Pressure-Volume (PV) Diagrams

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 9.4.B, 9.4.B.2, 9.4.B.2.i, 9.4.B.ii, 9.4.B.3

Mastery Objective(s): (Students will be able to…)

- Determine changes in heat, work, internal energy and entropy from a pressure-volume (PV) diagram.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain what is physically happening to a gas for each section of a PV diagram.

Tier 2 Vocabulary: internal, energy, heat, work

Notes:

P-V diagram: a graph that shows changes in pressure *vs.* changes in volume.

Recall that:

$$W = -\int P\,dV$$

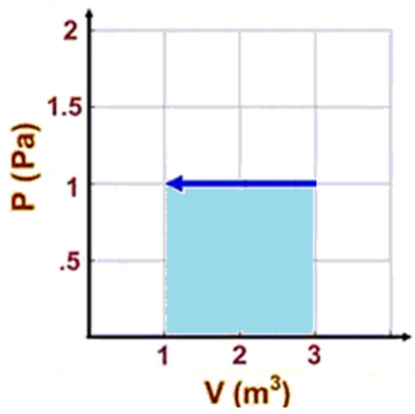
On a graph, the integral is the area "under the curve" (meaning the area between the curve and the *x*-axis).

Therefore, if we plot a graph of pressure *vs.* volume with pressure on the *y*-axis and volume on the *x*-axis, the integral would therefore be represented by the area between the curve (pressure) and the *x*-axis.

This means that *the work done by a thermodynamic change equals the area under a P-V graph*.[*]

[*] While the above explanation requires calculus, as stated earlier we will limit ourselves to areas that can be calculated using simple geometry equations. Note that some of these will result in situations that would not be realistically achievable in the "real world".

In the following example, suppose that a gas is compressed from 3 m^3 to 1 m^3 at a pressure of 1 Pa. (A pressure of 1 Pa is much smaller than you would encounter in any real problem; these numbers were chosen to keep the math simple.)



The pressure is P = 1 Pa, and the change in volume is $\Delta V = -2\ m^3$. Because pressure is constant, we can use $W = -P\Delta V = -(1)(-2) = +2\text{ J}$.

$P\Delta V$ is the area under the graph. Because it is a rectangular region, the area is the base of the rectangle times the height. The base is 2 m^3 and the height is 1 Pa, which gives an area of 2 J.

Note that the arrow showing the change points to the *left*, which indicates that the volume is *decreasing*. Because work must be put *into* the gas in order to compress it, this means that the work done ***on*** the gas will be positive.[*] This is where the negative sign comes from. $W = -P\Delta V$ means that:

- if work is done on the gas (work is positive), the gas is compressed and the change in volume is therefore negative.
- If work is done by the gas on the surroundings (work is negative), the gas expands and the change in volume is therefore positive.

We will look at the effects of changes in pressure *vs.* volume in four types of pressure-volume changes:

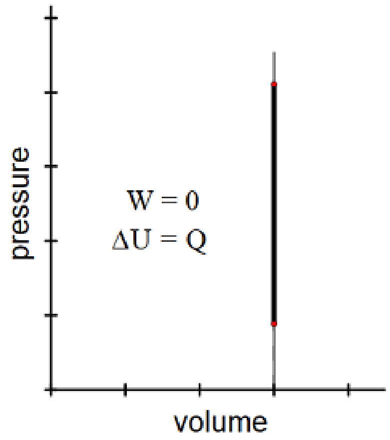
- isochoric (constant volume)
- isothermal (constant temperature)
- adiabatic (no heat loss)
- isobaric (constant pressure).

[*] Unless explicitly stated otherwise, positive work means work done ***on*** the gas, meaning that energy is added to the gas and the internal energy of the gas increases.

Isochoric

From Greek "iso" (same) and "khoros" (volume). An isochoric change is one in which volume remains constant, but pressure and temperature may vary.

An example is any rigid, closed container, such as a thermometer.



