Coulomb's Law

Unit: Electric Force, Field & Potential

NGSS Standards/MA Curriculum Frameworks (2016): HS-PS2-4

AP® Physics 2 Learning Objectives/Essential Knowledge (2024): 10.1.A, 10.1.A.2, , 10.1.A.3, 10.1.A.3.i, 10.1.A.3.ii, 10.1.A.4, 10.1.B, 10.1.B.1, 10.1.B.2, 10.1.B.3

Mastery Objective(s): (Students will be able to…)

- Solve problems using Coulomb's Law
- Quantitatively predict the effects on the electrostatic force when one of the variables (amount of electric charge or distance) in Coulomb's Law is changed.

Success Criteria:

- Variables are correctly identified and substituted correctly into the correct part of the correct equation.
- Algebra is correct and rounding to appropriate number of significant figures is reasonable.

Language Objectives:

- Explain how force and distance both affect the amount of force between two charged objects.

Tier 2 Vocabulary: charge

Labs, Activities & Demonstrations:

- Charged balloon or Styrofoam sticking to wall.
- Charged balloon pushing meter stick.
- Van de Graaff generator with negative electrode attached to inertia balance pan.

Notes:

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

+1 C is the charge of 6.2415×10^{18} protons.
−1 C is the charge of 6.2415×10^{18} electrons.

A single proton or electron therefore has a charge of $\pm 1.6022 \times 10^{-19}$ C. This amount of charge is called the elementary charge, because it is the charge of one elementary particle.

An object can only have an integer multiple of this amount of charge, because it is impossible* to have a charge that is a fraction of a proton or electron.

Because charged particles attract or repel each other, that attraction or repulsion must be a force, which can be measured and quantified. The force is directly proportional to the strengths of the charges, and inversely proportional to the square of the distance. The formula is:

$$F_e = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{r^2} = k\frac{q_1 q_2}{r^2}$$

where:

- F_e = electrostatic force of repulsion between electric charges. A positive value of F_e denotes that the charges are repelling (pushing away from) each other; a negative value of F_e denotes that the charges are attracting (pulling towards) each other.†
- ε_o = electric permittivity of free space $\approx 8.85\times 10^{-12}\ \frac{\text{F}}{\text{m}}$.
- k = electrostatic constant = $= \frac{1}{4\pi\varepsilon_o} \approx 9.0\times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$.
- q_1 , q_2 = the charges on objects #1 and #2 respectively
- r = distance (radius, because it goes outward in every direction) between the centers of the two charges

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Coulomb's Law has several parallels with Newton's Law of Universal Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

gravitational force

$$F_e = k\frac{q_1 q_2}{r^2}$$

electrostatic force

The situations are similar in that there are two objects, each exerting a force on the other separated by some distance, r. With the gravitational force, masses can only attract, which means F_g is always attractive. However, F_e can be either attractive or repulsive, because charges attract or repel, depending on whether they have opposite or like signs. If the charges have the same sign (repulsive), the value of F_e will be positive; if the charges have opposite signs (attractive), the value of F_e will be negative.

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* This is true for macroscopic objects. Certain quarks, which are the particles that protons and neutrons are made of, have charges of $\frac{1}{3}$ or $\frac{2}{3}$ of an elementary charge.

† It is unfortunate that a positive value for force denotes attraction in the gravitational force equation, but repulsion in the electrostatic force equation.

Sample problems:

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is 1.60×10^{-19} C, and the charge of a single electron is -1.60×10^{-19} C.

$$37.1\,\text{pm} = 3.71 \times 10^{-11}\,\text{m}$$

$$F_e = \frac{kq_1q_2}{r^2} = \frac{(8.99\times10^{9})(1.60\times10^{-19})(-1.60\times10^{-19})}{(3.71\times10^{-11})^2} = -1.67\times10^{-7}\,\text{N}$$

The value of the force is negative, which signifies that the force is attractive. However, rather than memorize whether a positive or negative indicates attraction or repulsion, it's easier to reason that the charges are opposite, so the objects attract. *Never memorize what you can understand!*

Q: Two charged particles, each with charge $+q$ (which means $q_1 = q_2 = q$) are separated by distance d. If the amount of charge on one of the particles is halved and the distance is doubled, what will be the effect on the force between them?

