $P_D = \dfrac{F}{A}$ for force gives $F = P_D A$. Substituting this into the above equation gives:

$$\tfrac{1}{2}mv^2 = F_{||}d = P_D A d$$

Rearranging the above equation to solve for dynamic pressure gives the following. Because volume is area times distance ($V = Ad$), we can then substitute $V$ for $Ad$:

$$P_D = \frac{\tfrac{1}{2}mv^2}{Ad} = \frac{\tfrac{1}{2}mv^2}{V}$$

Finally, rearranging $\rho = \dfrac{m}{V}$ to solve for mass gives $m = \rho V$. This means our equation becomes:

$$P_D = \frac{\tfrac{1}{2}mv^2}{V} = \frac{\tfrac{1}{2}\rho \cancel{V} v^2}{\cancel{V}} = \tfrac{1}{2}\rho v^2$$
$$P_D = \tfrac{1}{2}\rho v^2$$

Use this space for summary and/or additional notes:

## Bernoulli's Principle

Bernoulli's Principle, named for Dutch-Swiss mathematician Daniel Bernoulli states that the pressures in a moving fluid are caused by a combination of:

- The hydrostatic pressure: $P_H = \rho g h$
- The dynamic pressure: $P_D = \frac{1}{2}\rho v^2$
- The "external" pressure, which is the pressure that the fluid exerts on its surroundings. (This is the pressure we would measure with a pressure gauge.)

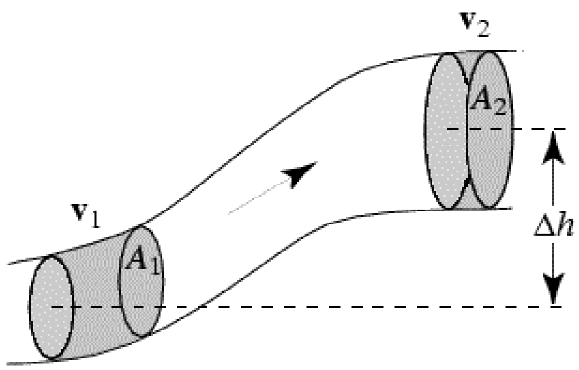
A change in any of these pressures affects the others, which means:

$$P_{ext.} + P_H + P_D = \text{constant}$$

$$P_{ext.} + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

The above equation is Bernoulli's equation.

For example, consider the following situation:




- The velocity of the fluid is changing (because the cross-sectional area is changing—remember the continuity equation $A_1 v_1 = A_2 v_2$ ). This means the dynamic pressure, $P_D = \frac{1}{2}\rho v^2$ is changing.
- The height is changing, which means the hydrostatic pressure, $P_H = \rho g h$ is changing.
- The external pressures will also be different, in order to satisfy Bernoulli's Law.

This means Bernoulli's equation becomes:

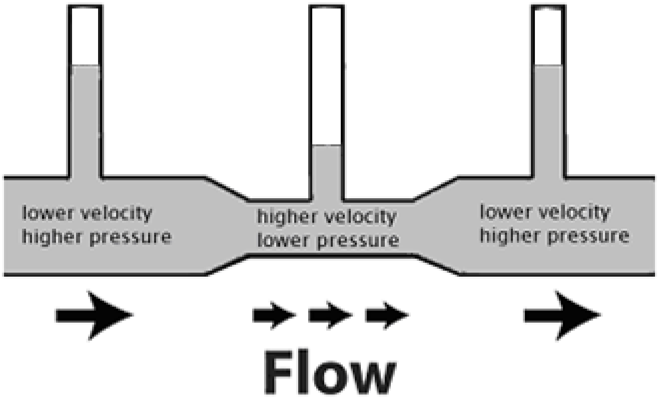
$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

Use this space for summary and/or additional notes:

Note particularly that Bernoulli's equation tells us that increasing the fluid velocity ($v$) increases the dynamic pressure. (If $v$ increases, then $P_D = \frac{1}{2}\rho v^2$ increases.)

This means if more of the total pressure is in the form of dynamic pressure, that means the hydrostatic and/or external pressures will be less.