$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2} \rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Because the volume is not changing, there is no way for the gas to displace anything. (Recall from Physics 1 that $W = \vec{F} \bullet \vec{d}$.) If there is no displacement, there is no work, which means $W = 0$.

$$\Delta U = Q + \cancelto{0}{W} = \frac{3}{2} nR\Delta T$$

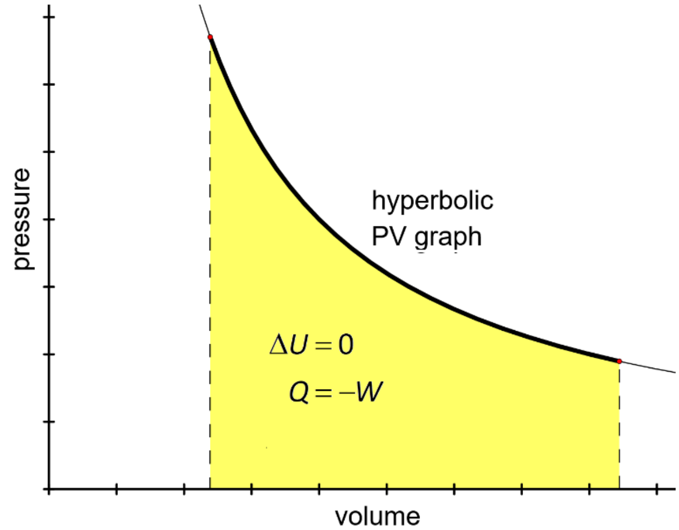
$$\Delta U = Q = \frac{3}{2} nR\Delta T$$

Another way to think of a constant volume change is that if you add heat to a rigid container of gas, none of the energy can be converted to work, so all of it must be converted to an increase in internal energy (*i.e.,* an increase in temperature).

Isothermal

Constant temperature. From Greek "iso" (same) and "thermotita" (heat). An isothermal change is one in which temperature remains constant, but pressure and volume may vary.

An example is any "slow" process, such as breathing out through a wide open mouth.



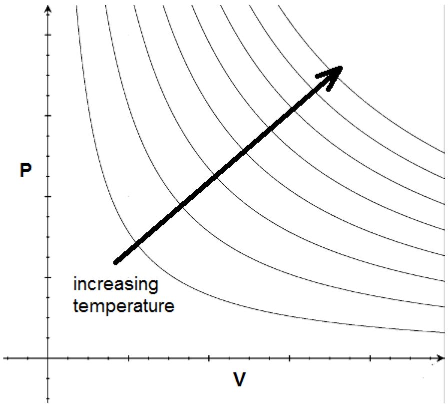
$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \rightarrow P_1 V_1 = P_2 V_2$$

Because $\Delta T = 0$ (definition of isothermal), this means

$$\Delta U = \frac{3}{2} nR\Delta T = 0$$

Further, because:

$$\Delta U = Q + W = 0$$
$$Q = -W$$

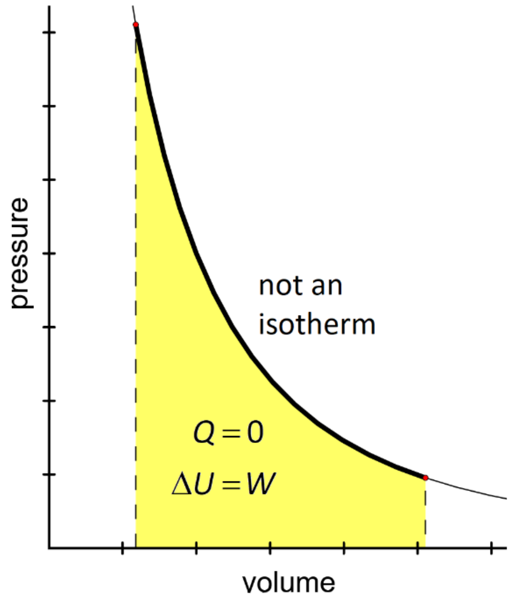


The P-V curve for an isothermal process is called an <u>isotherm</u>. Note that isotherms are hyperbolas, *i.e.,* they are solutions to the equation PV = constant. You may recall that allowing pressure and volume to change while keeping temperature constant is represented by Boyle's Law: $P_1 V_1 = P_2 V_2$

As temperature increases, the isotherm moves farther away from the origin.