A: To solve this problem, we first set up Coulomb's Law:

$$F_e = \frac{kq_1q_2}{r^2}$$

Now, we replace one of the charges with half of itself—let's say q_1 will become $(0.5\,q_1)$. Similarly, we replace the distance r with $(2r)$. This gives:

$$F_e = \frac{k(0.5q_1)q_2}{(2r)^2}$$

Simplifying and rearranging this expression gives:

$$F_e = \frac{0.5kq_1q_2}{4r^2} = \frac{0.5}{4}\cdot\frac{kq_1q_2}{r^2} = \frac{1}{8}\cdot\frac{kq_1q_2}{r^2}$$

Therefore, the new F_e will be $\frac{1}{8}$ of the old F_e.

An easier way to solve this problem is to do a "before and after" calculation. Set the value of every quantity in the "before" equation to 1:

$$F_e = \frac{kq_1q_2}{r^2} \quad \rightarrow \quad \frac{1\cdot 1\cdot 1}{1^2} = 1$$

For the "after" equation, replace quantities that change with their multipliers:

$$F_e = \frac{kq_1q_2}{r^2} \quad \rightarrow \quad \frac{1\cdot 1\cdot 0.5}{2^2} = \frac{0.5}{4} = \frac{1}{8}$$

The "before" value for F_e was 1, and the "after" value was $\frac{1}{8}$, which means the new force will be $\frac{1}{8}$ of the original force.

This method is sometimes called "Bertha's Rule of Ones," or the "factor of change" method.

Homework Problems

1. **(M)** What is the magnitude of the electric force between two objects, each with a charge of $+2.00\times10^{-6}$ C, which are separated by a distance of 1.50 m? Is the force attractive or repulsive?

   Answer: 0.016 N, repulsive

2. **(M)** An object with a charge of $+q_1$ is separated from a second object with an unknown charge by a distance *r*. If the objects attract each other with a force *F*, what is the charge on the second object?
   (*If you are not sure how to do this problem, do #3 below and use the steps to guide your algebra.*)

   Answer: $q_2 = -\frac{Fr^2}{kq_1}$

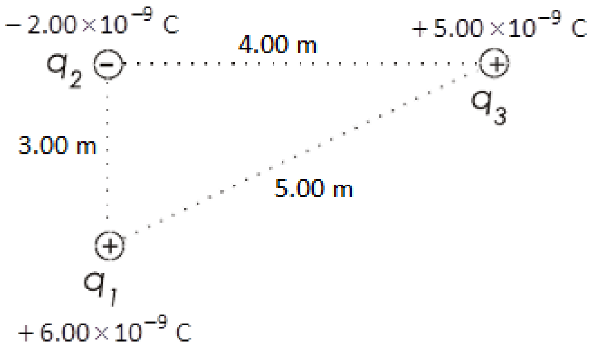
3. **(S)** An object with a charge of $+1.50\times10^{-2}$ C is separated from a second object with an unknown charge by a distance of 0.500 m. If the objects attract each other with a force of 1.35×10^{6} N, what is the charge on the second object?
   (*You must start with the equations in your* Physics Reference Tables. *You may only use the answer to question #2 above as a starting point if you have already solved that problem.*)

   Answer: -2.50×10^{-3} C

4. **(M)** The distance between an alpha particle (+2 elementary charges) and an electron (−1 elementary charge) is 2.00×10^{-25} m. If that distance is tripled, what will be the effect on the force between the charges?

Answer: The new F_e will be $\frac{1}{9}$ of the old F_e.

5. **(A)** Three elementary charges, particle q_1 with a charge of $+6.00\times10^{-9}$ C, particle q_2 with a charge of -2.00×10^{-9} C, and particle q_3 with a charge of $+5.00\times10^{-9}$ C, are arranged as shown in the diagram below.



What is the net force (magnitude and direction) on particle q_3?
(*Hint: this is a forces-at-an-angle problem like you saw in Physics 1.*)

Answer: 7.16×10^{-9} N at an angle of 65.2° above the *x*-axis.