Consider the following example:




This pipe is horizontal, which means $h$ is constant; therefore $\rho gh$ is constant. This means that if $\frac{1}{2}\rho v^2$ increases, then pressure ($P$) must decrease so that

$P_{ext.} + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$ .

Although Bernoulli published his principle in 1738, the application to fluids in constricted channels was not published until 1797 by Italian physicist Giovanni Venturi. The above apparatus is named after Venturi and is called a Venturi tube.

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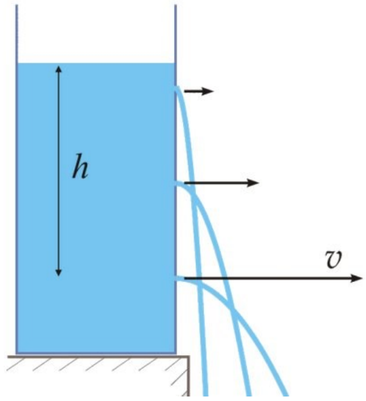
## Torricelli's Theorem

A special case of Bernoulli's Principle was discovered almost 100 years earlier, in 1643 by Italian physicist and mathematician Evangelista Torricelli. Torricelli observed that in a container with fluid effusing (flowing out) through a hole, the more fluid there is above the opening, the faster the fluid comes out.

Torricelli found that the velocity of the fluid was the same as the velocity would have been if the fluid were falling straight down, which can be calculated from the change of gravitational potential energy to kinetic energy:




$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh \rightarrow v^2 = 2gh \rightarrow v = \sqrt{2gh}$$

Torricelli's theorem can also be derived from Bernoulli's equation*:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

- The external pressures ($P_1$ and $P_2$) are both equal—atmospheric pressure—so they cancel.
- The fluid level is going down slowly enough that the velocity of the fluid inside the container ($v_1$) is essentially zero.
- Once the fluid exits the container, the hydrostatic pressure is zero $(\rho g h_2 = 0)$.

This leaves us with:

$$\cancel{\rho} g h_1 = \frac{1}{2}\cancel{\rho} v_2^2 \rightarrow 2gh_1 = v_2^2 \rightarrow \sqrt{2gh_1} = v_2$$

We could do a similar proof from the kinematic equation: $v^2 - v_o^2 = 2ad$

Substituting $a = g, d = h$, and $v_o = 0$ gives $v^2 = 2gh$ and therefore $v = \sqrt{2gh}$

Note: as described in Hydrostatic Pressure, starting on page 158, hydrostatic pressure is caused by the fluid ***above*** the point of interest, meaning that height is measured *upward*, not downward. In the above situation, the two points of interest for the application of Bernoulli's law are actually:

- inside the container next to the opening, where there is fluid above, but essentially no movement of fluid (v = 0, but h ≠ 0)
- outside the opening where there is no fluid above, but the jet of fluid is flowing out of the container (h = 0, but v ≠ 0)

* On the AP® Physics exam, you must start problems from equations that are on the formula sheet. This means you may not use Torricelli's Theorem on the exam unless you first derive it from Bernoulli's Equation.

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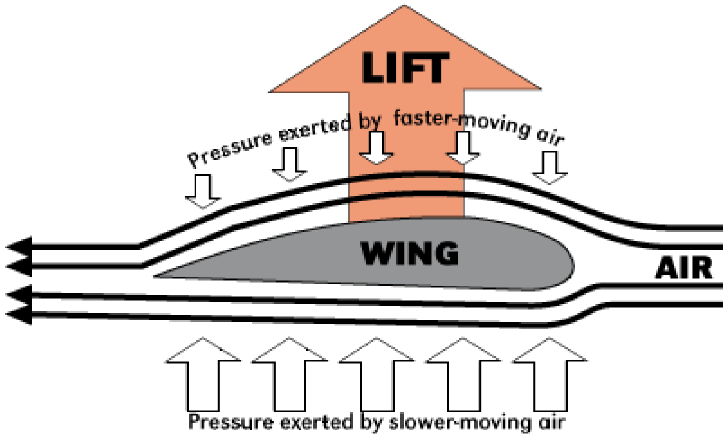
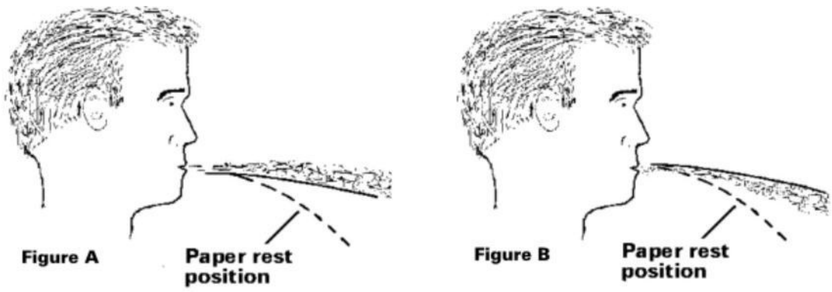
The decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:




(Of course, most of an airplane's lift comes from the fact that the wing is inclined with an angle of attack relative to its direction of motion, an application of Newton's third law.)

A common demonstration of Bernoulli's Law is to blow across a piece of paper:




The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

Use this space for summary and/or additional notes:

## Sample Problems:

Q: A fluid in a pipe with a *diameter* of 0.40 m is moving with a velocity of $0.30\,\frac{\text{m}}{\text{s}}$. If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: The cross-sectional area of the first pipe is:

$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126\text{ m}^2$

The cross-sectional area of the second pipe is:

$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314\text{ m}^2$

Using the continuity equation:

$$A_1 v_1 = A_2 v_2$$

$$A_1 v_1 = A_2 v_2 \; (0.126)(0.30) = (0.0314) v_2$$

$$v_2 = 1.2\,\tfrac{\text{m}}{\text{s}}$$

Q: A fluid with a density of $1250\,\frac{\text{kg}}{\text{m}^3}$ has a pressure of 45 000 Pa as it flows at $1.5\,\frac{\text{m}}{\text{s}}$ through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of $3.4\,\frac{\text{m}}{\text{s}}$ through it?

A:

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$

$$45\,000 + (1250)(10)(0) + \left(\tfrac{1}{2}\right)(1250)(1.5)^2 = P_2 + (1250)(10)(2.5) + \left(\tfrac{1}{2}\right)(1250)(3.4)^2$$

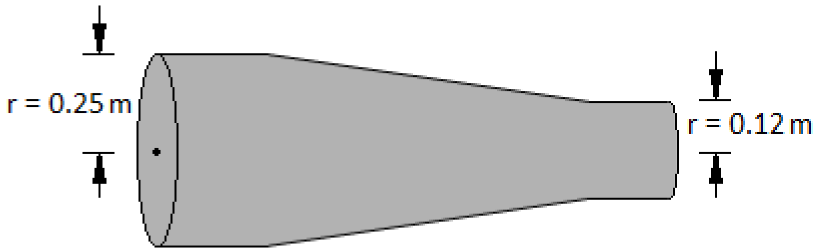
$$45\,000 + 1406 = P_2 + 31\,250 + 7225$$

$$P_2 = 7931\text{ Pa}$$

Use this space for summary and/or additional notes:

## Homework Problems

1. **(S)** A pipe has a radius of 0.25 m at the entrance and a radius of 0.12 m at the exit, as shown in the figure below:




If the fluid in the pipe is flowing at $5.2\,\frac{m}{s}$ at the inlet, then how fast is it flowing at the outlet?

Answer: $22.6\,\frac{m}{s}$

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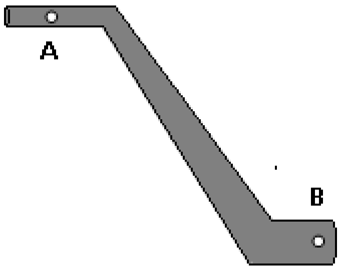
2. **(M)** At point A on the pipe to the right, the water's speed is $4.8\,\frac{m}{s}$ and the external pressure (the pressure on the walls of the pipe) is 52.0 kPa. The water drops 14.8 m to point B, where the pipe's cross-sectional area is twice that at point A.




a. Calculate the velocity of the water at point B.

Answer: $2.4\,\frac{m}{s}$

b. Calculate the external pressure (the pressure on the walls of the pipe) at point B.

Answer: 208 600 Pa or 208.6 kPa

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