Adiabatic

An adiabatic process is one in which there is no heat exchange with the environment. From Greek "a" (not) + "dia" (through) + "batos" (passable).



An example is any "fast" process, such as forcing air out through pursed lips or a bicycle tire pump.

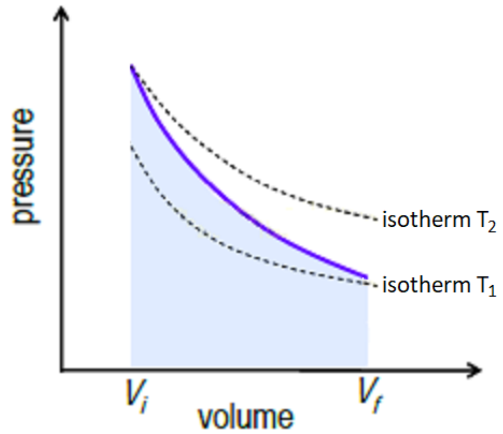
Because the definition of an adiabatic process is one for which $Q = 0$, this means:

$$Q = 0$$

$$\Delta U = \cancelto{0}{Q} + W = \tfrac{3}{2} nR\Delta T$$

$$\Delta U = W = \tfrac{3}{2} nR\Delta T$$

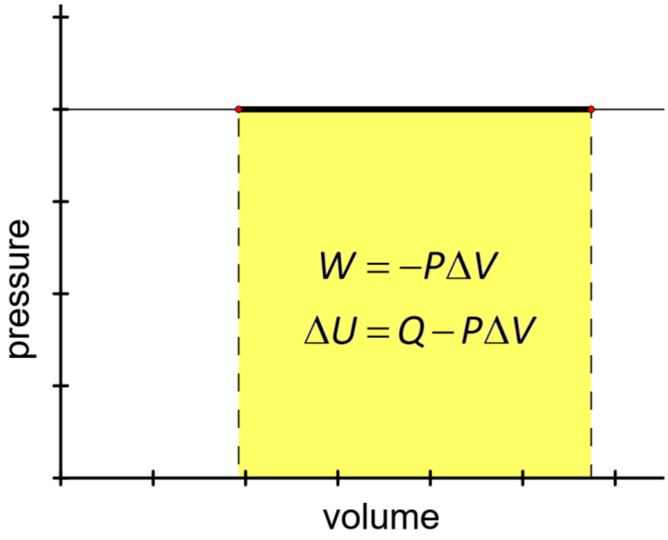
Note that adiabatic expansion (sudden increase in volume without time for heat transfer) results in a decrease in temperature, and adiabatic compression (sudden decrease in volume without time for heat transfer) results in an increase in temperature.



Isobaric

From Greek "iso" (same) and "baros" (weight). An isobaric change is one in which pressure remains constant, but volume and temperature may vary.

Some examples include a weighted piston, a flexible container in earth's atmosphere, or a hot air balloon.



$$\frac{\cancel{P_1} V_1}{T_1} = \frac{\cancel{P_2} V_2}{T_2} \quad \rightarrow \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Isobaric changes involve changes in both Q and W, because to change the volume of a gas while keeping pressure constant, you need to add or remove heat, but the resulting change in volume means that work is being done.

$$W = -\Delta(PV) = -P\Delta V$$
$$\Delta U = Q + W$$
$$\Delta U = Q + (-P\Delta V)$$
$$\Delta U = Q - P\Delta V$$

