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Efficiency

Unit: Thermodynamics

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-6

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): N/A

Mastery Objective(s): (Students will be able to…)

- Calculate the efficiency of a thermodynamic process.

Success Criteria:

- Correct equation is chosen.
- Solutions have the correct quantities substituted for the correct variables.
- Sign is correct for work (positive *vs.* negative).
- Algebra is correct.

Language Objectives:

- Explain how the efficiency of a process relates to the energy it uses and the work it produces.

Tier 2 Vocabulary: energy, heat, work

Notes:

efficiency (η): the ratio of the energy consumed by a device or process to the energy output by the device or process.

Assume that a heat engine starts with a certain temperature, which means a certain internal energy (U). The engine takes heat from a heat source at the incoming temperature T_{in}, does work (W), and exhausts heat at the higher temperature T_{out}. Assuming the internal energy of the machine itself stays constant, this means $\Delta U = 0$. Therefore, from the First law:

$$\Delta U = 0 = \Delta Q - \Delta W$$
$$0 = Q_{in} - Q_{out} - \Delta W$$
$$\Delta W = Q_{in} - Q_{out}$$

A 100% efficient heat engine would turn all of the heat into work, and would exhaust no heat ($Q_{out} = 0$, which would mean $\Delta W = Q_{in}$). Of course, real engines cannot do this, so we define efficiency, *e*, as the ratio of work out to heat in, *i.e.*:

$$e = \frac{\Delta W}{\Delta Q_{in}} = \frac{\Delta Q_{in} - \Delta Q_{out}}{\Delta Q_{in}} = \frac{\Delta Q_{in}}{\Delta Q_{in}} - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}}$$

Because the engine is doing work, $\Delta W > 0$, which means $0 \le e \le 1$. Furthermore, a consequence of the Second law is that some energy is always lost to the surroundings (entropy), which means $Q_{out} > 0$ and therefore $0 \le e < 1$.

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Sample Problem

Q: 80. J of heat is injected into a heat engine, causing it to do work. The engine then exhausts 20. J of heat into a cool reservoir. What is the efficiency of the engine?

A: $Q_{in} = 80\,\text{J}$ and $Q_{out} = 20\,\text{J}$. Therefore:

$$e = 1 - \frac{\Delta Q_{out}}{\Delta Q_{in}} = 1 - \frac{20}{80} = 1 - 0.25 = 0.75$$

Because efficiency is usually expressed as a percentage, we would say that the engine is 75% efficient.

The following table gives energy conversion efficiencies for common devices and processes. In all of these cases, the "lost" energy is converted to heat that is given off to the surroundings.

Energy Conversion Efficiency

Device/Process	Typical Efficiency
gas generator	up to 40%
coal/gas-fired power plant	45%
combined cycle power plant	60%
hydroelectric power plant	up to 90%
wind turbine	up to 59%
solar cell	6–40%; usually 15%
hydrogen fuel cell	up to 85%
internal combustion engine	25%
electric motor, small (10–200 W)	50–90%
electric motor, large (> 200 W)	70–99%
photosynthesis in plants	up to 6%
human muscle	14–27%
refrigerator	20%
refrigerator, energy-saving	40–50%
light bulb, incandescent	0.7–5%
light bulb, fluorescent	8–16%
light bulb, LED	4–15%
electric heater	100%
firearm	30%

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You may notice that an electric heater is 100% efficient, because all of its energy is converted to heat. However, this does not mean that electric heat is necessarily the best choice for your home, because the power plant that generated the electricity is probably only 45% efficient.

Heating Efficiency

Heating efficiency is calculated in a similar way. The difference is that the energy produced by the heater *is* Q_{out}, which means:

$$\eta = \frac{Q_{out}}{Q_{in}} = \frac{\text{usable heat out}}{\text{total energy in}}$$

"Usable heat out" means heat that is not lost to the environment. For example, if the boiler or furnace in your house is 70% efficient, that means 70% of the energy from the gas or oil that it burned was used to heat the steam, hot water or hot air that was used to heat your house. The other 30% of the energy heated the air in the boiler or furnace, and that heat was lost to the surroundings when the hot air went up the chimney.

Older boilers and furnaces (pre-1990s) were typically 70% efficient. Newer boilers and furnaces are around 80% efficient, and high-efficiency boilers and furnaces that use heat exchangers to collect the heat from the exhaust air before it goes up the chimney can be 90−97% efficient.

Efficiency of a Heat Pump

Carnot's theorem states that the maximum possible efficiency of a heat pump is related to the ratio of the temperature of the heat transfer fluid (liquid or gas) when it enters the heat pump to the temperature when it exits:

$$\eta \le 1 - \frac{T_{in}}{T_{out}}$$

Note that the Carnot equation is really the same as the efficiency equation in the previous section. Recall that for heating or cooling a substance:

$$Q = mC\Delta T = mC(T_{out} - T_{in})$$

The refrigerant is the same substance, which means mC is the same for the input as for the output, and it drops out of the equation.

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It is a little counter-intuitive that a higher temperature difference means the heat pump is more efficient, but you should think about this in terms of heat transfer. (See the section on Heat Transfer starting on page 45.) Recall from Fourier's Law of Conduction that a higher temperature difference means a higher rate of heat transfer from one side to the other. In other words, the more heat you pump into the refrigerant, the higher its temperature will be when it leaves the system, and therefore the more efficiently the pump is moving heat. Conversely, if $T_{out} = T_{in}$, then the heat pump cannot transfer any heat, and the efficiency is zero.

Sample Problem

Q: Refrigerant enters a heat pump at 20. °C (293 K) and exits at 300. °C (573 K). What is the Carnot efficiency of this heat pump?

A: Carnot's equation states that:

$$\eta = 1 - \frac{T_{in}}{T_{out}} = 1 - \frac{293}{573}$$

$$\eta = 1 - 0.51 = 0.49$$

I.e., this heat pump is 49% efficient.