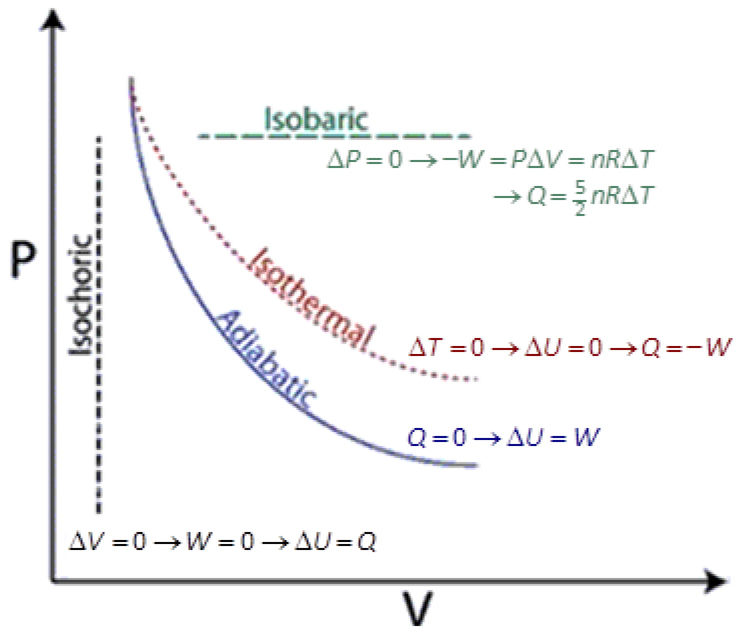
Adding $P\Delta V$ to both sides gives $Q = \Delta U + P\Delta V$.

Now, because $\Delta U = \frac{3}{2}nR\Delta T$ and $P\Delta V = nR\Delta T = \frac{2}{2}nR\Delta T$, that means:

$$Q = \Delta U + P\Delta V$$
$$\downarrow \qquad \downarrow$$
$$Q = \frac{3}{2}nR\Delta T + \frac{2}{2}nR\Delta T$$
$$Q = \frac{5}{2}nR\Delta T$$

This makes sense, because some of the heat is used to do the work of expanding the gas $(P\Delta V = nR\Delta T)$, and some of the heat is used to increase the temperature. $(\Delta U = \frac{3}{2}nR\Delta T)$.

If we wanted to compare all four processes on the same PV diagram, they would look like this:



Positive *vs.* Negative Work

In thermodynamics problems, whether work is represented by a positive or negative number depends on how the problem is stated.

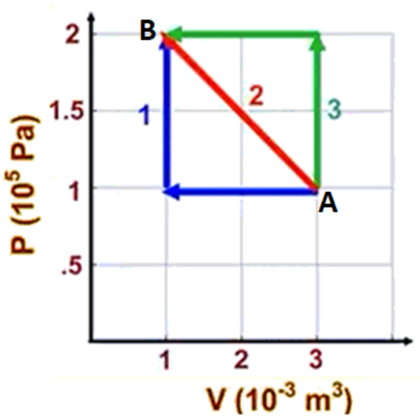
Work done ***on*** the gas: a positive number means work is coming from the surroundings into the gas.

Work done ***by*** the gas: a positive number means work is going from the gas out to the surroundings.

If the problem does not specify otherwise, the convention is to use a positive number to indicate work done on (*i.e.,* going into) the gas.

Sample Problem

Q: Calculate the work done as the pressure and volume of a gas are taken from point A to point B along each of paths 1, 2, and 3.



A: Process #1 is first isobaric (constant pressure), then isochoric (constant volume).

For the isobaric part of the process:

$$W = -P\Delta V$$
$$W = -(1\times10^{5})(1\times10^{-3} - 3\times10^{-3})$$
$$W = -(1\times10^{5})(-2\times10^{-3})$$
$$W = 2\times10^{2} = 200\,\text{J}$$

For the isochoric process, there is no change in volume, which means the gas does no work (because it cannot push against anything). Therefore $W = 0$.

The total work for process #1 is therefore 200 J.

Notice that the work is equal to the area under the PV graph, which is a rectangular area with a base of 2×10^{-3} m^3 and a height of 1×10^5 Pa.

$$W = (1\times10^{5})(2\times10^{-3}) = 200\text{ J}$$

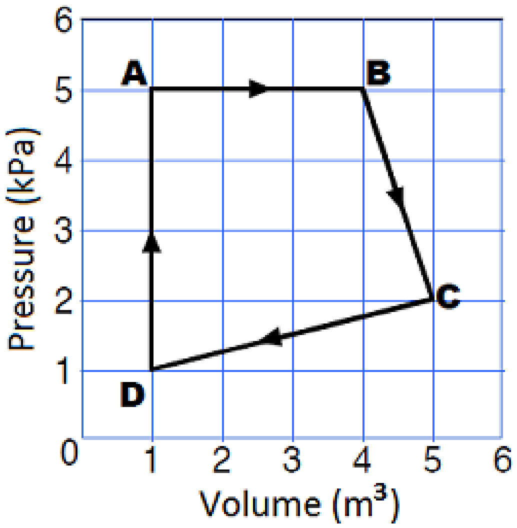
Note that because the arrow points to the left, this means the *volume is decreasing*. That means *work is being done on the gas*, which means the *work is represented by a positive number*. (We have to make this determination any time we use the graph to calculate the work.)

For process #2, the area is the 200 J square that we calculated for process #1 plus the area of the triangle above it, which is $\frac{1}{2}bh = \frac{1}{2}(2\times10^{-3})(1\times10^{5}) = 100\text{ J}$. Therefore, 200 J + 100 J = 300 J.

For process #3, the area under the curve is $W = (2\times10^{5})(2\times10^{-3}) = 400\text{ J}$.

Homework Problems*

Problems #1–8 refer to the following PV diagram, in which 2 moles of gas undergo the pressure and volume changes represented by the path from point A to B to C to D and back to A.



1. **(M)** Which thermodynamic process takes place along the path from point A to point B?

2. **(M)** Which thermodynamic process takes place along the path from point D to point A?

3. **(M)** How much work is done ***by the gas*** as it undergoes a change along the curve from point B → C? (Remember to use a positive number for work done on the gas by the surroundings, and a negative number for work done by the gas on the surroundings.)

   Answer: +3500 J

* These problems are from a worksheet by Tony Wayne.

4. **(S)** How much work is done ***on** the gas* as it undergoes a change along the curve from point C → D?

   Answer: +6000 J

5. **(S)** How much net work is done ***by** the gas* on the surroundings as it undergoes a change along the curve from point A → B → C → D → A?

   Answer: +12 500 J

6. **(S)** What is the temperature of the 2 moles of gas at point A?

   Answer: 300.8 K

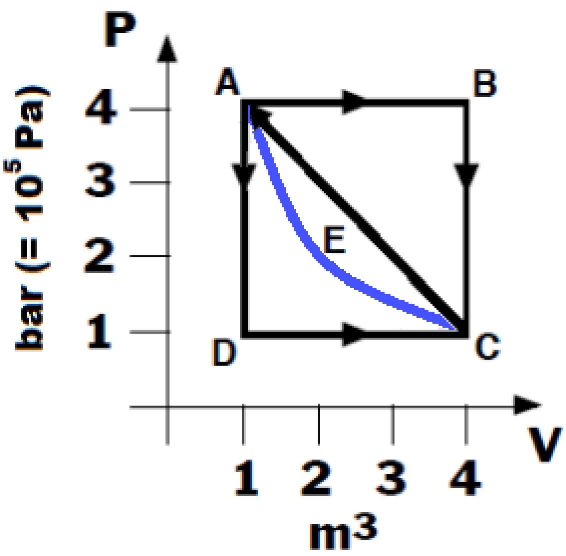
7. **(M)** What is the change in internal energy of the gas during the process from point D → A?

   Answer: 6000 J

8. **(M)** How much work is done on or by the gas during the process from point D → A?

   Answer: zero

Problems #9–13 refer to the following diagram:



9. **(S)** For which process(es) is $Q = \frac{5}{2} nR\Delta T$? Show calculations to justify your answer.

Answer: A → B and D → C

10. **(M)** For which process(es) is no work done? Explain.

Answer: A → D and B → C

11. **(M)** Which thermodynamic process takes place along path E?

12. **(M)** Calculate the heat exchanged in process A → B? Is heat added or released? Explain.

    Answer: 3×10^{6} J; heat is added because the temperature increases.

13. **(M)** Does path A → D → C → E → A require more or less work than path A → D → C → A? Explain.

14. **(S)** Calculate the work done ***by** the gas* in processes A → B → C → A and A → D → C → A.

    Answer: A → B → C → A: 450 000 J
    A → D → C → A: −450 000 J